
An angle can be seen as a rotation of a line about a fixed point. In other words, if I were to mark a point on a paper, then rotate a pencil around that point, I would be forming angles.

One complete rotation measures 360°. Half a rotation would then measure 180°. A quarter rotation would measure 90°.

Many skateboarders, skiers, ice skaters, etc. practice these kinds of moves. 2 full rotations equals 720° or \((360° \cdot 2)\). One and a half rotations is 540° or \((1.5 \cdot 360°)\). Be sure to mention these applications.

Let’s use a more formal definition.

An angle is formed by the union of two rays with a common endpoint, called the vertex. Angles can be named by the vertex.

\[ \angle V \]

This angle would be called “angle \(V\)”, shown as \(\angle V\). However, the best way to describe an angle is with 3 points: one point on each ray and the vertex. When naming an angle, the vertex point is always in the middle.

\[ \angle SNU \]

This angle can now be named three different ways: \(\angle SUN\), \(\angle NUS\), or \(\angle U\).

Angles are measured in degrees (°). Protractors are used to measure angles. Here are two interactive websites you might use to show students how to use this measuring tool.

http://www.amblesideprimary.com/ambleweb/mentalmaths/protractor.html
http://www.mathplayground.com/measuringangles.html

You can classify an angle by its measure. Acute angles are greater than 0°, but less than 90°. In other words, not quite a quarter rotation. Right angles are angles whose measure is 90°, exactly a
quarter rotation. *Obtuse* angles are greater than 90°, but less than 180°. That’s more than a quarter rotation, but less than a half turn. And finally, *straight* angles measure 180°.

### Classifying Lines

Two lines are *parallel lines* if they do not intersect and lie in the same plane. The symbol || is used to show two lines are parallel. Triangles (▲) or arrowheads (> or <) are used in a diagram to indicate lines are parallel.

Two lines are *perpendicular lines* if they intersect to form a right angle. The symbol ⊥ is used to state that two lines are perpendicular.

Two lines are *skew lines* if they do not lie in the same plane and do not intersect.

Lines \( r \) and \( t \) are skew lines.
7.G.B.5  Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.

**Angle Relationships**

*Adjacent* angles are two angles that have a common vertex, a common side (ray), and no common interior points.

In the examples to the right, notice $\angle 1$ and $\angle 2$ are adjacent.

In this next example, there are many more relationships:

- $\angle PQR$ and $\angle PQT$ are adjacent angles.
- $\angle PQT$ and $\angle TQS$ are adjacent angles.
- $\angle TQS$ and $\angle SQR$ are adjacent angles.
- $\angle SQR$ and $\angle RQP$ are adjacent angles.

*Vertical angles* are formed when two lines intersect—they are opposite each other. These angles always have the same measure. We call angles with the same measure *congruent* ($\cong$).

In the example to the right, the two yellow angles are vertical angles. They are formed by the same two lines, they sit opposite each other and they are congruent.

Also notice the two green angles are also vertical angles.

Angles will be shown as congruent by using tick marks. If angles are marked with the same number of tick marks, then the angles are congruent.

$$\angle TQP \cong \angle RQS$$
$$\angle TQS \cong \angle PQR$$
Two angles whose sum is $90^\circ$ are called **complementary angles**. For instance, if $m\angle P = 40^\circ$ and $m\angle Q = 50^\circ$, then $\angle P$ and $\angle Q$ are complementary angles. If $m\angle A = 30^\circ$, then the complement of $\angle A$ measures $60^\circ$.

Two angles whose sum is $180^\circ$ are called **supplementary angles**. If $m\angle M = 100^\circ$ and $m\angle S = 80^\circ$, then $\angle M$ and $\angle S$ are supplementary angles.

**Example:**

1. Name an angle that is complementary to $\angle 1$. $\angle 2$
2. Name an angle that is supplementary to $\angle 1$. $\angle 5$
3. If $m\angle 2 = 50^\circ$, what is the measure of $\angle 1$? $40^\circ$
4. Knowing the measure of $\angle 1$ from above, what is the measure of $\angle 4$? $40^\circ$
5. What is the sum of the measures of $\angle 1$, $\angle 2$, $\angle 3$, $\angle 4$ and $\angle 5$? $360^\circ$

**Example:** Explain why two obtuse angles cannot be supplementary to one another.

The sum of two obtuse angles $\neq 180^\circ$.

**Examples:** Determine if each of the following statements is **always true**, **sometimes true** or **never true**. For each answer that you choose ‘sometimes true’ show one example when the statement is true and one example of when the statement is not true.

Your examples should be diagrams with the angle measurements shown.

1. Two adjacent angles are supplementary. **sometimes true**
2. Two acute angles are supplementary. **never true**
3. Vertical angles are congruent. **always true**
4. Vertical angles are adjacent angles. **never true**
5. If the measure of an angle is called $x$, the measure of its complement is $180-x$. **always true**

When a pair of **parallel** lines is cut by a transversal, there is a special relationship between the angles. Discover what this relationship is!

1) Use the lines on a piece of graph paper or a piece of lined paper as a guide to draw a pair of parallel lines. You can also use both edges of your straightedge to create parallel lines on a blank sheet of paper.

2) Draw a transversal intersecting the parallel lines. Label the angles with numbers as in the diagram above.

3) Measure $\angle 1$. Calculate the measures of the other three angles that share the same vertex.
4) Measure $\angle 8$ and calculate the measures of the other three angles. Record your findings into the table.

<table>
<thead>
<tr>
<th>Angle</th>
<th>Measure</th>
<th>Angle</th>
<th>Measure</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\angle 1$</td>
<td></td>
<td>$\angle 5$</td>
<td></td>
</tr>
<tr>
<td>$\angle 2$</td>
<td></td>
<td>$\angle 6$</td>
<td></td>
</tr>
<tr>
<td>$\angle 3$</td>
<td></td>
<td>$\angle 7$</td>
<td></td>
</tr>
<tr>
<td>$\angle 4$</td>
<td></td>
<td>$\angle 8$</td>
<td></td>
</tr>
</tbody>
</table>

Look for patterns. Complete the conjecture below:

*If two parallel lines are cut by a transversal, then the corresponding angles are congruent.*

Hopefully, you discovered that the corresponding angles created when a transversal intersects parallel lines are congruent! This will save us time when finding angle measures.

**Example:** $m\angle 2 = 50^\circ$. Find the measure of all other angles.

- $m\angle 1 = 130^\circ$, since $\angle 1$ and $\angle 2$ are adjacent and supplementary
- $m\angle 3 = 130^\circ$, since $\angle 1$ and $\angle 3$ are vertical angles or $\angle 2$ and $\angle 3$ are adjacent and supplementary
- $m\angle 4 = 50^\circ$, since $\angle 2$ and $\angle 4$ are vertical angles or $\angle 1$ and $\angle 4$ are adjacent and supplementary
- $m\angle 5 = 130^\circ$, since $\angle 1$ and $\angle 5$ are corresponding angles
- $m\angle 7 = 130^\circ$, $m\angle 6 = 50^\circ$, and $m\angle 8 = 50^\circ$ (using same reasoning as above)

**Example:** Find the measure of each angle in the diagram below given $m\angle 1 = 95^\circ$ and $m\angle 5 = 25^\circ$.

