Claim 1: Concepts and Procedures Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.	
Content Domain: **The Number System**	
Target A [s]: Know that there are numbers that are not rational, and approximate them by rational numbers. (DOK Levels 1, 2) Tasks will ask students to approximate irrational numbers on a number line or as rational numbers with a certain degree of precision. This target may be combined with 8.EE Target B (e.g., by asking students to express the solution to a cube root equation as a point on the number line).	
Standards: 8.NS.A, 8.NS.1, 8.NS.2	**8.NS.A Know that there are numbers that are not rational, and approximate them by rational numbers.** **8.NS.1** Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion, which repeats eventually into a rational number. **8.NS.2** Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2). *For example, by truncating the decimal expansion of* $\sqrt{2}$ *show that* $\sqrt{2}$ *is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.*
Related Below-Grade and Above-Grade Standards for Purposes of Planning for Vertical Scaling: 7.NS.A, 7.NS.2, 7.NS.3 N-RN.A, N-RN.1, N-RN.2,N-RN.B, N-RN.3	**Related Grade 7 Standards** **7.NS.A Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.** **7.NS.2** Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers. a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (–1)(–1) = 1 and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts. b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If *p* and *q* are integers, then –*(p/q)* = (–*p*)/*q* = *p*/(–*q*). Interpret quotients of rational numbers by describing real-world contexts. c. Apply properties of operations as strategies to multiply and divide rational numbers. d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats. **7.NS.3** Solve real-world and mathematical problems involving the four operations with rational numbers.

	Related High School Standards **N-RN.A Extend the properties of exponents to rational exponents.** **N-RN.1** Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define* $5^{1/3}$ *to be the cube root of 5 because we want* $(5^{1/3})^3 = 5^{(1/3)3}$ *to hold, so* $(5^{1/3})^3$ *must equal 5.* **N-RN.2** Rewrite expressions involving radicals and rational exponents using the properties of exponents. **N-RN.B Use the properties of rational and irrational numbers.** **N-RN.3** Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.
DOK Levels:	1, 2
Achievement Level Descriptors:	
RANGE Achievement Level Descriptor (Range ALD) Target A: Know that there are numbers that are not rational and approximate them by rational numbers.	**Level 1** Students should be able to identify square roots of numbers less than 100; identify pi as not rational; and understand that every rational number has a decimal expansion.
	Level 2 Students should be able to identify approximate locations of familiar irrational numbers on a number line; identify numbers as rational or irrational; and convert between fractions and terminating decimals.
	Level 3 Students should be able to use rational approximations of irrational numbers to locate them on a number line and to make numerical comparisons; convert between fractions and repeating decimals; and compare rational numbers.
	Level 4 Students should be able to approximate irrational numbers to a specified level of precision and should be able to use the approximations to solve problems or estimate the value of an expression.
Evidence Required:	1. The student classifies real numbers as rational or irrational. 2. The student converts a repeating decimal into a fraction. 3. The student writes approximations of irrational numbers as rational numbers. 4. The student compares the sizes of irrational numbers by using rational approximations of irrational numbers. 5. The student approximates the locations of irrational numbers on the number line by using rational approximations of irrational numbers.
Allowable Response Types:	Multiple Choice, multiple correct response; Multiple Choice, single correct response; Matching Tables; Equation/Numeric; Drag and Drop; Graphing
Allowable Stimulus Materials:	rational numbers, irrational numbers, expressions involving irrational numbers, explanations of processes, number lines (showing tenths

	or hundredths), square roots, cube roots, pi, repeating bar, repeating and terminating decimals
Construct-Relevant Vocabulary:	rational number, irrational number, repeating decimal, terminating decimal, square root, pi (π)
Allowable Tools:	None
Target-Specific Attributes:	Irrational numbers should be square roots, cube roots, or pi (π). Calculators are not allowed for this target.
Non-Targeted Constructs:	
Accessibility Concerns:	Visual graphics may be difficult or not accessible for students who are blind or visually impaired. The simplest graphics should be used to minimize this issue. Students with dyscalculia may have difficulty with the calculations. Students with visual perceptual disabilities may struggle with answer choices that contain complex number sentences. Students who are visually impaired or blind may need enlarged or brailled text. Students with reading disabilities may struggle with the reading load of word problems. All vocabulary should be at or below grade level to minimize this issue. Students with reading disabilities may need to read the text aloud, or have access to trackers or maskers to follow along. Students with visual processing impairments may benefit from using a tracker or masker when reading. Drag and Drop response types may not be accessible for students who are visually impaired. Consider replacing these response types with multiple choice items for Braille versions. The accommodations listed here are suggestions and could be altered depending on what accommodations will be allowable.

