Lesson 2: Solving for Unknown Angles Using Equations

Student Outcomes

- Students solve for unknown angles in word problems and in diagrams involving complementary, supplementary, vertical, and adjacent angles.

Classwork

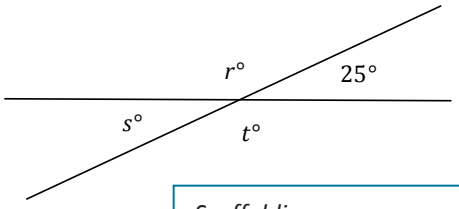
Opening Exercise (5 minutes)

MP.

Opening Exercise

Two lines meet at a point. In a complete sentence, describe the relevant angle relationships in the diagram. Find the values of , , and .

The two intersecting lines form two pairs of vertical angles; , and . Angles and are angles on a line and sum to .



Scaffolding:

Students may benefit from repeated practice drawing angle diagrams from verbal descriptions. For example, "Draw a diagram of two supplementary angles, where one has a measure of ." Students struggling to organize their solution to a problem may benefit from the five-part process of the Exit Ticket in Lesson 1, including writing an equation, explaining the connection between the equation and the situation, and assessing whether an answer is reasonable. This will build conceptual understanding.

In the following examples and exercises, students set up and solve an equation for the unknown angle based on the relevant angle relationships in the diagram. Model the good habit of always stating the geometric reason when you use one. This will be a requirement in Grade 10 geometry.

Example 1 (4 minutes)

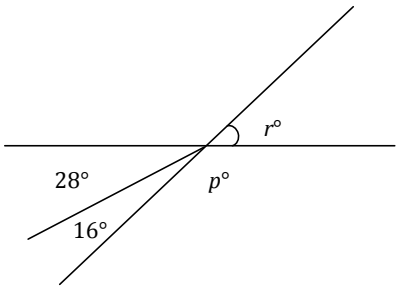
Example 1

Two lines meet at the vertex of a ray. In a complete sentence, describe the relevant angle relationships in the diagram. Set up and solve an equation to find the value of and .

The angle is vertically opposite from and equal to the sum of the angles with measurements and , or a sum of . Angles , , and are angles on a line and sum to .

vert.

on a line



Take the opportunity to distinguish the correct usage of

'supplementary' versus 'angles on a line' in this example. Remind students that 'supplementary' should be used in reference to two angles, whereas 'angles on a line' can be used for two *or more* angles.

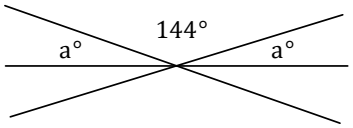
Exercise 1 (4 minutes)

Exercise 1

Three lines meet at a point. In a complete sentence, describe the relevant angle relationship in the diagram. Set up and solve an equation to find the value of .

The two angles and the angle are angles on a line and sum to .

∠s on a line



Example 2 (4 minutes)

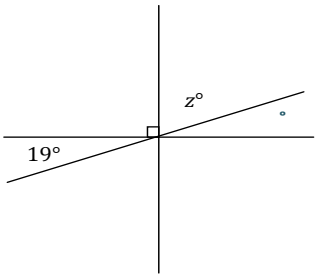
Encourage students to label diagrams as needed to facilitate their solutions. In this example, the label is added to the diagram to show the relationship of with . This addition allows for methodical progress towards the solution.

Example 2

Three lines meet at a point. In a complete sentence, describe the relevant angle relationships in the diagram. Set up and solve an equation to find the value of .

The angles and are complementary and sum to .
Let be the measure of the indicated angle.

vert.
complementary



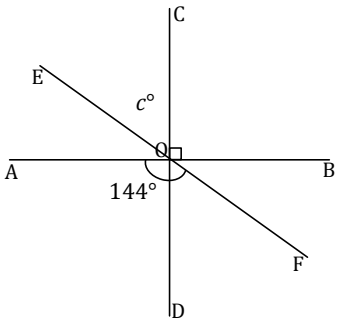
Exercise 2 (4 minutes)

Exercise 2

Three lines meet at a point; . In a complete sentence, describe the relevant angle relationships in the diagram. Set up and solve an equation to determine the value of .

Angle , formed by adjacent angles and , is vertical to and equal in measurement to .
The measurement of is (add).

vert.



Example 3 (4 minutes)

Example 3

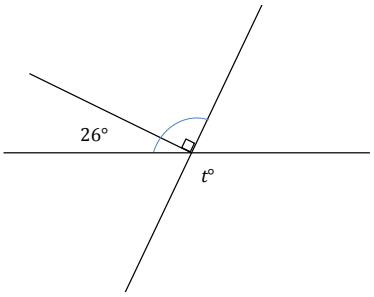
Two lines meet at the vertex of a ray. The ray is perpendicular to one of the lines as shown. In a complete sentence, describe the relevant angle relationships in the diagram. Set up and solve an equation to find the value of .

The measurement of the angle formed by adjacent angles of and is the sum of the adjacent angles. This angle is vertically opposite and equal in measurement to the angle .