Solution:

- $m\angle 3$ is $25^\circ$ because it is a vertical angle to $\angle 5$.
- $m\angle 2$ is $60^\circ$ because together with $\angle$'s 1 and 5 they form a straight angle. So $180^\circ - (95^\circ + 25^\circ) = 180^\circ - 120^\circ = 60^\circ$.
- $m\angle 4$ is $155^\circ$ because it is supplementary with $\angle 3$ so $180^\circ - 25^\circ = 155^\circ$.
- $m\angle 1 = 95^\circ$
- $m\angle 2 = 60^\circ$
- $m\angle 3 = 25^\circ$
- $m\angle 4 = 155^\circ$
- $m\angle 5 = 25^\circ$
**Example:** Find the $m\angle EDF$ and $m\angle IDH$. Explain your thinking for each answer.

Solution:
To find the $m\angle EDF$ students would need to know that this angle plus the $65^\circ$ and $95^\circ$ angles total $180^\circ$. We say they are **angles on a line**.

$$x + 65 + 95 = 180$$
$$x = 20$$
So $m\angle EDF = 20^\circ$.

To find the $m\angle IDH$ we could add $20^\circ$ and $95^\circ$ (joining the measures of $\angle EDF$ and $\angle FDG$ because together they form a vertical angle to $\angle IDH$).

Other students may take another approach and say that $\angle GDH$ is $65^\circ$ since it is vertical to $\angle EDI$ and then state that it is supplementary to $\angle IDH$ and subtract $180-65$. In any case $m\angle IDH = 115^\circ$.

Earlier we would have a question like… given the diagram below. Find the measure of $x$.

Students would have to identify them as complements and then

$$x + 46 = 90$$
$$x = 44$$

Now we want to begin using monomials and equations. So same type of problem, but the question would be… using the diagram below, find the measure of $x$ and the $m\angle CBD$.

Knowing the angles are complements we write:

$$2x + 46 = 90$$
$$2x = 44$$
$$x = 22$$

$x = 22$ and $m\angle CBD = 44^\circ$.
Example: Find the measure of $x$ and the $m \angle EGF$.

\[
3x + 51 = 180 \\
-51 & \quad -51 \\
3x & = 129 \\
3x & = \frac{129}{3} \\
x & = 43 \quad 3x = 3(43) = 129
\]

$x = 43$ and $m \angle EGF = 129^\circ$

As diagrams become more complex we may find examples like the following.

\[
\text{We can see that angles } x^\circ \text{ and } 22^\circ \text{ are supplementary and sum to } 180^\circ. \\
x + 22 = 180 \\
x + 22 - 22 = 180 \\
x = 158
\]

Using the diagram below, what we can easily see is there are three angles that total $360^\circ$. So we could sum all three and set them equal to 360.

Example: Find the measure of $m$.

\[
90 + 165 + m = 360 \\
255 + m = 360 \\
255 - 255 + m = 360 - 225 \\
m = 135
\]
In this diagram notice that $p$ is vertically opposite from and equal to the sum of angles with measurements $22^\circ$ and $26^\circ$, or a sum of $48^\circ$.

Here three lines meet at a point. Notice that the two $a^\circ$ angles and the angle $144^\circ$ are angles on a line and sum to $180^\circ$. We could write the equation $a + a + 144 = 180$.

Here again three lines meet at a point. Notice that angles $x$ and $y$ are complementary. Angle $x$ is a vertical angle to the angle marked $21^\circ$, so $x = 21^\circ$.

Here three lines meet at a point. Notice that part of angle $z$ is a right angle and the additional portion is vertical to the $48^\circ$ angle. So angle $z = 48^\circ + 90^\circ = 138^\circ$. 
Three lines meet at a point; $\angle AOC = 154^\circ$. What is the measure of $\angle DOF$?

Some students may just start writing equations and figuring out that $\angle AOB$ must be $64^\circ$ ($154 - 90 = 64$). Then $\angle DOE = 64^\circ$ since it is a vertical angle to $\angle AOB$. Finally $\angle DOF = 90 + 64 = 154^\circ$.

OR Others may have some difficulty here due to visualization skills. Have them begin by breaking down what they know.

Example:
\[
\begin{align*}
\angle FOE &= 90^\circ \\
\angle BOC &= 90^\circ \\
\angle AOB &\cong \angle DOE
\end{align*}
\]

OR Students they may want to use some colored pencils to lightly shade portions of the diagram. This will aid with their visualization skills.

Visually looking either at the colored portions OR focusing on the blank vertical angles, students should automatically see $\angle AOC = 154^\circ$ and $\angle DOF = 154^\circ$ because they are vertical angles, and vertical angels are congruent.
Example: In a pair of complementary angles, the measurement of the larger angle is four times the measure of the smaller angle. Find the measures of the two angles.

\[ x + 4x = 90 \]
\[ 5x = 90 \]
\[ \frac{5x}{5} = \frac{90}{5} \]
\[ x = 18 \]

\[ \text{Angle 1} = 18^\circ \]
\[ \text{Angle 2} = 4(18) = 72^\circ \]

Example: The measures of two supplementary angles are in the ratio of 4:5. Find the two angles.

\[ 4x + 5x = 180 \]
\[ 9x = 180 \]
\[ \frac{9x}{9} = \frac{180}{9} \]
\[ x = 20 \]

\[ \text{Angle 1} = 4(20) = 80^\circ \]
\[ \text{Angle 2} = 5(20) = 100^\circ \]

Example: The measure of a complement of an angle is 8° more than three times the angle. Find the measurements of the two angles.

\[ x + (3x + 8) = 90 \]
\[ 4x + 8 = 90 \]
\[ 4x + 8 - 8 = 90 \]
\[ 4x = 90 \]
\[ \frac{4x}{4} = \frac{90}{4} \]
\[ x = 20.5 \]

\[ \text{Angle 1} = 20.5^\circ \]
\[ \text{Angle 2} = 3(20.5) + 8 = 61.5 + 8 = 69.5^\circ \]
**Example:** Find the measurement of the two missing angles.

\[ s + 19 = 90 \]
\[ s + 19 - 19 = 90 \quad \text{complementary angles} \]
\[ s = 71 \]

\[ r + s + 19 + 46 = 180 \]
\[ r + 71 + 19 + 46 = 180 \quad \text{angles on a line} \]
\[ r + 136 = 180 \]
\[ r + 136 - 136 = 180 - 136 \]
\[ r = 44 \]

**Example:** Two lines meet at the common vertex of two rays. Set up and solve an appropriate equation for \( x \) and \( y \).

\[ 28 + y = 90 \]
\[ 28 - 28 + y = 90 - 28 \quad \text{complementary angles} \]
\[ y = 62 \]

\[ x + y = 180 \]
\[ x + 62 = 180 \quad \text{supplementary angles} \]
\[ x + 62 - 62 = 180 - 62 \]
\[ x = 118 \]

**Example:** Two lines meet at the common vertex of a ray. Set up and solve the appropriate equations to determine \( x \) and \( y \).

\[ x + 57 = 90 \]
\[ x + 57 - 57 = 90 \]
\[ x = 33 \]

\[ y + 57 = 180 \]
\[ y + 57 - 57 = 180 - 57 \]
\[ y = 123 \]
**Example:** The measurement of the complement of an angle exceeds the measure of the angle by 25%. Find the angle and its complement.

\[
x + (x + \frac{1}{4} x) = 90
\]
\[
x + \frac{5}{4} x = 90
\]
\[
x \frac{9}{4} x = 90
\]
\[
\left(\frac{4}{9}\right) \frac{9}{4} x = 90 \left(\frac{4}{9}\right)
\]
\[
x = 40
\]
\[
90 - x = 90 - 40 = 50
\]

The angle is 40° and its complement is 50°.

