Lesson 18: Writing, Evaluating, and Finding Equivalent Expressions with Rational Numbers

Student Outcomes

- Students create equivalent forms of expressions in order to see structure, reveal characteristics and make connections to context.
- Students compare equivalent forms of expressions and recognize that there are multiple ways to represent the context of a word problem.
- Students write and evaluate expressions to represent real-world scenarios.

Classwork

Exercise 1 (15 minutes)

MP.1 & MP.2

Students work with a partner or small group to determine the best cell phone plan when given the different prices and options for three different companies. Students are required to write an expression to represent each plan and evaluate each expression to determine which plan is most economical.

Prior to the exercise, recall the description of an expression:

- An expression can be a number or a letter (which can be raised to a whole number exponent) that represents a number. What are some examples?
  - $x, 3, x^2$
- An expression can be the product whose factors are any one of the entities described above. What are some examples?
  - $3 \cdot 2$
- An expression can also be the sum and/or difference of the products described above. What are some examples?
  - $3 \cdot 2 + x - 2$

Exercise 1

John's father asked him to compare several different cell phone plans and identify which plan will be the least expensive for the family. Use the information contained in the table below to answer the following questions.

Cell Phone Plans

Name of Plan	Monthly fee (Includes $1,500$ shared minutes)	Price per phone line x	Price per line for unlimited texting y	Price per line for internet access z
Company A	$70	$20	$15	$15
Company B	$90	$15	$10	$20
Company C	$200	$10	included in monthly fee	included in monthly fee

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All members of the family may not want identical plans; therefore, we will let x represent the number of phone lines, y represent the number of phone lines with unlimited texting, and z represent the number of phone lines with internet access.

Expression

Company A _____ $70 + 20x + 15y + 15z$ _____

Company B _____ $90 + 15x + 10y + 20z$ _____

Company C _____ $200 + 10x$ _____

Using the expressions above, find the cost to the family of each company's phone plan if:

a.   Four people want a phone line, four people want unlimited texting, and the family needs two internet lines.

Company A	Company B	Company C
$70 + 20x + 15y + 15z$	$90 + 15x + 10y + 20z$	$200 + 10x$
$70 + 20(4) + 15(4) + 15(2)$	$90 + 15(4) + 10(4) + 20(2)$	$200 + 10(4)$
$70 + 80 + 60 + 30$	$90 + 60 + 40 + 40$	$200 + 40$
240	230	240

Which cell phone company should John's family use? Why?

Company B since it is cheaper than the others for the given values.

b.   Four people want a phone line, four people want unlimited texting, and all four people want internet lines.

Company A	Company B	Company C
$70 + 20x + 15y + 15z$	$90 + 15x + 10y + 20z$	$200 + 10x$
$70 + 20(4) + 15(4) + 15(4)$	$90 + 15(4) + 10(4) + 20(4)$	$200 + 10(4)$
$70 + 80 + 60 + 60$	$90 + 60 + 40 + 80$	$200 + 40$
270	270	240

Which cell phone company should John's family use? Why?

Company C since it is cheaper than the other companies for the given values.

c.   Two people want a phone line, two people want unlimited texting, and the family needs two internet lines.

Company A	Company B	Company C
$70 + 20x + 15y + 15z$	$90 + 15x + 10y + 20z$	$200 + 10x$
$70 + 20(2) + 15(2) + 15(2)$	$90 + 15(2) + 10(2) + 20(2)$	$200 + 10(2)$
$70 + 40 + 30 + 30$	$90 + 30 + 20 + 40$	$200 + 20$
170	180	220

Which cell phone company should John's family use? Why?

Company A since it is cheaper than the other companies for the given values

Questions to follow exercise:

- Why is there no equal sign in the expressions?
  - □ *There is no equal sign because we are writing an expression and not an equation.*

- Each plan charges for four different options, yet there are only three variables in each expression. Why is this?
  - □ *There are only three variables used because the monthly fee is added to the options regardless of how many lines are purchased.*

- What would be the minimum cost for each plan?
  - □ *The minimum cost for each plan would be the monthly fee and no other options. Company A would be* $70*, company B would be* $90 *and company C would be* $200*.*

- What role did the expression play in your decision-making process?
  - □ *Writing an expression allowed us to evaluate and compare the different companies for many different values of each variable.*

- Describe the process you used to arrive at the total cost of each plan?
  - □ *First, the given values for each variable are substituted into the expression so every variable is replaced with the corresponding numerical value. After that, you do the arithmetic following order of operations.*

Exercise 2 (10 minutes)

Students continue to write and evaluate expressions from real-world problems, but also identify equivalent expressions during the process.

MP.1 & MP.2

In the same groups as Exercise 1, have students first individually read through the following problem, write an expression and evaluate the expression. Once completed, have students compare their results with those of their group. Once all members of the group agree upon the correct answer, the students should compare their solutions, looking for similarities and differences among the various methods used. If there are any differences have them discuss what they were. As a large group lead a discussion about the different possible ways of obtaining the same solution. If any person or group obtained the solution in different ways have them explain their process to the class.

