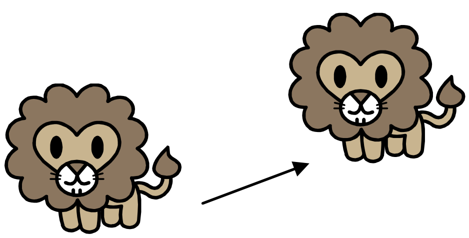
Pre-Algebra Notes – Unit 11 & 12: Transformations & Congruence

NVACS 8.G.A.2 *Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.*

NVACS 8.G.A.3 *Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates*.

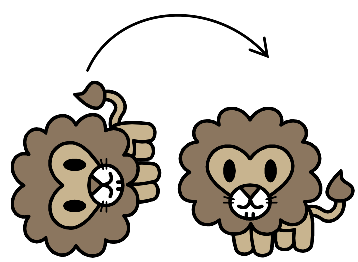
Geometry not only looks at figures, it also studies the movement of figures. If you move all the points of one figure to create a new geometric figure, you call the new figure the *image.* Each point of the image matches exactly with a corresponding point of the original figure; in other words, the image is congruent to the original figure. This changing of position is called a *transformation.*

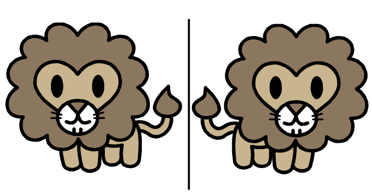
There are 4 types of transformations we are going to study: *translation, rotation, reflection, and dilation*. You may be more familiar with these as being called s*lide, turn, flip, and stretch/shrink.*



A *translation* is the simplest. If you copy a figure onto a piece of paper, than slide the paper along a straight path without turning it, your slide motion represents a translation.

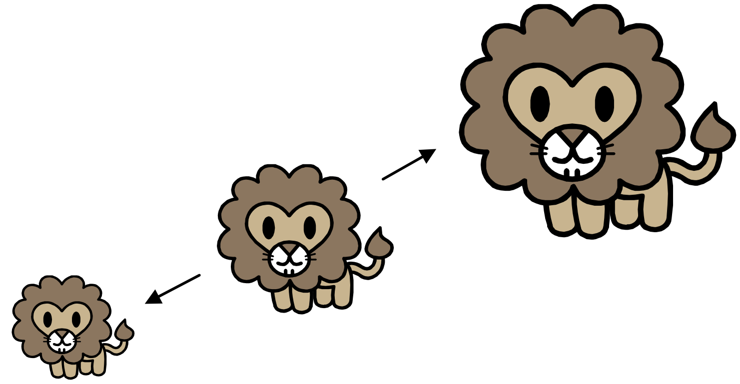
A *rotation* is a turning motion. Points of the original figure rotate an identical number of degrees around a fixed center point.





A *reflection* flips the figure across a line of reflection, creating a mirror image.

A *dilation* stretches or shrinks a figure, with respect to a fixed point.



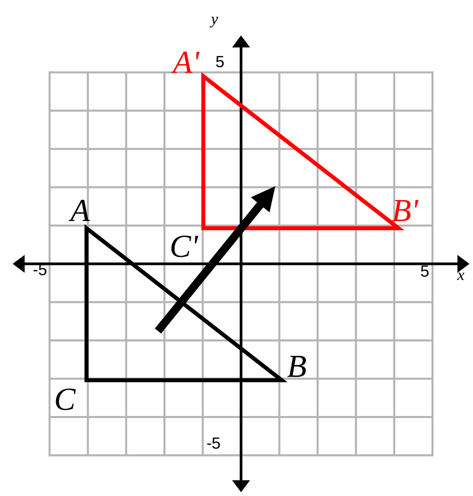
Review with students how they may relate the terms to ones they already know:

slide ⟶ translation

turn ⟶ rotation

flip ⟶ reflection

Transformations can also be shown on a coordinate grid. A *translation* will move each point like an ordered pair. For example, the translation shown moved the triangle right 3 units and up 4 units, which we could also show as $(x+3, y+4)$ or $(x,\ y) \rightarrow (x+3, y+4)$.



A prime mark (') is used with the label of an image point. The image of point *A* is shown as point *A*′ (read "*A* prime").

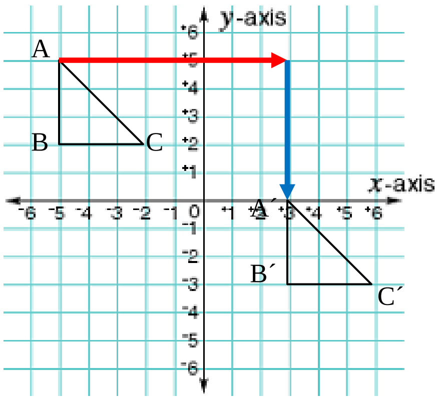
<u>Translations</u>: Slide

If they give you the algebraic transformation $(x, y) \rightarrow (x + 3, y – 5)$ just follow the directions. This says whatever x is, add 3. Whatever y is, subtract 5.

