Lesson 22: Solving Equations Using Algebra

Student Outcomes

- Students use algebra to solve equations (of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers); using techniques of making zero (adding the additive inverse) and making one (multiplying by the multiplicative inverse) to solve for the variable.
- Students identify and compare the sequence of operations used to find the solution to an equation algebraically, with the sequence of operations used to solve the equation with tape diagrams. They recognize the steps as being the same.
- Students solve equations for the value of the variable using inverse operations; by making zero (adding the additive inverse) and making one (multiplying by the multiplicative inverse).

Classwork

In this lesson, you will transition from solving equations using tape diagrams to solving equations algebraically by *making zero* (using the additive inverse) and *making one* (using the multiplicative inverse). Justify your work by identifying which algebraic property you used for each step in solving the problems. Explain your work by writing out how you solved the equations step by step and relate each step to those used with a tape diagram.

Example 1 (10 minutes): Yoshiro's New Puppy

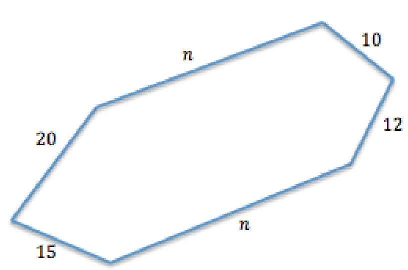
MP.1

Use this problem to emphasize the use of illustrating the problem and solving an algebraic problem with a tape diagram. Drawing the puppy yard will help show students the meaning of perimeter and make sense of the problem.

Example 1: Yoshiro's New Puppy

Yoshiro has a new puppy. She decides to create an enclosure for her puppy in her back yard. The enclosure is in the shape of a hexagon (six-sided polygon) with one pair of opposite sides running the same distance along the length of two parallel flowerbeds. There are two boundaries at one end of the flowerbeds that are 10 ft. and 12 ft., respectively, and at the other end, the two boundaries are 15 ft. and 20 ft., respectively. If the perimeter of the enclosure is 137 ft., what is the length of each side that runs along the flowerbed?

- What is the general shape of the puppy yard? Draw a sketch of the puppy yard.



Scaffolding:

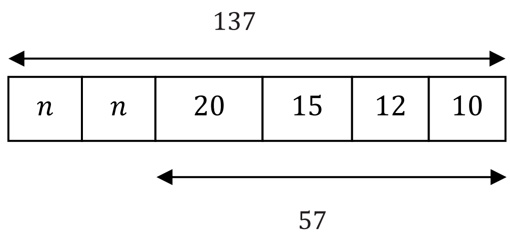
Have students write out in words what they will do to help them transition from words to algebraic symbols.

MP.4

- Write an equation that would model finding the perimeter of the puppy yard.
  - *The sum of the lengths of the sides = Perimeter*

    $n + n + 10 + 12 + 20 + 15 = 137$

- Model and solve this equation with a tape diagram.
  - *Sample response:*



$137 - 57 = 80; 80 \div 2 = 40$

Now review *making zero* in an equation and *making one* in an equation. Explicitly connect *making zero* and *making one* in the next question to the bar model diagram. Subtracting 57 from 137 in the bar diagram is the same as using the subtraction property of equality (i.e., subtracting 57 from both sides of the equation in order to make zero). Dividing 80 by 2 because we want to find the size of two equal groups that total 80 is the same as using the multiplicative property of equality (i.e., multiplying each side of the equation by $\frac{1}{2}$ to make one group of n).

- Use algebra to solve this equation.
  - *First, use the additive inverse to find out what the lengths of the two missing sides are together. Then, use the multiplicative inverse to find the length of one of the two equal sides. Sum of missing sides + Sum of known sides = Perimeter*

If: $2n + 57 = 137$	
Then: $2n + 57 - 57 = 137 - 57$	*Subtraction Property of Equality*
If: $2n + 0 = 80$	
Then: $2n = 80$	*Additive Identity*
If: $2n = 80$	
Then: $\frac{1}{2}(2n) = \frac{1}{2}(80)$	*Multiplication Property of Equality*
If: $1n = 40$	
Then: $n = 40$	*Multiplicative Identity*

- Does your solution make sense in this context? Why?
  - *Yes, 40 ft. makes sense because when you replace the two missing sides of the hexagon with 40 in the number sentence (* $40 + 40 + 10 + 12 + 20 + 15 = 137$*), the lengths of the sides reach a total of 137.*

Example 2 (10 minutes): Swim Practice

Example 2: Swim Practice

Jenny is on the local swim team for the summer and has swim practice four days per week. The schedule is the same each day. The team swims in the morning and then again for 2 hours in the evening. If she swims 12 hours per week, how long does she swim each morning?

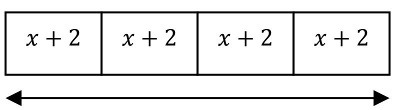
- Write an algebraic equation to model this problem. Draw a tape diagram to model this problem.
  - *Let x = number of hours of swimming each morning*

    "Model" days per week (number of hours swimming a.m. and p.m.) = hours of swimming total

    $$4 \qquad (x+2) \qquad = \ 12$$

Recall in the last problem that students used *making zero* first, and then *making one* to solve the equation. Explicitly connect *making zero* and *making one* in the previous statement to the tape diagram.

- Solve the equations algebraically and graphically with the help of the tape diagram.
  - *Sample response:*

 12 $12-8 = 4$ $\frac{4}{4} = 1$ *Jenny swims 1 hour each morning.*	*Algebraically* *If:* $4\,(x+2)=12$ *Then:* $\frac{1}{4}\left(4\,(x+2)\right)=\frac{1}{4}(12)$ *If:* $1(x+2)=3$ *Then:* $x+2=3$ *If:* $x+2=3$ *Then:* $x+2-2=3-2$ *If:* $x+0=1$ *Then:* $x=1$	*Multiplication property of equality using the multiplicative inverse of 4* *Multiplicative identity* *Subtraction property of equality for the additive inverse of 2* *Additive identity*

- Does your solution make sense in this context? Why?
  - *Yes, if Jenny swims 1 hour in the morning and 2 hours in the evening for a total of 3 hours per day and swims 4 days per week, then $3(4) = 12$ hours for the entire week.*

Exercises (15 minutes)

Exercises

Solve each equation algebraically using if-then statements to justify each step.

1. $5x + 4 = 19$

If: $5x + 4 = 19$

Then: $5x + 4 - 4 = 19 - 4$ — *Subtraction property of equality for the additive inverse of* 4

If: $5x + 0 = 15$

Then: $5x = 15$ — *Additive identity*

If: $5x = 15$

Then: $\frac{1}{5}(5x) = \left(\frac{1}{5}\right)15$ — *Multiplication property of equality for the multiplicative inverse of* 5

If: $1x = 3$

Then: $x = 3$ — *Multiplicative identity*

2. $15x + 14 = 19$

If: $15x + 14 = 19$

Then: $15x + 14 - 14 = 19 - 14$ — *Subtraction property of equality for the additive inverse of* 14

If: $15x + 0 = 5$

Then: $15x = 5$ — *Additive Identity*

If: $15x = 5$

Then: $\frac{1}{15}(15x) = \left(\frac{1}{15}\right)5$ — *Multiplication property of equality for the multiplicative inverse of* 15

If: $1x = \frac{1}{3}$

Then: $x = \frac{1}{3}$ — *Multiplicative identity*

3. **Claire's mom found a very good price on a large computer monitor. She paid $\$325$ for a monitor that was only $\$65$ more than half the original price. What was the original price?**

x: the original price of the monitor

If: $\frac{1}{2}x + 65 = 325$

Then: $\frac{1}{2}x + 65 - 65 = 325 - 65$ — *Subtraction property of equality for the additive inverse of* 65

If: $\frac{1}{2}x + 0 = 260$

Then: $\frac{1}{2}x = 260$ — *Additive identity*

If: $\frac{1}{2}x = 260$

Then: $(2)\frac{1}{2}x = (2)260$ — *Multiplication property of equality for the multiplicative inverse of* $\frac{1}{2}$

If: $1x = 520$

Then: $x = 520$ — *Multiplicative identity*

The original price was $\$520$*.*

4. $2(x+4)=18$

If: $2(x+4)=18$

Then: $\frac{1}{2}[2\ (x+4)]=\frac{1}{2}(18)$ — *Multiplication property of equality using the multiplicative inverse of* 2

If: $1(x+4)=\ 9$

Then: $x+4=9$ — *Multiplicative identity*

If: $x+4=9$

Then: $x+4-4=9-4$ — *Subtraction property of equality for the additive inverse of* 4

If: $x+0=5$

Then: $x=5$ — *Additive identity*

5. **Ben's family left for vacation after his Dad came home from work on Friday. The entire trip was 600 mi. Dad was very tired after working a long day and decided to stop and spend the night in a hotel after 4 hours of driving. The next morning, Dad drove the remainder of the trip. If the average speed of the car was 60 miles per hour, what was the remaining time left to drive on the second part of the trip? Remember: Distance = rate multiplied by time.**

m*: the number of miles driven on the second day*

$60\ (m+4)=600$

If: $60\ (m+4)=600$

Then: $\left(\frac{1}{60}\right)60\ (m+4)=\left(\frac{1}{60}\right)600$ — *Multiplication property of equality for the multiplicative inverse of* 60

If: $1\ (\ m+4)=10$

Then: $m+4=10$ — *Multiplicative identity*

If: $m+4=10$

Then: $m+4-4=10-4$ — *Subtraction property of equality for the additive inverse of* 4

If: $m+0=6$

Then: $m=6$ — *Additive identity*

There were 6 *hour left to drive.*

Closing (5 minutes)

- What do we mean when we say "solve the equation $6x-8=40$?"
  - *Find the value of the variable to make the number sentence true.*
- What property allows us to add 8 to both sides?
  - *Addition property of equality*
- What role does the additive inverse play in solving this equation, and how can you model its use with the tape diagram?
  - *The additive inverse allows us to make zero. This is demonstrated on the tape diagram when we subtract numerical values and there are no numerical values left.*
- What role does the multiplicative inverse play in solving this equation, and how can you model its use with the tape diagram?
  - *The multiplicative inverse allows us to make one. This is demonstrated on the tape diagram because you can see the number of equal boxes or equal parts.*

- What does this equation look like when modeled using a tape diagram?
  - *Answers will vary because it depends on what type of equation we are modeling.*

Lesson Summary

We work backwards to solve an algebraic equation. For example, to find the value of the variable in the equation $6x - 8 = 40$:

1. **Use the addition property of equality to add the opposite of -8 to each side of the equation to arrive at $6x - 8 + 8 = 40 + 8$.**
2. **Use the additive inverse property to show that $-8 + 8 = 0$; thus, $6x + 0 = 48$.**
3. **Use the additive identity property to arrive at $6x = 48$.**
4. **Then use the multiplication property of equality to multiply both sides of the equation by $\frac{1}{6}$ to get: $\left(\frac{1}{6}\right)6x = \left(\frac{1}{6}\right)48$.**
5. **Then use the multiplicative inverse property to show that $\frac{1}{6}(6) = 1$; thus, $1x = 8$.**
6. **Use the multiplicative identity property to arrive at $x = 8$.**

Exit Ticket (5 minutes)

Name ____ Date____________________

Lesson 22: Solving Equations Using Algebra

Exit Ticket

Susan and Bonnie are shopping for school clothes. Susan has $\$50$ and a coupon for a $\$10$ discount at a clothing store where each shirt costs $\$12$.

Susan thinks that she can buy three shirts, but Bonnie says that Susan can buy five shirts. The equations they used to model the problem are listed below. Solve each equation algebraically, justify your steps, and determine who is correct and why.

Susan's Equation	Bonnie's Equation
$12n + 10 = 50$	$12n - 10 = 50$

Exit Ticket Sample Solutions

Susan and Bonnie are shopping for school clothes. Susan has $\$50$ and a coupon for a $\$10$ discount at a clothing store where each shirt costs $\$12$.

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Susan's Equation	Bonnie's Equation
$12n + 10 = 50$	$12n - 10 = 50$

Bonnie is correct. The equation that would model this situation is $12n - 10 = 50$. Solving this equation would involve "Making zero" by adding 10. And by doing so, $12n - 10 + 10 = 50 + 10$, we arrive at $12n = 60$. So, if a group of shirts that cost $\$12$ each totals $\$60$, then there must be five shirts, since $\frac{60}{12}$ equals 5.

Bonnie's Equation:

$12n - 10 = 50$	
$12n - 10 + 10 = 50 + 10$	*Addition property of equality for the additive inverse of -10*
$12n + 0 = 60$	
$12n = 60$	*Additive identity*
$\left(\frac{1}{12}\right)12n = \left(\frac{1}{12}\right)60$	*Multiplication property of equality using the multiplicative inverse of 12*
$1n = 5$	
$n = 5$	*Multiplicative identity*

Susan's Equation:

$12n + 10 = 50$	
$12n + 10 - 10 = 50 - 10$	*Subtraction property of equality for the additive inverse of 10*
$12n + 0 = 40$	
$12n = 40$	*Additive identity*
$\left(\frac{1}{12}\right)12n = \left(\frac{1}{12}\right)40$	*Multiplication property of equality using the multiplicative inverse of 12*
$1n = 3\frac{1}{3}$	
$n = 3\frac{1}{3}$	*Multiplicative identity*

Problem Set Sample Solutions

For each problem below, explain the steps in finding the value of the variable. Then find the value of the variable, showing each step. Write if-then statements to justify each step in solving the equation.

1. $7(m+5) = 21$

Multiply both sides of the equation by $\frac{1}{7}$, then subtract 5 from both sides of the equation; $m = -2$.

If: $7(m+5) = 21$

Then: $\frac{1}{7}[7(m+5)] = \frac{1}{7}(21)$ — *Multiplication property of equality using the multiplicative inverse of 7*

If: $1\,(m+5) = 3$

Then: $m+5 = 3$ — *Multiplicative identity*

If: $m+5 = 3$

Then: $m+5-5 = 3-5$ — *Subtraction property of equality for the additive inverse of 5*

If: $m+0 = -2$

Then: $m = -2$ — *Additive identity*

2. $-2v+9 = 25$

Subtract 9 from both sides of the equation and then multiply both sides of the equation by $-\frac{1}{2}$; $v = -8$.

If: $-2v+9 = 25$

Then: $-2v+9-9 = 25-9$ — *Subtraction property of equality for the additive inverse of 9*

If: $-2v+0 = 16$

Then: $-2v = 16$ — *Additive identity*

If: $-2v = 16$

Then: $-\frac{1}{2}(-2v) = -\frac{1}{2}(16)$ — *Multiplication property of equality using the multiplicative inverse of -2*

If: $1\,v = -8$

Then: $v = -8$ — *Multiplicative identity*

3. $\frac{1}{3}y - 18 = 2$

Add 18 to both sides of the equation and then multiply both sides of the equation by 3; $y = 60$.

If: $\frac{1}{3}y - 18 = 2$

Then: $\frac{1}{3}y - 18 + 18 = 2 + 18$ *Addition property of equality for the additive inverse of* -18

If: $\frac{1}{3}y + 0 = 20$

Then: $\frac{1}{3}y = 20$ *Additive identity*

If: $\frac{1}{3}y = 20$

Then: $3\left(\frac{1}{3}y\right) = 3\,(20)$ *Multiplication property of equality using the multiplicative inverse of* $\frac{1}{3}$

If: $1\,y = 60$

Then: $y = 60$ *Multiplicative identity*

4. $6 - 8p = 38$

Subtract 6 from both sides of the equation and then multiply both sides of the equation by $-\frac{1}{8}$; $p = -4$.

If: $6 - 8p = 38$

Then: $6 - 6 - 8p = 38 - 6$ *Subtraction property of equality for the additive inverse of* 6

If: $0 + (-8p) = 32$

Then: $-8p = 32$ *Additive identity*

If: $-8p = 32$

Then: $\left(-\frac{1}{8}\right)(-8p) = \left(-\frac{1}{8}\right)32$ *Multiplication property of equality using the multiplicative inverse of* -8

If: $1\,p = -4$

Then: $p = -4$ *Multiplicative identity*

5. $15 = 5k - 13$

Add 13 to both sides of the equation and then multiply both sides of the equation by $\frac{1}{5}$; $k = 5.6$.

If: $15 = 5k - 13$

Then: $15 + 13 = 5k - 13 + 13$ *Addition property of equality for the additive inverse of* -13

If: $28 = 5k + 0$

Then: $28 = 5k$ *Additive identity*

If: $28 = 5k$

Then: $\left(\frac{1}{5}\right)28 = \left(\frac{1}{5}\right)5k$ *Multiplication property of equality using the multiplicative inverse of* 5

If: $5.6 = 1k$

Then: $5.6 = k$ *Multiplicative identity*