**Prep for 7.G.B.6**

Area of Rectangles, Squares and Rhombi

One way to describe the size of a room is by naming its dimensions. A room that measures 12 ft. by 10 ft. would be described by saying it’s a 12 by 10 foot room. That’s easy enough.

There is nothing wrong with that description. In geometry, rather than talking about a room, we might talk about the size of a rectangular region.

For instance, let’s say I have a closet with dimensions 2 feet by 6 feet. That’s the size of the closet.

By simply counting the number of squares that fit inside that region, we find there are 12 squares.

Someone else might choose to describe the closet by determining how many one foot by one foot tiles it would take to cover the floor. To demonstrate, let me divide that closet into one foot squares.

By multiplying the dimensions together, we can determine the area of a rectangle.

If I continue making rectangles of different dimensions, I would be able to describe their size by those dimensions, or I could mark off units and determine how many equally sized squares can be made.

Rather than describing the rectangle by its dimensions or counting the number of squares to determine its size, we could multiply its dimensions together.
Putting this into perspective, we see the number of squares that fits inside a rectangular region is referred to as the area. A shortcut to determine that number of squares is to multiply the base by the height. More formally area is defined as the space inside a figure or the amount of surface a figure covers. The Area of a rectangle is equal to the product of the length of the base and the length of a height to that base. That is \( A = bh \).

Most books refer to the longer side of a rectangle as the length \((l)\), the shorter side as the width \((w)\). That results in the formula \( A = lw \). So now we have 2 formulas for the areas of a rectangle that can be used interchangeably. The answer in an area problem is always given in square units because we are determining how many squares fit inside the region. Of course you will show a variety of rectangles to your students and practice identifying the base and height of those various rectangles.

**Example:** Find the area of a rectangle with the dimensions 3 m by 2 m.

\[
A = lw \\
A = 3 \cdot 2 \\
A = 6
\]

The area of the rectangle is 6 \(m^2\).

**Example:** The area of a rectangle is 16 square inches. If the height is 8 inches, find the base.

**Solution:**

\[
A = bh \\
16 = b \cdot 8 \\
2 = b
\]

The base is 2 inches.

**Example:** The area of the rectangle is 24 square centimeters. Find all possible whole number dimensions for the length and width.

<table>
<thead>
<tr>
<th>Area</th>
<th>Length</th>
<th>Width</th>
</tr>
</thead>
<tbody>
<tr>
<td>24</td>
<td>1</td>
<td>24</td>
</tr>
<tr>
<td>24</td>
<td>2</td>
<td>12</td>
</tr>
<tr>
<td>24</td>
<td>3</td>
<td>8</td>
</tr>
<tr>
<td>24</td>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>24</td>
<td>6</td>
<td>4</td>
</tr>
<tr>
<td>24</td>
<td>8</td>
<td>3</td>
</tr>
<tr>
<td>24</td>
<td>12</td>
<td>2</td>
</tr>
<tr>
<td>24</td>
<td>24</td>
<td>1</td>
</tr>
</tbody>
</table>

**Example:** Find the area of the rectangle.

9 ft. 2 yd.
Be careful! Area of a rectangle is easy to find, and students may quickly multiply to get an answer of 18. This is wrong because the measurements are in different units. We must first convert feet into yards, or yards into feet.

Since 1 yard = 3 feet we start with \[
\frac{\text{yards}}{\text{feet}} \rightarrow \frac{1}{3} = \frac{x}{9}
\]

\[
9 = 3x
\]

\[
x = 3
\]

We now have a rectangle with dimensions 3 yd. by 2 yd.

\[A = lw\]

\[A = (3)(2)\]

\[A = 6\]

The area of our rectangle is 6 square yards.

If we were to have a square whose sides measure 5 inches, we could find the area of the square by putting it on a grid and counting the squares as shown below.

Counting the squares is a viable method but eventually students will begin to see the area can be computed more efficiently. If they multiply the base times the height, then again \[A = bh\] or some will see the area as the square of one side, so \[A = s^2\].

In this example, \[A = bh = 5 \cdot 5 = 25 \text{units}^2\] or \[A = s^2 = 5^2 = 5 \cdot 5 = 25 \text{units}^2\].

Next, if we begin with a 6 x 6 square and cut from one corner to the other side (as show in light blue) and translate that triangle to the right, it forms a new quadrilateral called a rhombus (plural they are called rhombi). Since the area of the original square was 6 x 6 or 36 square units then the rhombus has the same area, since the parts were just rearranged. So we know the rhombus has an area of 36 square units. Again we find that the Area of the rhombus = \[bh\].
Be sure to practice with a number of different rhombi so your students are comfortable with identifying the height versus the width of the figures.

**Area of Parallelograms, Triangles and Trapezoids**

If I were to cut one corner of a rectangle and place it on the other side, I would have the following:

We now have a parallelogram. Notice, to form a parallelogram, we cut a piece of a rectangle from one side and placed it on the other side. Do you think we changed the area? The answer is no. All we did was rearrange it; the area of the new figure, the parallelogram, is the same as the original rectangle.

So we have the **Area of a parallelogram = bh**.

![Parallelogram Diagram](image)

**Example:** The height of a parallelogram is twice the base. If the base of the parallelogram is 3 meters, what is its area?

*First, find the height. Since the base is 3 meters, the height would be twice that or 2(3) or 6 m. To find the area,*

\[
A = bh \\
A = 3 \cdot 6 \\
A = 18
\]

*The area of the parallelogram is 18 m\(^2\).*

**NOTE:** If students are having difficulty with correctly labeling areas with **square units** considering having them write the units within the problem. For example, instead of

\[
A = bh \\
A = 3 \cdot 6 \\
A = 18 \\
18 \text{ square meters}
\]

Say aloud \(3 \cdot 6 = 18\) and **meters \cdot meters** = **square meters**
We have established that the area of a parallelogram is $A = bh$. Let’s see how that helps us to understand the area formula for a triangle and trapezoid.

**For this parallelogram, its base is 4 units and its height is 3 units. Therefore, the area is $4 \cdot 3 = 12$ units².**

If we draw a diagonal, it cuts the parallelogram into 2 triangles. That means one triangle would have one-half of the area or 6 units². Note the base and height stay the same. So for a triangle, $A = \frac{1}{2}bh$, or $\frac{1}{2}(4)(3) = 6$ units².

**For this parallelogram, its base is 8 units and its height is 2 units. Therefore, the area is $8 \cdot 2 = 16$ units².**

If we draw a line strategically, we can cut the parallelogram into 2 congruent trapezoids. One trapezoid would have an area of one-half of the parallelogram’s area (8 units²). Height remains the same. The base would be written as the sum of $b_1$ and $b_2$. For a trapezoid, $A = \frac{1}{2}(b_1 + b_2)h$.

**Triangles**

As demonstrated above, one way to introduce the area formula of a triangle is to begin with a rectangle, square, rhombus and/or a parallelogram and show that cutting the figure using a diagonal produces 2 congruent triangles. This aids students in remembering the **Area formula of a Triangle** is $A = \frac{1}{2}bh$ or $A = \frac{bh}{2}$. 
Example: Find the area of each of the following triangles.

a. 
\[ A = \frac{1}{2} \times 17 \times 6 \]
\[ A = 51 \text{ square cm} \]

b. 
\[ A = \frac{1}{2} \times 6 \times 8 \]
\[ A = 24 \text{ squares inches} \]

c. 
\[ A = \frac{1}{2} \times 12 \times 8 \]
\[ A = 48 \text{ square mm} \]

Solutions:

Important things here:
- In each example, students must determine which dimensions given are the base and height.
- In example a, the Commutative Property of multiplication was employed to take half of the even number to make the arithmetic easy.
- In example b, students must note that the height and width are both 8 in.
- In example c, the height must be given outside the triangle. Why?