Let be the measure of the indicated angle.

add

vert



Exercise 3 (4 minutes)

Exercise 3

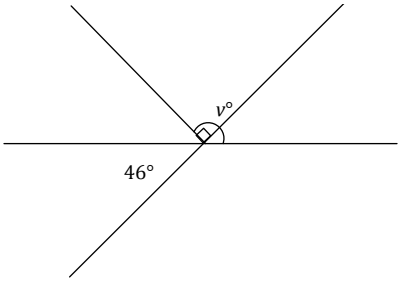
Two lines meet at the vertex of a ray. The ray is perpendicular to one of the lines as shown. In a complete sentence, describe the relevant angle relationships in the diagram. You may add labels to the diagram to help with your description of the angle relationship. Set up and solve an equation to find the value of .

One possible response: Let be the angle vertically opposite and equal in measurement to . The angles and are adjacent angles, and the angle they form together is equal to the sum of their measurements.

Let be the measure of the indicated angle.

vert

add



Example 4 (4 minutes)

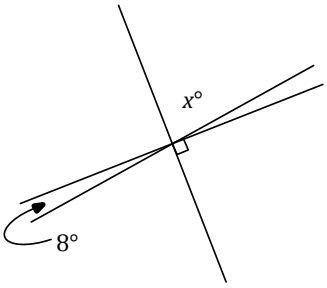
Example 4

Three lines meet at a point. In a complete sentence, describe the relevant angle relationships in the diagram. Set up and solve an equation to find the value of . Is your answer reasonable? Explain how you know.

The angle is vertically opposite from the angle formed by the right angle that contains and shares a common side with an angle.

add and vert

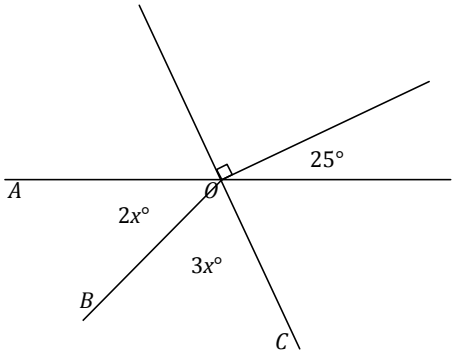
The answer is reasonable because the angle marked by is close to appearing as a right angle.



Exercise 4 (4 minutes)

Exercise 4

Two lines meet at the common vertex of two rays. In a complete sentence, describe the relevant angle relationships in the diagram. Set up and solve an equation to find the value of . Find the measurements of and .



is vertically opposite from the angle formed by adjacent angles and .

add and vert

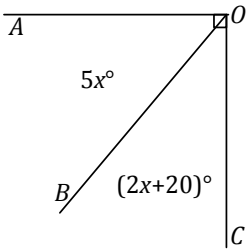
Exercise 5 (4 minutes)

Exercise 5

a. In a complete sentence, describe the relevant angle relationships in the diagram. Set up and solve an equation to find the value of . Find the measurements of and .

and are complementary and sum to .

complementary



b. Katrina was solving the problem above and wrote the equation . Then she rewrote this as . Why did she rewrite the equation in this way? How does this help her to find the value of ?

She grouped the quantity on the right-hand side of the equation similarly to that of the left-hand side. This way, it is clear that the quantity on the right-hand side must be equal to the quantity on the right-hand side.

MP.

Closing (1 minute)

- In every unknown angle problem, it is important to identify the angle relationship(s) correctly in order to set up an equation that will yield the unknown value.
- Check your answer by substituting and/or measuring to be sure it is correct.

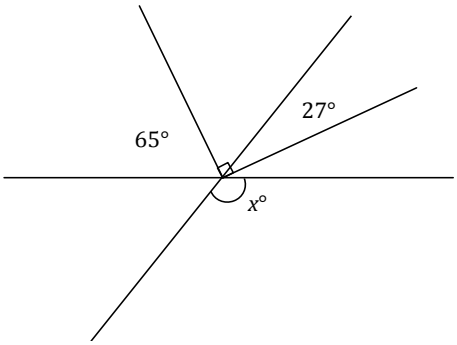
Exit Ticket (3 minutes)

Name _____ Date _____

Lesson 2: Solving for Unknown Angles Using Equations

Exit Ticket

Set up and solve an equation to find the value of . Explain why your answer is reasonable.

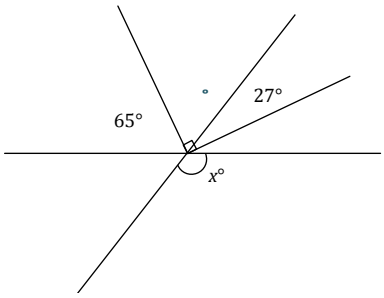


Exit Ticket Sample Solutions

Set up and solve an equation to find the value of . Explain why your answer is reasonable.

There are multiple solutions to the problem. Two possible solutions are shown below.

or

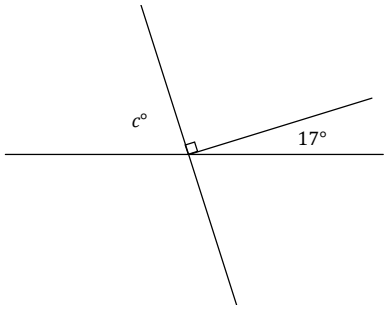


The answers seem reasonable because a rounded value of as and a rounded value of its adjacent angle as , yields a sum of , which is close to the calculated answer.