Task Model 1

Response Type: Matching Tables

DOK Level 1

8.NS.1
Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion, which repeats eventually into a rational number.

Evidence Required:
1. The student classifies real numbers as rational or irrational.

Tools: None

Prompt Features: The student classifies numbers as rational or irrational.

Stimulus Guidelines: Item difficulty can be adjusted via these methods:

- Rational numbers are positive; irrational numbers are pi or $\sqrt{2}$.
- Rational numbers can be positive or negative; irrational numbers are in the form $\sqrt{x}$, where x is less than 90.
- Rational numbers are positive and negative integers, mixed numbers, fractions, decimals, or repeating decimals; irrational numbers include numbers with pi and radicals.

TM1
Stimulus: The student is presented with a table of four to five rational and irrational numbers.

Example Stem: Determine for each number whether it is a rational or irrational number.

Number	Rational	Irrational
$\frac{4}{7}$		
$\sqrt{30}$		
$\frac{21}{\sqrt{4}}$		
π		
-27		

Rubric: (1 point) The student correctly classifies each number (e.g., irrational numbers are $\sqrt{30}$ and π, all others are rational).

Response Type: Matching Tables

Task Model 2 **Response Type: Equation/Numeric** **DOK Level 1** **8.NS.1** Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion, which repeats eventually into a rational number. **Evidence Required:** 2. The student converts repeating decimals to fractions. **Tools:** None	**Prompt Features:** The student converts a repeating decimal to a fraction. **Stimulus Guidelines:** Item difficulty can be adjusted via these methods: • Decimals with one repeating digit in the tenths place that may have a whole number (ex. $2.\overline{2}$). • Decimals with multiple repeating digits starting in the tenths place (ex. $0.\overline{24}$, $8.\overline{125}$) decimals begin to repeat in the hundredth or thousandth place (ex. $0.0\overline{42}$, $3.07\overline{6}$). **TM2** **Stimulus:** The student is presented with a decimal with a repeating bar over the last digit(s). **Example Stem:** Enter a fraction equivalent to $0.\overline{2}$. Use only whole numbers for numerators and denominators. **Rubric:** (1 point) Student enters an equivalent fraction to the repeating decimal (e.g., 2/9). **Response Type:** Equation/Numeric

Task Model 3
 	
Response Type: Multiple Choice, single correct response
 	
DOK Level 1
 	
8.NS.2
Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2).
 	
Evidence Required:
3. The student writes approximations of irrational numbers as rational numbers.
 	
Tools: None
 	
Development Notes: An item measuring the "explain" part of this target and standard may be assessed in Claim 3.
	Prompt Features: The student approximates the value of an irrational number to the nearest whole number.
	
	Stimulus Guidelines:
	• Irrational numbers should involve square roots, cube roots, or pi (π).
	• The expression could include addition, subtraction, multiplication, and division operations.
	• Item difficulty can be adjusted via these methods:
	  ○ An irrational number between $\sqrt{2}$ and $\sqrt{100}$.
	  ○ An irrational number or expression and the degree of precision is to the whole number.
	  ○ An irrational expression and the degree of precision is to the tenths.
	
	TM3a
	Stimulus: The student is presented with an irrational number.
	
	Example Stem: Which number is the closest approximation to the value of $\sqrt{167}$?
	
	A. 12
	B. 13
	C. 83
	D. 84
	
	Answer Choices: Distractors include incorrect rounding up or down and misinterpreting the square root sign as "divide by 2".
	
	Rubric: (1 point) The student correctly identifies the closest approximation (e.g., B).
	
	Response Type: Multiple Choice, single correct response

Task Model 3
 	
Response Type: Equation/Numeric
 	
DOK Level 2
 	
8.NS.2
Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2).
 	
Evidence Required:
3. The student writes approximations of irrational numbers as rational numbers.
 	
Tools: None
 	
Development Notes: An item measuring the "explain" part of this target and standard may be assessed in Claim 3.
	Prompt Features: The student approximates the value of an irrational expression.
	
	Stimulus Guidelines:
	• Irrational numbers should involve square roots, cube roots, or pi (π).
	• The expression could include addition, subtraction, multiplication, and division operations.
	• Item difficulty can be adjusted via these methods:
	  ○ An irrational number between $\sqrt{2}$ and $\sqrt{100}$.
	  ○ An irrational number or expression and the degree of precision is to the whole number.
	  ○ An irrational expression and the degree of precision is to the tenths.
	