Example: A triangular piece of fabric has an area of 54 square inches. The height of the triangle is 6 inches. What is the length of the triangle’s base?

Beginning with the formula and substituting in the values we know, we get:

\[ A = \frac{1}{2} \times b \times h \]
\[ 54 = \frac{1}{2} \times b \times 6 \]
\[ 54 = 3b \]
\[ 18 = b \]

18 in²

Reflection:
When can you use two side lengths to find the area of a triangle? In this situation, does it matter which side is the base and which side is the height?

Solution: When you have a right triangle you use two side lengths to find the area of a triangle. It doesn’t matter which side is the base or height.
Trapezoids

As demonstrated on page 5, the formula used to find the **Area of a trapezoid** is \( A = \frac{1}{2}(b_1 + b_2)h \).

We must be sure students see this formula in several ways. Be sure to teach the other forms

**Area of a trapezoid** is \( A = \frac{(b_1 + b_2)h}{2} \) and **Area of a trapezoid** is \( A = \text{(Average of the bases)} \cdot \text{height} \).

**Example:**

\[ \text{a.} \quad 4 \text{ ft} \]

\[ \begin{array}{c}
\text{4 ft} \\
\hline
\text{6 ft}
\end{array} \]

\[ \text{b.} \quad 20 \text{ mm} \]

\[ \begin{array}{c}
\text{8 mm} \\
\hline
\text{24 mm}
\end{array} \]

\[ \text{c.} \quad 2 \text{ cm} \]

\[ \begin{array}{c}
\text{2 cm} \\
\hline
\text{8 cm}
\end{array} \]

**Solutions:**

\[ \text{a.} \]

\[ A = \frac{(b_1 + b_2)h}{2} \]

\[ A = \frac{20 + 24}{2} \cdot (6) \]

\[ A = \frac{44}{2} \cdot (6) \]

\[ A = 22 \cdot (6) \]

\[ A = 132 \]

\[ \text{20 ft}^2 \]

\[ \text{b.} \]

\[ A = \frac{1}{2}(b_1 + b_2)h \]

\[ A = \frac{1}{2}(5 + 2) \cdot (8) \]

\[ A = \frac{1}{2}(7) \cdot (8) \]

\[ A = 4 \cdot (7) \]

\[ A = 28 \]

\[ \text{132 mm}^2 \]

\[ \text{c.} \]

\[ A = \frac{1}{2}(b_1 + b_2)h \]

\[ \text{The height is 10 cm.} \]

\[ \text{28 cm}^2 \]

**Notice:**

- In example a, it was easy to mentally find the average of 6 and 4 (the bases) thus the use of the formula \( A = \text{(Average of the bases)} \cdot \text{height} \).
- In example c, students will need to correctly identify the bases to solve this problem so be sure to give students exposure to this orientation of the trapezoid.

**Example:** A trapezoid has an area of 200 cm\(^2\) and bases of 15 cm and 25 cm. Find the height.

**Solution:**

\[ A = \frac{1}{2}(b_1 + b_2)h \]

\[ 200 = \frac{1}{2}(15 + 25)h \]

\[ 200 = \frac{1}{2}(40)h \]

\[ 200 = 20h \]

\[ 10 = h \]
**7.G.B.4**  
**Know the formulas for the area and circumference of a circle and use them to solve problems:** give an informal derivation of the relationship between the circumference and area of a circle.

### Circles: Circumference and Area

A *circle* is defined as all points in a plane that are equal distance (called the *radius*) from a fixed point (called the *center* of the circle). The distance across the circle, through the center, is called the *diameter*. Therefore, a diameter is twice the length of the radius, or \( d = 2r \).

We called the distance around a polygon the perimeter. The distance around a circle is called the *circumference*. There is a special relationship between the circumference and the diameter of a circle. Let’s get a visual to approximate that relationship. Take a can with 3 tennis balls in it. Wrap a string around the can to approximate the circumference of a tennis ball. Then compare that measurement with the height of the can (which represents three diameters). You will discover that the circumference of the can is a little greater than the three diameters (height of the can).

There are many labs that will help students understand the concept of \( \pi \). Most involve measuring the distance around the outside of a variety of different size cans, measuring the diameter of each can and dividing to show that the relationship is “3 and something”. You may want to use a table like the following to organize student work:

<table>
<thead>
<tr>
<th>Can #</th>
<th>Circumference</th>
<th>Diameter</th>
<th>Circumference ÷ Diameter = ( \pi )</th>
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Once students record the measurements for a variety of cans, students should find the quotient of the circumference and the diameter is **close to** 3.14. Some quotients are not exact, or are “off”, due to measurement errors or lack of precision.

So initially get students to comprehend the *circumference* \( \approx \) *diameter x 3*.

This should help convince students that this ratio will be the same for every circle. We can then introduce that \( \frac{C}{d} = \pi \) or \( C = \pi d \). Then, since \( d = 2r \), we can also write \( C = 2\pi r \).

Please note that \( \pi \) is an irrational number (never ends or repeats). Mathematicians use \( \pi \) to represent the exact value of the circumference/diameter ratio.

**Example:** If a circle has a diameter of 4 m, what is the circumference? Use 3.14 to approximate \( \pi \). State your answer to the nearest 0.1 meter.

Using the formula:

\[
C = \pi d
\]

\[
C \approx (3.14)(4) \quad \text{The circumference is about 12.6 meters.}
\]

\( C \approx 12.56 \)
Many standardized tests ask students to leave their answers in terms of \( \pi \). Be sure to practice this!

**Example:** If a circle has a radius of 5 feet, find its circumference. Do not use an approximation for \( \pi \).

Using the formula: \( C = 2\pi r \)

\[ C = 2 \cdot \pi \cdot 5 \]
\[ C = 10\pi \]

The circumference is about 10\( \pi \) feet.

**Example:** If a circle has a circumference of 12\( \pi \) inches, what is the radius?

Using the formula: \( C = 2\pi r \)

\[ 12\pi = 2\pi r \]
\[ 6 = r \]

The radius is 6 inches.

**Example:** A circle has a circumference of 88 inches. Using \( \pi \approx \frac{22}{7} \), find the diameter.

Using the formula: \( C = \pi d \)

\[ 88 = \frac{22}{7} d \]

Using the formula:

\[ 88 \cdot \frac{7}{22} = \frac{22}{7} \cdot \frac{7}{22} d \]
\[ 28 = d \]

The diameter is 28 inches.

**Example:** A circle has a circumference of 24 m. Using \( \pi \approx 3.14 \), find the diameter. Round your answer to the nearest whole number.

Using the formula: \( C = \pi d \)

\[ 24 \approx (3.14)d \]

\[ \frac{24}{3.14} \approx \frac{(3.14)d}{3.14} \]
\[ 7.6 \approx d \]

The diameter is about 8 meters.

7.G.B.4 Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.

**The Relationship Between Circumference and Area of a Circle**
You can demonstrate the formula for finding the area of a circle. First, draw a circle; cut it out. Fold it in half; fold in half again. Fold in half two more times, creating 16 wedges when you unfold the circle. Cut along these folds.

