

**Revolution** – full circle, turn, full turn or rotation.

1 revolution = 1 full circle = 1 turn = 1 *rotation* = 1 circle = 360°

**Degree** – used in mathematics to measure angles,  $\frac{1}{360}$  of the circumference of a circle.

1 full circle = 360°

**It’s an obvious fact that circles should have 360 degrees. Right?**

Wrong. Most of us have *no idea* why there’s 360 degrees in a circle. We memorize a magic number as the “size of a circle” and set ourselves up for confusion when studying advanced math or physics, with their so called “radians”.

Here’s a theory about how degrees came to pass:

- Humans noticed that constellations moved in a full circle every year
- Every day, they were off by a tiny bit (“a degree”)
- Since a year has about 360 days, a circle had 360 degrees

But, but... why not *365 degrees* in a circle?

Cut ‘em some slack: they had *sundials* and didn’t know a year should have a convenient 365.242199 degrees like you do.

360 is close enough for government work. It fits nicely into the Babylonian base-60 number system, and divides well (by 2, 3, 4, 6, 10, 12, 15, 30, 45, 90... so you get the idea).

**Radians** – used in advanced mathematics to measure angles.

*Invented in the 1700’s by mathematicians who wanted to define angles rationally, radians were determined from the circumference of a circle with a radius of 1. One **radian** is just under 57.3°.*

1 full circle =  $2\pi$  radians

“Radians make math easier!” the experts say, without a simple reason why.

Imagine watching a friend run the track from a position inside the track, degrees measure the amount you need to turn your head to see him run. Radians measure angles by distance traveled.

But absolute distance isn’t that useful, since going 10 miles is a different number of laps depending on the track. So we divide by radius to get a normalized angle:

$$\text{Radian} = \frac{\text{distance traveled}}{\text{radius}}$$

You’ll often see this as

$$\theta = \frac{s}{r}$$

or angle in radians (theta) is arc length (s) divided by radius (r).


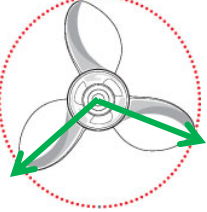


In plain English, 1 radian is what you get when the length of the segment of the circle’s circumference being measured is equal to the length of the radius.

Radians are a count of distance in terms of “radius units”; think of “radian” as shorthand for that concept.



**Fill in the radian measures to compare revolutions, degrees and radians:**

DIAGRAM	WORDS	REVOLUTIONS	DEGREES	RADIANS
...?...	no turn	0	0°	0
	quarter turn	1/4	90°	_____
	half turn	1/2	180°	_____
	three-quarter turn	3/4	270°	_____
	full turn	1	360°	_____
	twelfth turn	1/12	30°	_____
	eighth turn	1/8	45°	_____
	sixth turn	1/6	60°	_____

	fifth turn	1/5	72°	_____
	third turn	1/3	120°	_____
	two turns	2	720°	_____
	three turns	3	1080°	_____

**Conversions:**

In one full circle, there are 360 degrees or  $2\pi$  radians. Therefore we know that  $2\pi$  radians = 360 degrees. Since the angles within the circle divide the  $2\pi$  radians proportionately, we can use simple proportions to convert between degrees and radians.

$$\frac{\text{degree measure}}{360} = \frac{\text{radian measure}}{2\pi}$$

**Example:** Convert  $270^\circ$  to radians.

$$\frac{270}{360} = \frac{x}{2\pi}$$

fill in known and unknown values

$$540\pi = 360x$$

find cross-product

$$\frac{540}{360}\pi = x$$

solve for  $x$

$$\frac{3}{2}\pi = x$$

simplify

$$\text{SO... } 270^\circ = \frac{3}{2}\pi \text{ radians}$$



**Example:** Convert from  $\frac{2}{3}\pi$  radians to degrees.

$$\frac{x}{360} = \frac{\frac{2}{3}\pi}{2\pi}$$

$$2\pi x = 240\pi$$

$$x = \frac{240\pi}{2\pi}$$

fill in known and unknown values

find cross-product

solve for  $x$

$$x = 120$$

simplify

SO...  $\frac{2}{3}\pi$  radians =  $120^\circ$

We can also use the conversion equations:

$$\text{degree measure} = \text{radian measure} \left( \frac{180}{\pi} \right) \quad \text{OR} \quad \text{radian measure} = \text{degree measure} \left( \frac{\pi}{180} \right)$$

**Compile your results here, you may need to calculate some conversions:**

**{degrees, radians}**