	TM3b
	Stimulus: The student is presented with an expression that contains an irrational number.
	
	Example Stem: Enter the approximate value of $2\sqrt{47}$ to the nearest tenth.
	
	Rubric: (1 point) Student gives the correct approximation at the specified degree of approximation (e.g., 13.6 or 13.7).
	
	Response Type: Equation/Numeric

Task Model 3
 	
Response Type: Multiple Choice, single correct response
 	
DOK Level 2
 	
8.NS.2
Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2).
 	
Evidence Required:
3. The student writes approximations of irrational numbers as rational numbers.
 	
Tools: None
 	
Development Notes: An item measuring the "explain" part of this target and standard may be assessed in Claim 3.
	Prompt Features: The student identifies the range in which the value of an expression with an irrational number falls.
	
	Stimulus Guidelines:
	• Irrational numbers should involve square roots, cube roots, or pi (π).
	• The expression could include addition, subtraction, multiplication, and division operations.
	• Item difficulty can be adjusted via these methods:
	  ○ An irrational number between $\sqrt{2}$ and $\sqrt{100}$.
	  ○ An irrational number or expression and the degree of precision is to the whole number.
	  ○ An irrational expression and the degree of precision is to the tenths.
	
	TM3c
	Stimulus: The student is presented with an expression that contains an irrational number.
	
	Example Stem: Which range contains the value of $\sqrt{(16 + 9 + 20)}$?
	
	A. between 6.6 and 6.8
	B. between 7.5 and 7.7
	C. between 16.8 and 17.0
	D. between 22.4 and 22.6
	
	Answer Choices: Ranges may be whole numbers or decimals to the tenths. The distractors are ranges that are produced by errors in order of operations, errors in rounding, trying to remove perfect squares from the addends, or interpreting the square root as "divide by 2."
	
	Rubric: (1 point) Student selects the correct range (e.g., A).
	
	Response Type: Multiple Choice, single correct response

Task Model 4
 	
Response Type: Multiple Choice, multiple correct response
 	
DOK Level 2
 	
8.NS.2
Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2).
 	
Evidence Required:
4. The student compares the sizes of irrational numbers by using rational approximations of irrational numbers.
 	
Tools: None
	Prompt Features: The student compares the sizes of irrational numbers.
	
	Stimulus Guidelines:
	• Four or five expressions are to be given as options.
	• At least two of the options must be correct responses.
	• Item difficulty can be adjusted via these methods:
	  ○ Stem and answer choices are rational/irrational numbers such as fractions, decimals to the tenths, radicals up to $\sqrt{30}$, exponents, or numbers with pi (π).
	  ○ Stem and answer choices are a combination of integers, fractions, decimals to the hundredths place, radicals up to $\sqrt{120}$, exponents, and numbers with pi that contains an irrational number.
	  ○ Stem and answer choices are integers, fractions, decimals, radicals, exponents, or numbers with pi involving one operation that contains an irrational number.
	
	TM4a
	Stimulus: The student is presented with an expression containing an irrational number.
	
	Example Stem: Select **all** expressions that have a value greater than $3 + \sqrt{4.25}$.
	
	A. 4π
	B. $\frac{10}{\sqrt{3}}$
	C. $2 + \sqrt{2}$
	D. $5.7 - \frac{6}{\sqrt{20}}$
	
	Rubric: (1 point) Student selects all the appropriate expressions (e.g., A and B).
	
	Response Type: Multiple Choice, multiple correct response

Task Model 4

Response Type: Matching Tables

DOK Level 1

8.NS.2
Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2).

Evidence Required:
4. The student compares the sizes of irrational numbers by using rational approximations of irrational numbers.

Tools: None

Prompt Features: The student selects true statements about the comparison of irrational numbers.

Stimulus Guidelines: Item difficulty can be adjusted via these methods:

- Rational/irrational numbers may be written as fractions, decimals to the tenths, radicals up to $\sqrt{30}$, exponents, or numbers with pi (π).
- Inequalities may contain one operation with an irrational number.

TM4b
Stimulus: The student is presented with an inequality involving an irrational number.

Example Stem: Select True or False to indicate whether each comparison is true.

	True	False
$\frac{4}{7} > \sqrt{19}$		
$\sqrt{40} > 7$		
$\frac{20}{\sqrt{30}} > \frac{2}{3}$		

Rubric: (1 point) Student selects True or False correctly (e.g., F, F, T). In each inequality, one of the sides must contain an irrational number.

Response Type: Matching Tables

Task Model 5

Response Type: Matching Tables

DOK Level 2

8.NS.2
Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2).