Rearrange the wedges, alternating the pieces tip up and down (as shown), to look like a parallelogram.

\[
\text{radius (} r \text{)}
\]

This is \( \frac{1}{2} \) of the distance around the circle or \( \frac{1}{2} \) of \( C \). We know that

\[
C = 2\pi r, \text{ so } \frac{1}{2} C = \left( \frac{1}{2} \right) 2\pi r
\]

\[
\frac{1}{2} C = \pi r
\]

The more wedges we cut, the closer it would approach the shape of a parallelogram. No area has been lost (or gained). Our “parallelogram” has a base of \( \pi r \) and a height of \( r \). We know from our previous discussion that the area of a parallelogram is \( bh \). So we now have the area of a circle:

\[
A = bh
\]

\[
A = (\pi r)(r)
\]

\[
A = \pi r^2
\]

**Example:** Find the area of the circle to the nearest square meter if the radius of the circle is 12 m. Use \( \pi \approx 3.14 \).

**Using the formula:**

\[
A = \pi r^2
\]

\[
A \approx (3.14)(12)^2
\]

\[
A \approx 452.16
\]

**The area of the circle is about 452 square meters.**
**Example:** Find the area of the circle if the radius is 7 meters. Use \( \pi = \frac{22}{7} \).

\[
A = \pi r^2
\]

Using the formula:
\[
A = \frac{22}{7} \cdot 7
\]

\[
A = 22
\]

The area of the circle is 22 square meters.

**Example:** Find the area of the circle if the diameter is 10 inches. Leave your answer in terms of \( \pi \).

Using the formula:
\[
A = \pi r^2
\]

\[
A = \pi (5)^2
\]

\[
A = 25\pi
\]

The area of the circle is \(100\pi \) square inches.

**Example:** The circle to the right has a diameter of 12 cm. Calculate the area of the shaded region. Leave your answer in terms of \( \pi \).

\[
\text{Area of the quarter circle} = \frac{1}{4} \pi (6)^2
\]

\[
\frac{1}{4} \pi (36)
\]

\[
9\pi
\]

**Reflection - Check for Understanding**

For which plane figure(s) does the formula \( A = bh \) work?
For which plane figure(s) does the formula \( A = lw \) work?

For which plane figure(s) does the formula \( A = \frac{1}{2}bh \) work?

For which plane figure(s) does the formula \( A = s^2 \) work?
For which plane figure(s) does the formula \( A = \pi r^2 \) work?
For which plane figure(s) does the formula \( A = (\text{Average of the bases}) \text{height} \) work?

For which plane figure(s) does the formula \( A = \frac{1}{2}(b_1 + b_2)h \) work?
7.G.B.6 Solve real-world and mathematical problems involving area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

Area of Irregular Figures

Students should also practice finding the area of irregular figures by composing and decomposing into shapes they know.

**Example:** Find the area of the polygon.

![Polygon Diagram]

**Solutions:** Method 1 (Whole to Part)

1. Create a rectangle using the outside borders of the original figure. *(Composing)*
2. Find the area of the large “new” rectangle. *(Whole)*
3. Subtract the negative (white) spaces. *(Parts)*

**NOTE:** Students will need to compute outer side length and width (shown in red).

\[ A = \text{Area} = 26 \text{ square units} \]

Method 2 *(Part to Whole)*

1. Break the polygon into smaller figures (rectangles, squares, triangles, etc. that you know). *(Decomposing)*
2. Find the area of each piece. *(Parts)*
3. Add the areas of the pieces. *(Whole)*
NOTE: Students will need to compute the outer side length and width (shown in red).

**Example:** The dimensions of a church window are shown below. Find the area of the window to the nearest square foot.

We are given the diameter, so the radius would be half of the 10 feet or 5 feet.

\[
A = bh + \frac{1}{2} \pi r^2
\]

\[
A = 10 \cdot 8 + \frac{1}{2} (3.14)(5)^2
\]

\[
A = 80 + 1.57(25)
\]

\[
A = 80 + 39.25
\]

\[
A = 119.25
\]

The area of the church window is about 119 square feet to the nearest foot.
Example: Find the area of the shaded region. Leave in terms of $\pi$.

$$A = \bigcirc - \bigcirc$$

$$A = \pi r_1^2 - \pi r_2^2 \quad \text{where } r_1 \text{ is the radius of the bigger circle and } r_2 \text{ is the area of the smaller circle}$$

$$A = \left(\pi \cdot 14^2\right) - \left(\pi \cdot 10^2\right)$$

$$A = 196\pi - 100\pi$$

$$A = 96\pi$$

The area of the shaded region is $96\pi$ square cm ($96\pi$ cm\(^2\)).

Example: The square in the figure below has a side length of 14 inches. Find the area of the shaded region.

a. Leave in terms of $\pi$.

Solution: Steps may vary

$$A = \square - \bigcirc$$

$$A = bh - \pi r^2$$

$$A = 14 \times 14 - \pi (7)(7)$$

$$A = 196 - 49\pi$$

b. Find the area using $\pi \approx \frac{22}{7}$.

Area of the shaded region = \square - \bigcirc

Solution: Steps may vary

$$s^2 - \pi r^2$$

$$14^2 - \frac{22}{7} (7)^2$$

$$A = 196 - 22(7)$$

$$196 - 154$$

$$42$$

Solutions: a. $(196-49\pi)$ square inches

b. 42 square inches

It will be interesting to see how students view this diagram and solve the problem. Many may not see it as the area of the square minus the area of a circle. They may see four separate quarter circles and not one full circle.
Example: Find the area of the shaded region.

\[ A = bh - \pi r^2 \]
\[ A = 100 \times 100 - 3.14 \times 50^2 \]
\[ A = 10,000 - 3.14 \times (2,500) \]
\[ A = 10,000 - 7,850 \]
\[ A = 2,150 \]

The area of the shaded region is 2,150 mm\(^2\).

Example: Find the area of the shaded region.

\[ A = bh + \pi r^2 \]
\[ A = 25 \times 10 + 3.14 \times 5^2 \]
\[ A = 250 + 3.14 \times 25 \]
\[ A = 250 + 78.5 \]
\[ A = 328.5 \]

The area of the shaded region is 328.5 yd\(^2\).

Some real word problems.

Example: John wants to expand his backyard garden. Currently the garden is 4 ft long and 5 feet wide. He wants to increase the area of the garden by at least 50%. If he extends the length by 3 feet and the width by 2 feet, will the new garden satisfy the conditions? Show all your work and explain your thinking.

Example: A rectangular pool is 30 feet long and 20 feet wide. A 3 foot walkway is to be built around the perimeter of the pool. Find the area of the sidewalk.

The owner would like the walkway to be built out of pavers that are 9 inches long, 4 inches wide and 4 inches deep. Pavers cost $0.95 each. How much will it cost to build the walkway out of pavers? Show all your work and explain your thinking.
7.G.A.2 Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.

NOTE: A Core Principles newsletter on this standard is a “must” read. Newsletters may be found in the Core Principles folder on InterAct, or on the CPD K-12 Mathematics blog at cpdmath.wordpress.com.

Teachers should begin having students make simple freehand sketches with simple examples and discuss when the conditions determine a unique triangle, more than one triangle, or no triangle.

It is important to understand the idea of unique triangles. When drawing a triangle under a given condition, the triangle will either be identical or non-identical to the original triangle. If only one triangle can be drawn under the condition, we say the condition determines a unique triangle. A triangle drawn under a condition that is known to determine a unique triangle will be identical to the original triangle.

Following are the conditions that determine unique triangles:

- Three sides condition
- Two sides and included angle condition
- Two angles and included side condition
- Two angles and the side opposite a given angle condition
- Two sides and a non-included angle, provided the angle is 90° or greater
- Two sides and a non-included angle, provided the side adjacent to the angle is shorter than the side opposite the angle

**Example:** Sketch a triangle with three congruent side lengths. unique triangle
(This may be a great discussion point with your class, as some students may say they can sketch different triangles with different side lengths—we need to point out that this is an equilateral triangle so they could be enlarged or reduced (dilated).