Exercise 2

Three friends went to the movies. Each purchased a medium-sized popcorn for p dollars and a small soft drink for s dollars.

a. Write the expression that represents the total amount of money (in dollars) the three friends spent at the concession stand.

$3(p + s)$

b. If the concession stand charges 6.50 for a medium-sized popcorn and 4.00 for a small soft drink, how much did the three friends spend on their refreshments all together?

One possible solution is shown here, more solution methods are shown in the discussion below.

$3(p + s)$
$3(6.50 + 4)$
$3(10.50)$
31.50

They spent $31.50.$

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Questions to follow the exercise:

- What information did you use to write the expression?
    - □ *You needed to know what the variables were and what they represented. You also needed to know or figure out how many people were getting each item.*

- John created the expression $3p + 3s$ to represent the total cost of the refreshments while Sally used the expression $3(p + s)$. Are they both correct? If so, what do the expressions tell us about the two different ways in which John and Sally calculate the cost of the refreshments?
    - □ *Yes, both expressions are correct. John calculated the cost of three drinks and the cost of three popcorns and added them together. Sally calculated the amount each friend spent and then multiplied by the number of friends.*

- Compare the four samples of student work. What are the differences between the four methods? (This is where groups or individuals can share their work if it matches any of these.)

Student 1:	Student 2:	Student 3:	Student 4:
$3(p + s)$	$3(p + s)$	$3(p + s)$	$3(p + s)$
$3(6.50 + 4)$	$3(6.50 + 4)$	$3(6.50 + 4)$	$(p + s) + (p + s) + (p + s)$
$3(10.50)$	$3(10.50)$	$3(6.50) + 3(4)$	$(6.50 + 4) + (6.50 + 4) + (6.50 + 4)$
$\$31.50$	$3(10 + 0.50)$	$19.50 + 12$	$10.50 + 10.50 + 10.50$
	$30 + 1.50$	31.50	31.50
	31.50		

While discussing the differences in the four methods, clearly describe that the methods above are beginning with the same expression, but that each method demonstrates a different way of evaluating the same expression.

- The next time the three friends went to the movies they each purchased a small-sized soft drink but decided to share one medium-sized popcorn. Write the expression that describes the amount the group spent at the concession stand. How does this expression differ from the one you created before?
    - □ *Expression $3s + p$. Only the value of the soda would be multiplied by 3, and not the popcorn. There would be no need to apply the distributive property.*

Exercise 3 (10 minutes)

Exercise 3

Complete the table below by writing equivalent expressions to the given expression and evaluating each expression with the given values.

Equivalent Expressions			
EXAMPLE: Evaluate $x = 2$, $y = -1$	$4(x + 2y)$ $4(2 + 2(-1))$ $4(0)$ 0	$4x + 8y$ $4(2) + 8(-1)$ $8 + (-8)$ 0	$4x + 4y + 4y$ $4(2) + 4(-1) + 4(-1)$ $8 + (-4) + (-4)$ 0

		$5(3-4y)$	$15-20y$	$15-10y-10y$
1.	**Evaluate** $y = 1$	$5(3-4(1))$ $5(3-4)$ $5(-1)$ -5	$15-20(1)$ $15-20$ $15+(-20)$ -5	$15-10(1)-10(1)$ $15-10-10$ $5-10$ $5+(-10)$ -5
2.	**Evaluate** $x = 5,$ $y = -2$	$-3x+12y$ $-3(5)+12(-2)$ $-15+(-24)$ -39	$3(-1x+4y)$ $3(-1(5)+4(-2))$ $3(-5+(-8))$ $3(-13)$ -39	$-x-x-x+4y+4y+4y$ $-5-5-5+4(-2)+4(-2)+4(-2)$ $-5+(-5)+(-5)+(-8)+(-8)+(-8)$ -39
3.	**Evaluate** $x = -\frac{1}{2},$ $y = 1$	$8x-6y$ $8\left(-\frac{1}{2}\right)-6(1)$ $-4-6$ $-4+(-6)$ -10	$2(4x-3y)$ $2\left(4\left(-\frac{1}{2}\right)-3(1)\right)$ $2(-2-3)$ $2(-2+(-3))$ $2(-5)$ -10	$-2x+10x-6y$ $-2\left(-\frac{1}{2}\right)+10\left(-\frac{1}{2}\right)-6(1)$ $1-5-6$ $1+(-5)+(-6)$ -10

Questions and Discussion to follow the exercise:

- Looking at the equivalent expressions you created, can you see any benefit of using one over the other?
  - *Answers will vary.*
- Would it matter which equivalent expression you use if you were asked to evaluate the expression?
  - *No, the expressions are equivalent because when evaluated, all of the expressions equal the same rational number.*
- For each equivalent expression written in the table, discuss with your elbow partner why each expression is equivalent.

Provide time for students to discuss, then have them share their responses with the class.

  - *Sample explanations should include students applying the distributive property.*

Have students identify a context that the expression could be modeling with their partners.