If they give you two figures, take one point such as A and draw an arrow right or left first to follow the *x*-axis, then move up or down to follow the *y*-axis to get to A'. The prime marker (A') tells you it is the transformed figure.

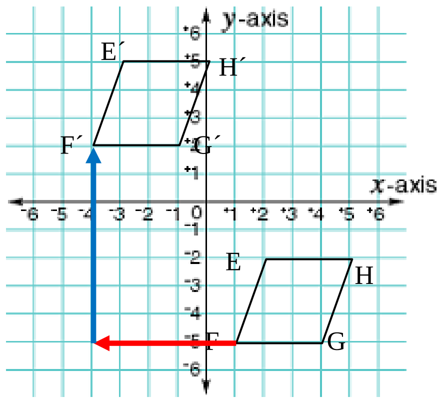
1. Words: **8 right**, **5 down**____

Algebra: $(x, y) \rightarrow$ **$(x + 8, y - 5)$**



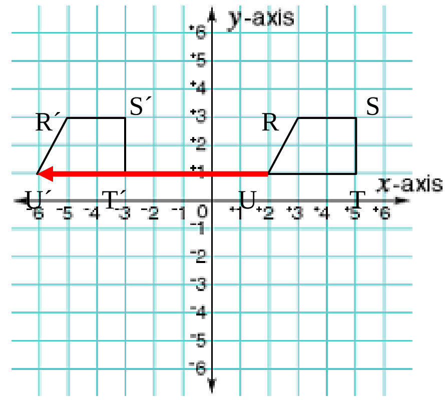
2. Words: **5 left**, **7 up**_____

Algebra: $(x, y) \rightarrow$ **$(x - 5, y + 7)$**

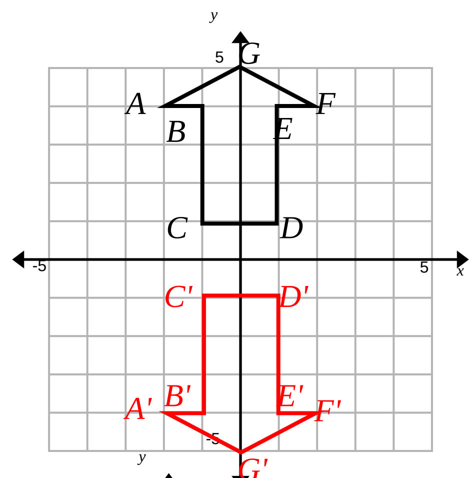


3. Words: **8 left**, **stay**______

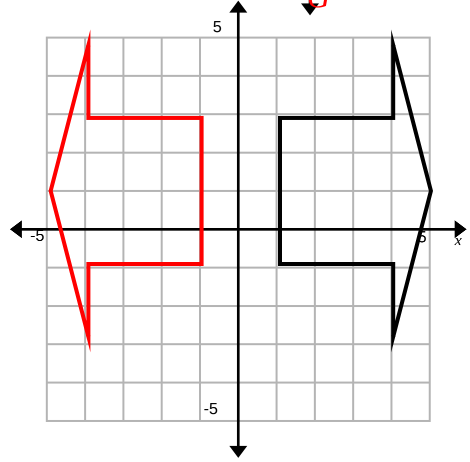
Algebra: $(x, y) \rightarrow$ **$(x - 8, y)$**



A *reflection* can be shown on a coordinate grid by reflecting the figure across an axis.



Across the *x*-axis, the figure will follow the rule $(x, -y)$.

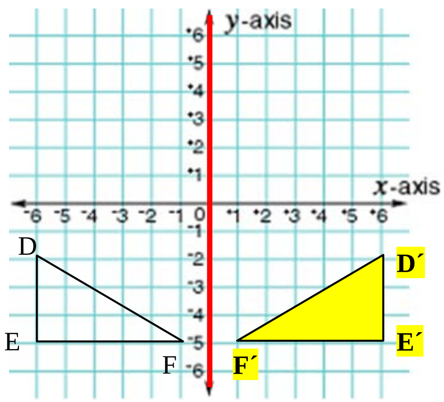


Across the *y*-axis, the figure will follow the rule $(-x, y)$.

Reflections: Flip

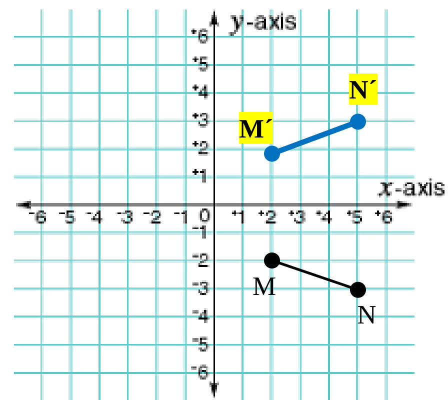
When they tell you what the line of reflection is, the first step is to highlight the line. Start at a point such as A and count how many units it is to the line then count that many past the line to get the reflected point.

