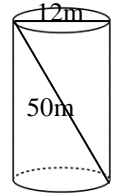




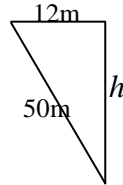
Pythagorean Theorem in 3D

Pythagoras' theorem is often used to find lengths in three-dimensional problems. In these problems we sometimes need to apply it twice.

Example A: A 50m rope is attached inside an empty cylindrical wheat silo of diameter 12m as shown. How high (h) is the wheat silo?



Solution: Pullout the triangle we are considering and apply the Pythagorean Theorem.



$$h^2 + 12^2 = 50^2$$

$$h^2 + 144 = 2500$$

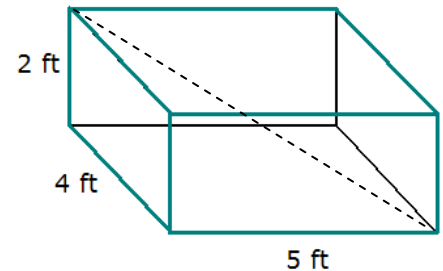
$$h^2 = 2356$$

$$h = \sqrt{2356}$$

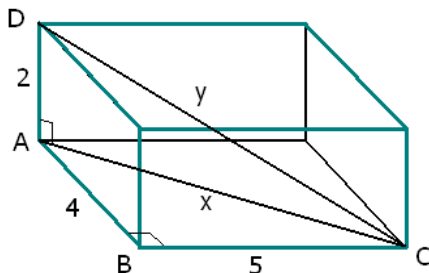
$$h \approx 48.5$$

So, the wheat silo is about 48.5m high.

Example B: Find the length of the longest line segment (called the main diagonal) in the rectangular prism shown on the picture to the right.



Solution: We will apply the Pythagorean Theorem twice. First, label the points and sides that will be used on the picture.



We will find x using the Pythagorean Theorem in triangle ABC . Then we can find y using the Pythagorean Theorem in triangle ACD .

$$\begin{aligned} \text{For } \triangle ABC, \quad & 4^2 + 5^2 = x^2 \\ & 16 + 25 = x^2 \\ & 41 = x^2 \end{aligned}$$

We do not have to simplify x^2 as we need it in the Pythagorean Theorem: For $\triangle ACD$, $2^2 + 41 = y^2$

$$45 = y^2$$

$$\sqrt{45} = \sqrt{y^2}$$

$$\sqrt{45} = y$$

The length of the main diagonal would be $\sqrt{45}$ feet (or approximately 6.7 feet).

Note: Our result is actually $\sqrt{2^2 + 4^2 + 5^2}$. Indeed, we can see that the length of the main diagonal in a rectangular prism with sides x , y , and z is $L = \sqrt{x^2 + y^2 + z^2}$. This is sometimes called the 3-dimensional Pythagorean Theorem.

For the following problems make a sketch. Show your work! Estimate lengths to the nearest tenth.

Problem 1. A cone has a slant height of 17cm and a base radius of 8cm. How high is the cone?

Problem 2. Find the length of the longest nail that could fit entirely within a cylindrical can of radius 3cm and height 8cm.

Problem 3. A cube has sides of length 3cm. Find the length of a diagonal of the cube. Show your work.

Problem 4. Determine the length of the longest piece of timber which could be stored in a rectangular shed 6m by 5m by 2m high.