Closing (5 minutes)

- What is an expression? Describe the steps to evaluating it.
  - *An expression is a number or letter (that may have a whole number exponent) that represents a number. Expressions can also be products or sums.*
- To evaluate an expression, substitute numerical values into the expression and then follow order of operations to solve.
- How do you determine if expressions are equivalent?
  - *Equivalent expressions have the same numerical value for every number substituted into the problem.*

Lesson Summary

- An expression is a number or a letter, which can be raised to a whole number exponent. An expression can be a product whose factors are any one of the entities described above. An expression can also be the sum or difference of the products described above.

- To evaluate an expression, replace each variable with its corresponding numerical value. Using order of operations, the expression can be written as a single numerical value.

- When numbers are substituted into all the letters in an expression and the results are the same, then the expressions are equivalent.

Exit Ticket (5 minutes)

Name _____ Date_____

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Exit Ticket

Bradley and Louie are roommates at college. At the beginning of the semester, they each paid a security deposit of A dollars. When they move out, their landlord will deduct from this deposit any expenses (B) for excessive wear and tear and refund the remaining amount. Bradley and Louie will share the expenses equally.

- Write an expression that describes the amount each roommate will receive from the landlord when the lease expires.
- Evaluate the expression using the following information: Each roommate paid a $125 deposit, and the landlord deducted $50 total for damages.

Exit Ticket Sample Solutions

Bradley and Louie are roommates at college. At the beginning of the semester, they each paid a security deposit of A dollars. When they move out, their landlord will deduct from this deposit any expenses (B) for excessive wear and tear, and refund the remaining amount.

- Write an expression that describes the amount each roommate will receive from the landlord when the lease expires.

- Evaluate the expression using the following information: Each roommate paid a $125 deposit and the landlord deducted $50 total for damages.

Deposit each person paid: A

Total damages: B

Each roommate receives: $A - \dfrac{B}{2}$

$A = 125, B = 50$

$$A - \frac{B}{2}$$
$$125 - \frac{50}{2}$$
$$125 - 25$$
$$100$$

Problem Set Sample Solutions

1. Sally is paid a fixed amount of money to walk her neighbor's dog every day after school. When she is paid each month, she puts aside $20 to spend and saves the remaining amount. Write an expression that represents the amount Sally will save in 6 months if she earns m dollars each month. If Sally is paid $65 each month, how much will she save in 6 months?

m = monthly pay

$6(m - 20)$

$6m - 120$

For $m = 65$

$6(m - 20)$	*or*	$6(m - 20)$
$6(65 - 20)$		$6(65 - 20)$
$6(45)$		$390 - 120$
$270		$270

2. A football team scored 3 touchdowns, 3 extra points, and 4 field goals.

   a. Write an expression to represent the total points the football team scored.

   t = number of points for a touchdown

   e = number of points for the extra point

   f = number of points for a field goal.

   $3t + 3e + 4f$

b. Write another expression that is equivalent to the one written above.

Answers may vary. Sample response: $3t + 3e + 2f + 2f$

c. If each touchdown is worth 6 points, each extra point is 1 point, and each field goal is 3 points, how many total points did the team score?

$3t + 3e + 4f$

$3(6) + 3(1) + 4(3)$

$18 + 3 + 12$

33

3. Write three other expressions that are equivalent to $8x - 12$.

Answers may vary.

$4(2x - 3)$

$6x + 2x - 12$

$8(x - 1) - 4$

$-12 + 8x$

4. Profit is defined as earnings less expenses (earnings – expenses). At the local hot-air balloon festival, the Ma & Pops Ice Cream Truck sells ice cream pops, which cost them $0.75 each, but are sold for $2 each. They also paid $50 to the festival's organizers for a vendor permit. The table below shows the earnings, expenses, and profit earned when 50, 75, and 100 ice cream pops were sold at the festival.

Number of Pops Sold	Earnings	Expenses	Profit
50	$50(2) = 100$	$50(0.75) + 50$ $37.5 + 50 = 87.5$	$100 - 87.5 = 12.50$
75	$75(2) = 150$	$75(0.75) + 50$ $56.25 + 50 = 106.25$	$150 - 106.25 = 43.75$
100	$100(2) = 200$	$100(0.75) + 50$ $75 + 50 = 125$	$200 - 125 = 75$

a. Write an expression that represents the profit (in dollars) Ma & Pop earned by selling ice cream pops at the festival.

p represents the number of pops sold

$2p - 0.75p - 50$

b. Write an equivalent expression.

$1.25p - 50$

c.   **How much did Ma & Pops Ice Cream Truck profit if it sold 20 ice cream pops? What does this mean? Explain why this might be the case?**

$1.25p - 50$

$1.25(20) - 50$

$25 - 50$

-25

They did not make any money; they lost $25. A possible reason is it could have been cold or rainy and people were not buying ice cream.

d.   **How much did Ma & Pops Ice Cream truck profit if it sold 75 Ice Cream Pops? What does this mean? Explain why this might be the case?**

$1.25p - 50$

$1.25(75) - 50$

$93.75 - 50$

43.75

They made a profit of $43.75. Possible reasons are the weather could have been warmer and people bought the ice cream, or people just like to eat ice cream no matter what the weather is.