**Example:** Sketch a triangle with two congruent side lengths. more than one triangle

**Example:** Sketch a triangle with three congruent angles. unique triangle

**Example:** Sketch a triangle with no congruent side lengths. more than one triangle

**Example:** Sketch a triangle with no congruent angles. more than one triangle

**Example:** Sketch a triangle with two obtuse angles. no triangle

**Example:** Sketch a right triangle where one leg is double the length of the other. unique triangle
Example: Sketch a quadrilateral with no congruent sides. more than one quadrilateral

Example: Sketch a quadrilateral with 2 congruent side lengths but no parallel sides. more than one quadrilateral

As students go through examples like those above, in addition to conversations about whether each figure is unique, whether the figure can be drawn in more than one way or no figure can be drawn, it is important to discuss what those “more than one ways” would look like and if there is no figure that can be drawn, why no figure can be sketched.

As students progress, more precise measurements can be given and tools should be incorporated. If possible, the use of tracing paper or patty paper is suggested so different figures can be compared by laying papers on top of one another. Incorporating Geometer’s SketchPad would be ideal if technology is available.

Constructions are taught in “Labs” or “Extensions” in your Holt textbook on pages 456 – 457 and in McDougall Littel on pages 527-528. At minimum students should be taught how to bisect a line segment, construct an angle congruent to a given angle, bisect an angle, and construct parallel and perpendicular lines.

The following are tutorials that will demonstrate these skills step-by-step:

*How to bisect a line with compass and straightedge or ruler - Math Open Reference*
*How to construct congruent angles with compass and straightedge - Math Open Reference*
*How to bisect an angle with compass and straightedge - Math Open Reference*
*How to construct perpendicular lines - Math Open Reference*
*How to construct parallel lines - Math Open Reference*

Use a ruler, protractor and/or compass, to construct the following:

Example: a triangle with given lengths: 2 inches, 3 inches and 4 inches a unique triangle

Example: given angle measures: 30°, 60° and 90°. a unique triangle

Example: given lengths: 2 inches, 3 inches and an included angle of 50°. a unique triangle

Example: given lengths: 2 inches, 3 inches and a non-included angle of 50°.
   (A non-included angle is the angle NOT between the two given sides,) more than one triangle

Example: given lengths: 2 inches, 3 inches and 7 inches. no triangle
Note: no triangle can be drawn with these side lengths because the sum of the lengths of any two sides of a triangle must be greater than the length of the third side. Since 2 + 3 <7, no triangle can be constructed.

Example: given angle measurements: 40°, 50° and 60°. no triangle
Note: no triangle can be drawn with these angle measurements because the sum of the angle measures $\neq 180^\circ$. ($40^\circ + 50^\circ + 60^\circ \neq 180^\circ$)

**Example:** Draw $\triangle ABC$ so that $\angle A = 30^\circ$, $\angle B = 60^\circ$ and the length of a side is 10 cm. Draw as many different triangles that can be draw with these measurements.

**Example:** Draw vertical angles so that one angle is $130^\circ$. Label each angle formed with its measurement.

**Example:** Draw complementary angles so that one angle measures $25^\circ$. Label each angle with its measurement.

**Example:** Draw supplementary angles so that one angle measures $106^\circ$. Label each angle with its measurement.

**Example:** Draw 3 concentric circles.

**Example:** Draw three distinct segments of lengths 2 cm, 4 cm, 5 cm. Use your compass to draw three circles, each with a radius of one of the drawn segments. Label each radius with its measurement.

**Example:** Draw a square MNOP with side lengths equal to 4 cm. Label sides and angle measurements.

**Example:** Draw a rectangle DEFG so that $DE = FG = 6$ cm and $EF = GD = 4$ cm.

**Example:** Draw three adjacent angles so that $m\angle r = 35^\circ$, $m\angle s = 90^\circ$ and $m\angle t = 50^\circ$.

**Example:** Draw an isosceles triangle ABC. Begin by drawing $\angle A$ with a measurement of $80^\circ$. Use the rays of $\angle A$ as the equal legs of the triangle. Label all angle measurements.
An artist used silver wire to make a square that has a perimeter of 40 inches. She then used copper wire to make the largest circle that could fit in the square, as shown below.

How many more inches of silver wire did the artist use compared to copper wire? (Use \( \pi = 3.14 \)) Show all work necessary to justify your response.

**Sample Top-Score Response:**

Each side of the square has a length of \( 40 \times \frac{1}{4} = 10 \) inches.

The radius of the circle is \( \frac{10}{2} = 5 \) inches, so the circumference of the circle is \( 2 \times \pi \times 5 = 10 \times 3.14 = 31.4 \) inches.

The perimeter of the square minus the circumference of the circle is \( 40 - 31.4 = 8.6 \) inches.
Consider a circle that has a circumference of $28\pi$ centimeters (cm).

**Part A**
What is the area, in cm$^2$, of this circle? Show all work necessary to justify your response.

**Part B**
What would be the measure of the radius, in cm, of a circle with an area that is 20% greater than the circle in **Part A**? Show all work necessary to justify your response.
Sample Top-Score Response:

**Part A**
First, I found the radius: \( r = \frac{26\pi}{2\pi} = 14 \) cm. Then I found the area:
\[
A = \pi(14^2) = 196\pi \text{ cm}^2. \quad \text{OR} \quad A \approx (3.14)(14^2) \approx 615.44 \text{ cm}^2.
\]

**Part B**
First, I multiplied the area of the circle in Part A by 1.20 (which is 20% more than the original): \( A = 196\pi(1.20) = 235.2\pi \text{ cm}^2. \) Then I found the radius by solving the area formula for \( r \):
\[
235.2\pi = \pi r^2
\]
\[
235.2 = r^2
\]
\[
15.34 \approx r
\]

Scoring Rubric:

Responses to this item will receive 0–4 points, based on the following:

4 points: The student shows a thorough understanding of how to solve a real-world problem involving the area and circumference of a circle by using proportional relationships. The student correctly answers both parts and shows sufficient work to justify both answers.

3 points: The student shows a solid understanding of how to solve a real-world problem involving the area and circumference of a circle by using proportional relationships. The student correctly answers both parts but only shows sufficient work to justify one answer. OR The student shows sufficient strategy to justify both answers, but makes a computational error that leads to one incorrect answer.

2 points: The student shows a partial understanding of how to solve a real-world problem involving the area and circumference of a circle by using proportional relationships. The student correctly answers Part A and shows sufficient work to justify the answer. OR The student correctly answers Part B based on an incorrect answer to Part A and shows sufficient work to justify the answer. OR The student shows sufficient strategy to justify both answers, but makes minor computational errors that lead to two incorrect answers.

1 point: The student shows a limited understanding of how to solve a real-life problem involving the area and circumference of a circle by using proportional relationships. The student correctly answers Part A. OR The student correctly answers Part B based on an incorrect answer to Part A. OR The student shows sufficient strategy to justify an answer to one part, but either answers incorrectly or does not provide an answer.

0 points: The student shows inconsistent or no understanding of how to solve a real-life problem involving the area and circumference of a circle by using proportional relationships.
Part A

Determine if each of these statements is always true, sometimes true, or never true. Circle your response.