Problem Set Sample Solutions

1. Two lines meet at the vertex of a ray. Set up and solve an equation to find the value of .

on a line



Scaffolding:

Students struggling to organize their solution may benefit from prompts such as the following: Write an equation to model this situation. Explain how your equation describes the situation. Solve and interpret the solution. Is it reasonable?

Scaffolded solutions:

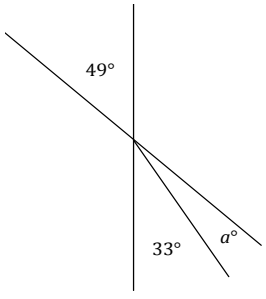
a. Use equation above.

b. The angle marked , the right angle, and the angle with measurement are angles on a line and their measurements sum to .

c. Use solution above. The answer seems reasonable because it looks like it has a measurement a little lesson than a angle.

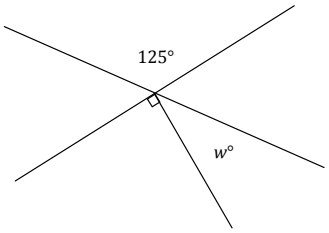
2. Two lines meet at the vertex of a ray. Set up and solve an equation to find the value of . Explain why your answer is reasonable.

add and vert



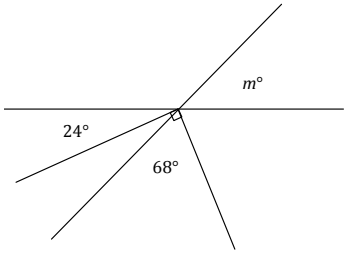
The answers seem reasonable because a rounded value of as and a rounded value of its adjacent angle as , yields a sum of , which is close to the rounded value of the measurement of the vertical angle.

3. Two lines meet at the vertex of a ray. Set up and solve an equation to find the value of .



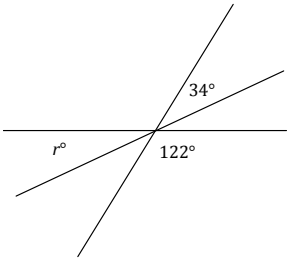
add and vert

4. Two lines meet at the common vertex of two rays. Set up and solve an equation to find the value of .



add and vert

5. Three lines meet at a point. Set up and solve an equation to find the value of .

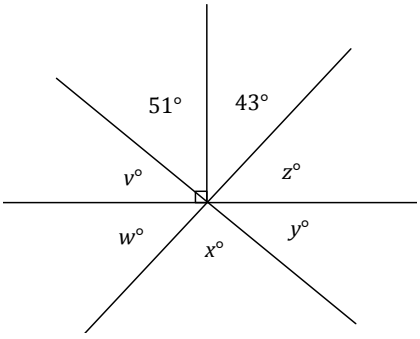


on a line and vert

6. Three lines meet at the vertex of a ray. Set up and solve an equation to find the value of each variable in the diagram.

complementary

on a line



vert

vert

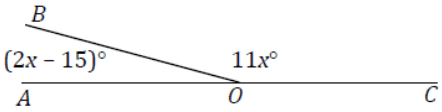
vert

7. Set up and solve an equation to find the value of . Find the measurement of and of .

Scaffolding:

Students struggling to organize their solution may benefit from prompts such as the following:

- Write an equation to model this situation. Explain how your equation describes the situation. Solve and interpret the solution. Is it reasonable?



supplementary

The measurement of
The measurement of

Scaffolded solutions:

a. Use equation above.

b. The marked angles are angles on a line and their measurements sum to .

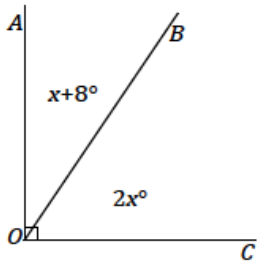
c. The answers seem reasonable because once is substituted, the respective angle measurements seem appropriate since is acute and is obtuse.

8. Set up and solve an equation to find the value of . Find the measurement of and of .

complementary

The measurement of – –

The measurement of – –

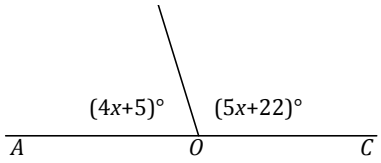


9. Set up and solve an equation to find the value of . Find the measurement of

on a line

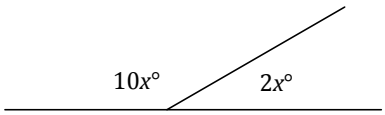
The measurement of

The measurement of



10. Write a verbal problem that models the following diagram. Then solve for the two angles.

One possible response: Two angles are supplementary. The measurement of one angle is five times the other. Find the measurements of both angles.



on a line

The measurement of Angle 1:

The measurement of Angle 2: