Lesson 21: If-Then Moves with Integer Number Cards

Student Outcomes

- Students understand that if a number sentence is true and we make any of the following changes to the number sentence, the resulting number sentence will be true:
  i. Adding the same number to both sides of the equation

     If $a = b$, then $a + c = b + c$.

  ii. Subtracting the same number from both sides of the equation

     If $a = b$, then $a - c = b - c$.

  iii. Multiplying each side of the equation by the same number

     If $a = b$, then $a(c) = b(c)$.

  iv. Dividing each side of the equation by the same nonzero number

     If $a = b$ and $c \neq 0$, then $a \div c = b \div c$.
- Students revisit the integer game to justify the above referenced if-then statements.

Classwork

Exploratory Challenge (25 minutes): Integer Game Revisited

Pass out three integer number cards to each student, using integers from -2 to 2. Have students, on their student pages, record their cards and their total score (sum). The scores will be between -6 and 6, inclusive. If there are more than 13 students, at least two will have the same score.

Have students find a classmate with the same score, and have them sit next to each other. Students with tied scores should compare their initial cards, noting they are probably different cards with the same sum.

MP.2

Select a pair of students with equal sums and have them write their cards and scores on the board. Continue playing the game with the following changes. Have students, in their student materials, describe the event, record their new sums, and write overall conclusions using if-then statements based on each event of the game.

Exploratory Challenge: Integer Game Revisited

Let's investigate what happens if a card is added or removed from a hand of integers.

My Cards:

My Score:

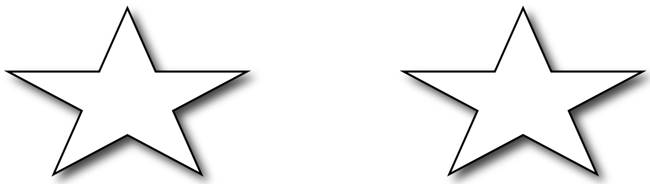
MP.2

Event 1

Give each pair of students two more integer cards (one for each student) containing the same positive value and ask them to record the change and the resulting score. (For instance, a "3" card is given to each partner, both of whom had a previous card total of -1, and both students determine that their card totals remain equal, as now they each have a score of 2.) Repeat this process with one minor change, this time both students receive one integer card containing the same negative value. Have students record their new scores and, after comparing with their partners, write a conclusion using an if-then statement.

Event 1

My new score:



Conclusion:

Possible solution:

	Partner 1			Partner 2		
Original cards:	-1	2	-2	0	1	-2
Original score:		-1			-1	
Event 1 (both partners receive the card 2 and -1)						
New score:		1 *and* -2			1 *and* -2	

Conclusion: *If the sums are equal then a negative or positive number added to the sums will remain equal.*

Series of questions leading to the conclusion:

- Were your scores the same when we began?
  - *Yes.*
- Did you add the same values to your hand each time?
  - *Yes.*
- Did the value of your hand change each time you added a new card?
  - *Yes.*
- Was the value of your hand still the same as your partner's after each card was added?
  - *Yes.*
- Why did the value of your hand remain the same after you added the new cards?
  - *We started with the same sum; therefore, when we added a new card, we had equivalent expressions, which resulted in the same sum.*
- Since your original cards were different but your original sum was the same, write a conclusion that was exemplified by this event.
  - *If the original sums were equal you can add a number, either positive or negative, and the sums will remain equal.*

Event 2

Pick either the same pair of students or another pair who have original sums that are equal AND have at least one identical card. If possible, pick two groups to go to the board. One group will have an identical positive card, the other will have an identical negative card.

> **Event 2**
>
> **My new score:**
>
> **Conclusion:**

If there are two students without the same scores, then use the example that follows.

Student 1: $-2, -1, 2$

Student 2: $0, -2, 1$

Instruct students to remove the identical card from their partner's hand and record their new score. In the student materials, students are asked to describe the event, record their new scores, compare their scores to their partners', write numerical expressions based on the cards, and write overall conclusions based on the event.

- Compare each of your cards to your partner's. Do you have the exact same two cards remaining?
  - *Probably not*
- Compare your new sum to your partner's new sum. What happened?
  - *The sums stayed the same.*
- Write a conclusion that explains what happens when the sums of your cards were the same when the same card is removed.
  - *If the original sums were equal, you can subtract a number, either positive or negative, and the sums will remain equal.*

Sample solution:

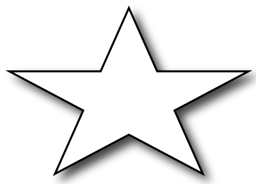
	Partner 1	*Partner 2*
	$\mathbf{-2, -1, 2}$	$\mathbf{0, -2, 1}$
Score	$\mathbf{-1}$	$\mathbf{-1}$
Remove identical cards, remove -2.		
New score:	$\mathbf{1}$	$\mathbf{1}$
Numerical expression:	$\mathbf{-2 + -1 + 2 - -2}$	$\mathbf{0 + -2 + 1 - -2}$
Conclusion:	*If the original sums are equal, you can subtract a number, either positive or negative, and the sums will remain equal.*	

Event 3

Instruct students to look at their original three cards. Double or triple (if there are enough cards) each student's cards with cards matching their original cards. In the student materials, students are asked to describe the event, write the sum as a numerical expression, record the new score, compare it to their partner's, and write an overall conclusion based on the event.

Event 3

My new score:



Expression:

Conclusion:

Possible solution:

Original cards	$1, \quad 2, \quad 2$		
Score	5		
Triple the cards.			
New score	15		
Numerical expression:	$1+2+2+1+2+2+1+2+2$	*or*	$3(1+2+2)$
Conclusion:	15		15

If the original sums are equal, you can multiply or divide the sum by a number and the result will remain the same.

- Compare your original sum to your new sum. What happened?
  - *It is doubled or tripled (if enough cards).*
- Compare your new sum to your partner's new sum. What happened?
  - *They are the same.*
- Look at your numerical expression to find the sum. For students who used only addition or repeated addition, look to see how you could have multiplied. For students who multiplied, what property is applied to get the solution?
  - *Repeated addition could be written as multiplication. The distributive property is then applied to simplify the expression.*
- Write a conclusion about the effects of multiplying a sum by a number.
  - *If the sums of two sets of numbers are equal, then when those numbers are multiplied by another number, the sums will be multiplied by the same number and remain equal.*

Event 4

Select a pair of students with sums of either 4 or -4 to come to the board. Now give them both integer cards with the same non-zero value. Instruct the students to divide the original sum by the new card. In their student materials, students are to describe the event, write a numerical expression, and write a conclusion based on the results shown at the front of the class.

- Compare your cards to your partner's. What can you conclude about your original cards and sum?
  - *Original cards are probably different, but the sums are the same.*
- Compared to your partner's, what happened to the sum when you divided by the same integer card?
  - *The sums are different from the original sums but remain equal to each other.*
- Write a conclusion that describes the effects of dividing equal sums by an identical number.
  - *If the sums are the same, then the quotient of the sums will remain equal when both are divided by the same rational number.*

Scaffolding:

This is an additional option for teachers with proficient students.

- Instruct one person from the pair to put together as many cards as possible so that the sum of the numbers on the cards is between −2 and 2. Have students make the following trade: If one person has a card equal to the value of the new sum, then trade the one card whose value is the sum for ALL of the other cards giving that sum. Calculate the new sum of remaining original cards with ALL of the new cards. In the student materials, students are to describe the event and summarize the results.

Event 4

Expression:

Conclusion:

Possible solution:

	Partner 1	*Partner 2*
Original cards:	$2,\ 2,\ 0$	$2,\ 1,\ 1$
Score:	4	4
Given card value:	-2	-2
Quotient:	-2	-2
Numerical expression:	$\frac{(2\ +\ 2\ +\ 0)}{-2}$	$\frac{(2+1+1)}{-2}$

Conclusion: ***If the sums are the same, then the quotient of the sums will remain equal when both are divided by the same rational number.***

Discussion

Discuss the overall conclusions that if two quantities are equal, then you can add, subtract, multiply, or divide a number to both quantities and the resulting quantities will be equal.

- Explain why the sum remains the same if you received many more cards.
  - *The cards you received in total were equal to the card you traded. You may have received many more cards, but the overall sum did not change because what you gave away was the same as what you gained.*

Exercises (10 minutes)

Have students complete the first row of the table individually and then compare their results with a partner.

Exercises

1. The table below shows two hands from the Integer Game and a series of changes that occurred to each hand. Part of the table is completed for you. Complete the remaining part of the table, then summarize the results.

	Hand 1	Result	Hand 2	Result
Original	$1+(-4)+2$	-1	$0+5+(-6)$	-1
Add 4	$(1+(-4)+2)+4$	3	$(0+5+(-6))+4$	3
Subtract 1	$((1+(-4)+2)+4)-1$	2	$((0+5+(-6))+4)-1$	2
Multiply by 3	$3(((1+(-4)+2)+4)-1)$	6	$3(((0+5+(-6))+4)-1)$	6
Divide by 2	$(3(((1+(-4)+2)+4)-1))\div 2$	3	$(3(((0+5+(-6))+4)-1))\div 2$	3

Since the sums of each original hand are the same, cards can be added, subtracted, multiplied, and divided and the sums will remain the equal to each other.

Perform each of the indicated operations to each expression, compare the new results, and write a conclusion.

- Does it matter if you perform the operation to the original numerical expression or to the original answer?
  - *It does not matter. Doing it both ways would be a good check.*

2. Complete the table below using the multiplication property of equality.

	Original expression and result	Equivalent expression and result
	$3+(-5)=-2$	$-4+2=-2$
Multiply both expressions by -3	$-3(3+-5)=-3(-2)=6$	$-3(-4+2)=-3(-2)=6$
Write a conclusion using if-then	*If* $3+-5=-4+2$, *then* $-3(3+5)=-3(-4+2)$	

Closing (2 minutes)

Describe additional questions.

- While playing the Integer Game, you and your partner each add a card with the same value to your hand. After doing this, you and your partner have the same score. How is this possible?
  - *This is only possible if we started with equivalent sums.*
- While playing the Integer Game, you and your partner have equal scores before and after removing a card from each of your hands. How is this possible?
  - *This is only possible if both partners removed the same card from their hand.*

Lesson Summary

- **If a number sentence is true, $a = b$, and you add or subtract the same number from both sides of the equation, then the resulting number sentence will be true.**
- **If a number sentence is true, $a = b$, and you multiply both sides of the equation by the same number, then the resulting number sentence will be true.**
- **If a number sentence is true, $a = b$, and you divide both sides of the equation by the same non-zero number, then the resulting number sentence will be true.**

Exit Ticket (8 minutes)

Name ____ Date____________________

Lesson 21: If-Then Moves with Integer Number Cards

Exit Ticket

Compare the two expressions: Expression 1: $6 + 7 + -5$

Expression 2: $-5 + 10 + 3$

1. Are the two expressions equivalent? How do you know?

2. Subtract -5 from each expression. Write the new numerical expression, and write a conclusion as an if-then statement.

3. Add 4 to each expression. Write the new numerical expression, and write a conclusion as an if-then statement.

4. Divide each expression by -2. Write the new numerical expression, and write a conclusion as an if-then statement.

Exit Ticket Sample Solutions

Compare the two expressions. **Expression 1:** $6 + 7 + -5$

Expression 2: $-5 + 10 + 3$

1. **Are the two expressions equivalent? How do you know?**

   Yes the expressions are equivalent because expression 1 is equal to 8 and expression 2 is equal to 8, as well. When two expressions evaluate to the same number they are equivalent.

2. **Subtract -5 from each expression. Write the new numerical expression, and write a conclusion as an if-then statement.**

   Expression 1: $6 + 7 + -5 - (-5)$ *Expression 2:* $-5 + 10 + 3 - (-5)$

   13 13

   If $6 + 7 + -5 = -5 + 10 + 3$, *then* $6 + 7 + -5 - (-5) = -5 + 10 + 3 - (-5)$.

   If expression 1 $=$ *expression* 2, *then* $($*expression* $1 - (-5)) = ($*expression* $2 - (-5))$.

3. **Add 4 to each expression. Write the new numerical expression, and write a conclusion as an if-then statement.**

   Expression 1: $6 + 7 + -5 + 4$ *Expression 2:* $-5 + 10 + 3 + 4$

   12 12

   If $6 + 7 + -5 = -5 + 10 + 3$, *then* $6 + 7 + -5 + 4 = -5 + 10 + 3 + 4$.

   If expression 1 $=$ *expression* 2, *then* $($*expression* $1 + 4) = ($*expression* $2 + 4)$.

4. **Divide each expression by -2. Write the new numerical expression, and write a conclusion as an if-then statement.**

   Expression 1: $(6 + 7 + -5) \div -2$ *Expression 2:* $(-5 + 10 + 3) \div -2$

   $8 \div -2$ $8 \div -2$

   -4 -4

   If $6 + 7 + -5 = -5 + 10 + 3$, *then* $(6 + 7 + -5) \div -2 = (-5 + 10 + 3) \div -2$

   If expression 1 $=$ *expression* 2, *then* $($*expression* $1 \div -2) = ($*expression* $2 \div -2)$.

Problem Set Sample Solutions

This problem set provides students with additional practice evaluating numerical expressions and applying different moves while seeing the effect on number sentences.

1. **Evaluate the following numerical expressions**

   a. $2 + (-3) + 7 = 6$

   b. $-4 - 1 = -5$

   c. $-\frac{5}{2} \times 2 = -5$

   d. $-10 \div 2 + 3 = -2$

   e. $\left(\frac{1}{2}\right)(8) + 2 = 6$

   f. $3 + (-4) - 1 = -2$

2. **Which expressions from Exercise 1 are equal?**

   Expressions (a) and (e) are equivalent.

   Expressions (b) and (c) are equivalent.

   Expressions (d) and (f) are equivalent.

3. **If two of the equivalent expressions from Exercise 1 are divided by 3, write an if-then statement using the properties of equality.**

   If $2 + (-3) + 7 = \left(\frac{1}{2}\right)(8) + 2$, *then* $(2 + (-3) + 7) \div 3 = \left(\left(\frac{1}{2}\right)(8) + 2\right) \div 3.$

4. **Write an if-then statement if -3 is multiplied by the following equation: $-1 - 3 = -4$.**

   If $-1 - 3 = -4$, *then* $-3(-1 - 3) = -3(-4)$

5. **Simplify the expression.** $5 + 6 - 5 + 4 + 7 - 3 + 6 - 3$

   $= 17$

   Using the expression, write an equation. $5 + 6 - 5 + 4 + 7 - 3 + 6 - 3 = 17$

   Rewrite the equation if 5 is added to both expressions. $5 + 6 - 5 + 4 + 7 - 3 + 6 - 3 + 5 = 17 + 5$

   Write an if-then statement using the properties of equality. *If* $5 + 6 - 5 + 4 + 7 - 3 + 6 - 3 = 17$, *then* $5 + 6 - 5 + 4 + 7 - 3 + 6 - 3 + 5 = 17 + 5$