1. The sum of the measures of two complementary angles is 90°.
   Always True  Sometimes True  Never True

2. Vertical angles are also adjacent angles.
   Always True  Sometimes True  Never True

3. Two adjacent angles are complementary.
   Always True  Sometimes True  Never True

4. If the measure of an angle is represented by \(x\), then the measure of its supplement is represented by \(180 - x\).
   Always True  Sometimes True  Never True

5. If two lines intersect, each pair of vertical angles are supplementary.
   Always True  Sometimes True  Never True

Part B

For each statement you chose as “Sometimes True,” provide one example of when the statement is true and one example of when the statement is not true. Your examples should be a diagram with the angle measurements labeled. If you did not choose any statement as “Sometimes True,” write “None” in the work space below.
Sample Top-Score Response:

Part A
- Statement 1 - Always True
- Statement 2 - Never True
- Statement 3 - Sometimes True
- Statement 4 - Always True
- Statement 5 - Sometimes True

Part B
- Statement 3 - Example of True (two adjacent angles that have a sum of 90°)
- Statement 4 - Example of Not True (two adjacent angles that have a sum of 80°)
- Statement 5 - Example of True (two intersecting lines with all angle measurements of 90°)
- Example of Not True (two lines that intersect with no right angles)

Scoring Rubric:

Responses to this item will receive 0–3 points, based on the following:

3 points: The student has thorough understanding of facts about supplementary, complementary, vertical, and adjacent angles and can justify these facts using examples. The student evaluates all five statements correctly and provides correctly labeled drawings for Statements 3 and 5 if discrete angles are used for the justification.

2 points: The student has good understanding of facts about supplementary, complementary, vertical, and adjacent angles and can justify these facts using examples. The student evaluates all five statements correctly and provides drawings for Statements 3 and 5 but does not include angle measures if discrete angles are used for the justification. OR The student evaluates all five statements correctly and provides a correct drawing for either Statement 3 or Statement 5 but not both. OR The student correctly evaluates Statements 3 and 5 and provides correctly labeled drawings for Statements 3 and 5 but incorrectly evaluates one of the other statements.

1 point: The student has limited understanding of facts about supplementary, complementary, vertical, and adjacent angles and how to justify these facts using examples. This is shown by the following: Answered all 5 statements correctly, but either does not provide drawings for statements 3 and 5. OR The student answered some questions incorrectly, but has either question 3 or 5 correct with supporting drawings and/or angle measures.

0 points: The student shows inconsistent or no understanding of facts about supplementary, complementary, vertical, and adjacent angles and how to justify these facts using examples.
## 2014 SBAC Questions

### Task Model 1

**Response Type:** Equation/Numeric  
**DOK Level:** 1

#### 7.G.4
Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.

#### Evidence Required:
1. The student solves real-life and mathematical problems for the circumference and area of circles.

#### Tools:
Calculator

<table>
<thead>
<tr>
<th>Prompt Features: The student is prompted to give the area of circles for mathematical problems.</th>
</tr>
</thead>
</table>

**Stimulus Guidelines:**
- Context should be familiar to students 12-14 years old.
- Unit label is a measurement of length.
- Item difficulty can be adjusted via these example methods:
  - Radius is a whole number, decimal, fraction, including mixed numbers
  - Diameter is a whole number, decimal, fraction, including mixed numbers
  - Number of computational steps
  - Partial areas or circumferences

**TM1a**

**Stimulus:** The student is presented with the radius, diameter or circumference of a circle in a mathematical context.

**Example Stem:** The radius of a circle is 7.5 centimeters.

Enter the area of the circle, in square centimeters. Round your answer to the nearest hundredth.

**Rubric:** (1 point) The student enters the correct area in a range of correct values (e.g., 176.63 – 176.79).

**Response Type:** Equation/Numeric
Task Model 1

**Response Type:** Equation/Numeric

**DOK Level:** 2

7.G.4 Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.

**Evidence Required:**
1. The student solves real-life and mathematical problems for the circumference and area of circles.

**Tools:** Calculator

**Prompt Features:** The student is prompted to give the area of circles for real-life problems.

**Stimulus Guidelines:**
- Context should be familiar to students 12–14 years old.
- Unit label is a measurement of length.
- Item difficulty can be adjusted via these example methods:
  - Radius is a whole number, decimal, fraction, including mixed numbers.
  - Diameter is a whole number, decimal, fraction, including mixed numbers.
  - Number of computational steps.
  - Partial areas or circumferences.

**TM1b Stimulus:** The student is presented with the radius, diameter or circumference of a circle in a real-life context.

**Example Stem 1:** A circular table top has a radius of 3 feet.

Enter the area, in square feet, of the table top. Round your answer to the nearest hundredth.

**Example Stem 2:** Jill buys two circular pizzas.

![Pizza Diagram]

The small pizza has an 8-inch diameter.

The medium pizza has a 12-inch diameter.

How much greater, in square inches, is the area of the medium pizza than the small pizza? Round your answer to the nearest hundredth.

**Rubric:** (1 point) The student enters the correct area in a range of correct values (e.g., 28.26 – 28.305; 62.80 – 62.90).

**Response Type:** Equation/Numeric
<table>
<thead>
<tr>
<th>TaskModel 1</th>
<th>Prompt Features: The student is prompted to give the circumference of a circle.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Response Type: Equation/Numeric</td>
<td>Stimulus Guidelines:</td>
</tr>
<tr>
<td>DOK Level 2</td>
<td>• Unit label is a measurement of length.</td>
</tr>
<tr>
<td>7.G.4</td>
<td>• Item difficulty can be adjusted via these example methods:</td>
</tr>
<tr>
<td></td>
<td>o Radius is a whole number, decimal, fraction, including mixed numbers.</td>
</tr>
<tr>
<td></td>
<td>o Diameter is a whole number, decimal, fraction, including mixed numbers.</td>
</tr>
<tr>
<td></td>
<td>o Number of computational steps.</td>
</tr>
<tr>
<td></td>
<td>o Partial areas or circumferences.</td>
</tr>
</tbody>
</table>

**Evidence Required:**
1. The student solves real-life and mathematical problems for the circumference and area of circles.

**Tools:** Calculator

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<table>
<thead>
<tr>
<th>Example Stimuli:</th>
<th>The student is presented with the radius or diameter of a circle in a real-life or mathematical context.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Example Stem:</strong></td>
<td>The radius of a circle is 7 centimeters.</td>
</tr>
<tr>
<td><strong>Stimulus:</strong></td>
<td>Enter the circumference of the circle, in centimeters. Round your answer to the nearest hundredth.</td>
</tr>
<tr>
<td><strong>Rubric:</strong></td>
<td>(1 point) The student enters the correct circumference in a range of correct values (e.g., 43.96 - 44.03).</td>
</tr>
<tr>
<td><strong>Response Type:</strong></td>
<td>Equation/Numeric</td>
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</tbody>
</table>

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<thead>
<tr>
<th>Example Stimuli:</th>
<th>The student is prompted to give the radius of a circle given its circumference.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Example Stem:</strong></td>
<td>The circumference of a circle is 31.4 inches.</td>
</tr>
<tr>
<td><strong>Stimulus:</strong></td>
<td>Enter the radius of the circle, in inches. Round your answer to the nearest whole number.</td>
</tr>
<tr>
<td><strong>Rubric:</strong></td>
<td>(1 point) The student enters the correct radius (e.g., 5).</td>
</tr>
<tr>
<td><strong>Response Type:</strong></td>
<td>Equation/Numeric</td>
</tr>
</tbody>
</table>
Task Model 1

Response Type: Equation/Numeric

DOK Level 2

7.G.4 Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.