1) What are the coordinates of D´, E´, and F´ after D, E, and F are reflected across the *y*-axis? Draw the image.



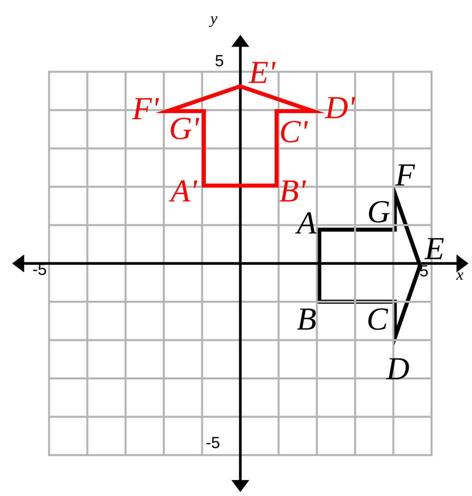
D (–6, –2) → D´ (6, –2)
E (–6, –5) → E´ (6, –5)
F (–1, –5) → F´ (1, –5)

2) What are the coordinates of M´ and N´ after $\overline{MN}$ is reflected across the *x*-axis? Draw the image.



M (2, –2) → M´ (2, 2)
N (5, –3) → N´ (5, 3)

A *rotation* can be shown on a coordinate grid by moving figure about a point. Unless it is indicated otherwise, rotation occurs around the origin. This example shows a rotation of 90° counterclockwise about the origin.



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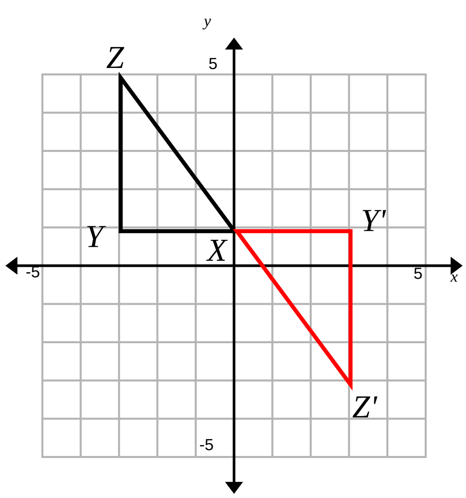


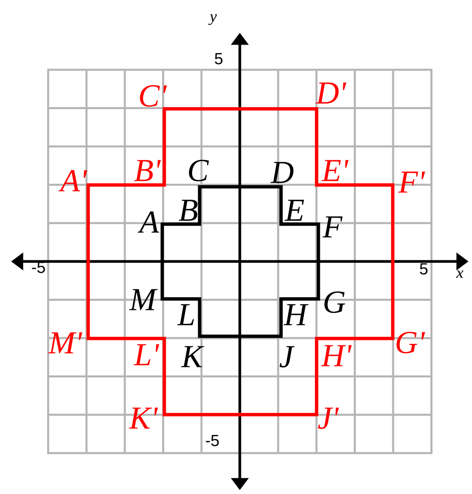
A rotation can also occur around a given point. This example rotated the triangle 180° about point *X*.

<u>Rotations</u>: Turn

Start at a point such as A and draw whatever angle they give in the direction given to A' and use guess and check method.

A *dilation* can be shown on a coordinate grid by stretching or shrinking the figure about a point. Unless it is indicated otherwise, the center of dilation is the origin. A scale factor is the ratio of a side length of the image to the corresponding side lenth of the original figure. This example shows a dilation about the origin, with a scale factor of 2.

In dilation, the figure will follow the rule $(kx,\ ky)$, where k is the scale factor.

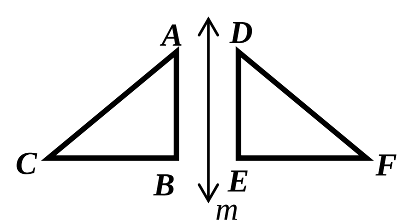


Transformations that can be used to show two figures are *congruent*: translations, reflections, and rotations.

Two figures are similar if the second figure can be obtained from the first by a sequence of transformations, including dilation. (Next unit of study)

Corresponding Parts of Congruent Figures

If two figures are congruent, their corresponding sides are congruent and their corresponding angles are congruent. For instance, in the figure to the right, triangle *ABC* is congruent to triangle *DEF* because ΔDEF is the image of ΔABC reflected over line *m*. So, $\Delta ABC \cong \Delta DEF$.



We now can list the corresponding (and congruent) parts of the triangles.

Congruent Sides:

$\overline{AB} \cong \overline{DE};\ \overline{BC} \cong \overline{EF};\ \overline{CA} \cong \overline{FD}$

Congruent Angles:

$\angle A \cong \angle D;\ \angle B \cong \angle E;\ \angle C \cong \angle F$