Evidence Required:
5. The student approximates the locations of irrational numbers on the number line by using rational approximations of irrational numbers.

Tools: None

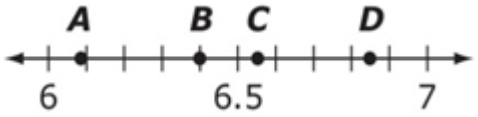
Prompt Features: The student identifies true statements about approximations of irrational numbers.

Stimulus Guidelines: Item difficulty can be adjusted via these methods:

- Using irrational numbers below or above $\sqrt{100}$.
- Using expressions of irrational numbers such as $2\sqrt{2}$.

TM5a
Stimulus: The student is presented with labeled points on a number line.

Example Stem: This number line shows four points labeled *A*, *B*, *C*, and *D*.

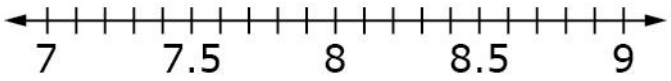


Select True or False for each statement about the number line.

Statement	True	False
The value of $\sqrt{40}$ is located between Point *A* and Point *B*.		
The value of$\sqrt{42}$ is located to the right of Point *D*.		
The value at Point *C* is less than $\sqrt{45}$.		

Rubric: (1 point) Student correctly determines each statement as being either True or False (e.g., T, F, T).

Response Type: Matching Tables

Task Model 5
 	
Response Type: Graphing
 	
DOK Level 1
 	
8.NS.2
Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2).
 	
Evidence Required:
5. The student approximates the locations of irrational numbers on the number line by using rational approximations of irrational numbers.
 	
Tools: None
	Prompt Features: The student is prompted to plot the approximate value of an irrational number onto a number line.
	
	Stimulus Guidelines:
	• Irrational numbers should involve square roots, cube roots, or pi (π).
	• Item difficulty can be adjusted via these methods:
	  ○ approximation should be to whole numbers
	  ○ approximation should be made to the tenths.
	
	TM5b
	Stimulus: The student is presented with a number line and an irrational number.
	
	Example Stem: Use the Add Point tool to approximate the value of $\sqrt{78}$ to the nearest tenth on the number line.
	
	 
	
	Interaction: Student will use the Add Point tool to graph a point on a number line containing snap-to regions at every tic mark. Add Point and Delete tools should be provided.
	
	Rubric: (1 point) Student plots a point at the correct approximation (e.g., 8.8).
	
	Response Type: Graphing

Task Model 5

Response Type: Drag and Drop

DOK Level 1

8.NS.2
Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2).

Evidence Required:
5. The student approximates the locations of irrational numbers on the number line by using rational approximations of irrational numbers.

Tools: None

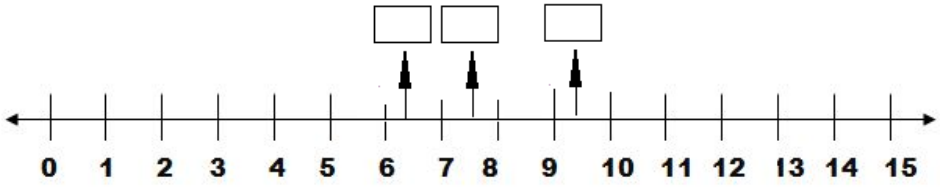
Prompt Features: The student is prompted to drag irrational numbers onto a number line to show the approximate locations of the values.

Stimulus Guidelines:

- Irrational numbers should involve square roots, cube roots, or π.
- The expression could involve any of the four operations.
- Items difficulty can be adjusted via these methods:
  - Expressions are π or in the form of $\sqrt{x}$ where x is a positive integer less than 30.
  - Expressions involving addition, subtraction, or multiplication to π or $\sqrt{x}$ where x is a positive integer less than 30.
  - Expressions may involve division or more than one operation, including an expression under a radical.

TM5c
Stimulus: The student is presented with a number line and three expressions containing irrational numbers.

Example Stem: Drag each expression to the number line to show the approximate value.



$2\sqrt{14}$ 2π $3\sqrt{10}$

Interaction: A palette on the bottom should be given for the three one-time use irrational expressions. Students should drag the expressions into the appropriate boxes on the number line. There should be an arrow from the number line to the box indicating exactly at which tic mark the expression should be placed.

Rubric: (2 points) Student places all three expressions in the correct location (e.g., 2π, $2\sqrt{14}$, $3\sqrt{10}$).
(1 point) Student places two expressions in the correct location.

Response Type: Drag and Drop