Evidence Required: The student solves real-life and mathematical problems for the circumference and area of circles.

Tools: Calculator

Prompt Features: The student is prompted to give a fractional part of the area of a circle for both real-life and mathematical problems.

Stimulus Guidelines:
- Context should be familiar to students 12–14 years old.
- Unit label is a measurement of length.
- Item difficulty can be adjusted via these example methods:
  - Radius is a whole number, decimal, fraction, including mixed numbers.
  - Diameter is a whole number, decimal, fraction, including mixed numbers.
  - Number of computational steps.
  - Partial areas or circumferences.

Example Stem 1: A corner shelf has a radius of 10.5 inches and represents \( \frac{1}{4} \) of a circle, as shown.

\[
\begin{align*}
\text{10.5 in} \\
\text{10.5 in}
\end{align*}
\]

Enter the area of the shelf, in square inches. Round your answer to the nearest hundredth.

Example Stem 2: The circumference of the circle is approximately 100.48 centimeters. The shaded region is \( \frac{2}{13} \) of the whole circle.

\[
\begin{align*}
\text{Enter the area of the shaded region, in square centimeters. Round your answer to the nearest hundredth.}
\end{align*}
\]

Rubric: (1 point) The student enters the correct area in a range of correct values (e.g., 86.55 – 86.60; 240.77 – 241.15).

Response Type: Equation/Numeric
Task Model 2  
Response Type: Matching Tables  
DOK Level 1  
7.G.5  
Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.  
Evidence Required:  
2. The student solves real-life and mathematical problems involving angle measures requiring writing and solving equations.  
Tools: Calculator  

Prompt Features: The student solves real-life and mathematical problems involving angle measure including problems requiring writing and solving equations.  

Stimulus Guidelines:  
- Measures of certain angles in the figure can be shown.  
- Measures of angles shown in the figure should be less than 180°.  
- Angle measures can be whole numbers or decimals to the tenths place.  

TM2a  
Stimulus: The student is given a figure involving supplementary, complementary, vertical, and/or adjacent angles that contains a missing angle measure.  

Example Stem: Lines $XU$ and $WY$ intersect at point $A$.  

Based on the diagram, determine whether each statement is true. Select True or False for each statement.  

<table>
<thead>
<tr>
<th>Statement</th>
<th>True</th>
<th>False</th>
</tr>
</thead>
<tbody>
<tr>
<td>An angle supplementary to $\angle WAX$ measures 50°.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>An angle complementary to $\angle WAX$ measures 40°.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>The angle vertical to $\angle YAU$ measures 50°.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Rubric: (1 point) Student correctly identifies each statement as being either true or false (e.g., T, T, T). True choices will be correct angle measures such as False choices will be incorrect angle measures about the computation and comparative statements of the angles.  

Response Type: Matching Tables
Grade 7 Mathematics Item Specification C1 TF

Task Model 2
Response Type: Matching Tables
DOK Level 2
7.G.5
Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.

Evidence Required:
2. The student solves real-life and mathematical problems involving angle measure including problems requiring writing and solving equations.

Tools: Calculator

Prompt Features: The student solves real-life and mathematical problems involving angle measure including problems requiring writing and solving equations.

Stimulus Guidelines:
- Measures of certain angles in the figure can be shown.
- Measures of angles shown in the figure should be less than 180.
- Angle measures can be whole numbers or decimals to the tenths place.

TM2b
Stimulus: The student is given a figure involving supplementary, complementary, vertical, and/or adjacent angles that contains a missing angle measure.

Example Stem: Lines UX and WY intersect at point A.

Based on the diagram, determine whether each statement is true. Select True or False for each statement.

<table>
<thead>
<tr>
<th>Statement</th>
<th>True</th>
<th>False</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \angle XAZ = 180^\circ - \angle ZAY - \angle YAU )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \angle WAZ = \angle WAY - \angle ZAY )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \angle WAU = \angle XAZ - \angle ZAY )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Rubric: (1 point) Student correctly identifies each statement as being either true or false (e.g., T, T, F).

Response Type: Matching Tables
Prompt Features: The student gives the solution to a multi-step problem involving supplementary, complementary, vertical, and/or adjacent angles.

Stimulus Guidelines:
- Variables used represent missing angle measure.
- Angles in the figure can be identified by variables.
- Item difficulty can be adjusted via these example methods:
  - Angle measures are whole numbers.
  - Angle measures are decimals.
  - Angle measures include variables.

TM2c
Stimulus: The student is provided a figure showing supplementary, complementary, vertical, and/or adjacent angles.

Example Stem: Consider this figure.

```
\[ \angle WYZ \]

Enter the measure of \( \angle WYZ \), in degrees.

Rubric: (1 point) The student enters the correct value (e.g., 56).

Response Type: Equation/Numeric
```
**Task Model 2**

**Response Type:** Matching Tables

**DOK Level 2**

7.G.5

Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.

**Evidence Required:**
2. The student solves real-life and mathematical problems involving angle measure including problems requiring writing and solving equations.

**Tools:** Calculator

**Prompt Features:** The student solves an equation for an unknown angle in a figure involving supplementary, complementary, vertical, and/or adjacent angles.

**Stimulus Guidelines:**
- Variables used represent missing angle measure.
- Angles in the figure can be identified by variables.
- Item difficulty can be adjusted via these example methods:
  - Angle measures are whole numbers.
  - Angle measures are decimals.
  - Angle measures include variables.

**Example Stem:** The base of a hexagon lies on ray AB as shown.

```
A
  \[ 3x + 20^\circ \]
  \[ 2x + 10^\circ \]
B
```

Based on the diagram, determine whether each equation is true. Select True or False for each statement.

<table>
<thead>
<tr>
<th>Statement</th>
<th>True</th>
<th>False</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ 3x + 20^\circ = 110^\circ ]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>[ 2x + 10^\circ = 70^\circ ]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>[ 5x + 30^\circ = 90^\circ ]</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Rubric:** (1 point) The student correctly determines each statement as being either true or false (e.g., T, T, T, F).

**Response Type:** Matching Tables
Example Stem 1: This is the floor plan of Julie’s bedroom.

Enter the amount of carpet, in square feet, needed to completely cover Julie’s bedroom floor.

Rubric: (1 point) The student enters the correct area (e.g., 46 1/2).

Response Type: Equation/Numeric
Example Stem 2: The figure shown is created by joining three rectangles.

Enter the area of the figure, in square centimeters. Round to the nearest hundredth.

Rubric: (1 point) The student enters the correct area (e.g., 46.75).

Response Type: Equation/Numeric
**Task Model 7**

**DOK Levels 2, 3**

**Target 6:** Determine conditions under which an argument does and does not apply

**Task Expectations:** The student must determine whether a proposition/conjecture is true for all cases, true for some cases, or not true for any case. In some problems the student must provide justification to support the conclusions.

**Example Item 1 (Grade 7):**
Primary Target 3G (Content Domain 3), Secondary Target 1F (CCSS 7.G.5), Tertiary Target 3A

Determine whether each statement is true for all cases, true for some cases, or not true for any case.

<table>
<thead>
<tr>
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<th>True for some</th>
<th>Not true for any</th>
</tr>
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<td>Two vertical angles form a linear pair.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>If two angles are supplementary and congruent, they are right angles.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>The sum of two adjacent angles is 90°.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>If the measure of an angle is 35°, then the measure of its complement is 55°.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>The measure of an exterior angle of a triangle is greater than every interior angle of the triangle.</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Rubric:** (1 point) The student can classify each statement correctly (e.g., see table above).

**Response Type:** Hot Spot

<table>
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