Lesson 13: Inequalities

Student Outcomes

- Students understand that an inequality is a statement that one expression is less than (or equal to) or greater than (or equal to) another expression, such as $2x + 3 < 5$ or $3x + 50 \geq 100$.
- Students interpret a solution to an inequality as a number that makes the inequality true when substituted for the variable.
- Students convert arithmetic inequalities into a new inequality with variables (e.g., $2 \times 6 + 3 > 12$ to $2m + 3 > 12$) and give a solution, such as $m = 6$, to the new inequality. They check to see if different values of the variable make an inequality true or false.

Lesson Notes

This lesson reviews the conceptual understanding of inequalities and introduces the "why" and "how" of moving from numerical expressions to algebraic inequalities.

Classwork

Opening Exercise (12 minutes): Writing Inequality Statements

Opening Exercise: Writing Inequality Statements

Tarik is trying to save $\$265.49$ to buy a new tablet. Right now, he has $\$40$ and can save $\$38$ a week from his allowance.

Write and evaluate an expression to represent the amount of money saved after …

2 weeks

$40 + 38(2)$
$40 + 76$
116

3 weeks

$40 + 38(3)$
$40 + 114$
154

4 weeks

$40 + 38(4)$
$40 + 152$
192

5 weeks

$40 + 38(5)$
$40 + 190$
230

6 weeks

$40 + 38(6)$
$40 + 228$
268

7 weeks	$40 + 38(7)$ $40 + 266$ 306
8 weeks	$40 + 38(8)$ $40 + 304$ 344

When will Tarik have enough money to buy the tablet?

From 6 weeks and onward

Write an inequality that will generalize the problem.

$38w + 40 \geq 265.49$ ***Where w represents the number of weeks it will take to save the money.***

Discussion

- Why is it possible to have more than one solution?
  - *It is possible because the minimum amount of money Tarik needs to buy the tablet is* $\$265.49$*. He can save more money than that, but he cannot have less than that amount. As more time passes, he will have saved more money. Therefore, any amount of time from* 6 *weeks onward will ensure he has enough money to purchase the tablet.*
- So, the minimum amount of money Tarik needs is $\$265.49$, and he could have more but certainly not less. What inequality would demonstrate this?
  - *Greater than or equal to*

MP.7

- Examine each of the numerical expressions previously and write an inequality showing the actual amount of money saved compared to what is needed. Then, determine if each inequality written is true or false.
  - 2 *weeks:* $116 \geq 265.49$ *False*
  - 3 *weeks:* $154 \geq 265.49$ *False*
  - 4 *weeks:* $192 \geq 265.49$ *False*
  - 5 *weeks:* $230 \geq 265.49$ *False*
  - 6 *weeks:* $268 \geq 265.49$ *True*
  - 7 *weeks:* $306 \geq 265.49$ *True*
  - 8 *weeks:* $344 \geq 265.49$ *True*
- How can this problem be generalized?
  - *Instead of asking what amount of money was saved after a specific amount of time, the question can be asked: How long will it take Tarik to save enough money to buy the tablet?*
- Write an inequality that would generalize this problem for money being saved for w weeks.
  - $38w + 40 \geq 265.49$.
- Interpret the meaning of the 38 in the inequality $38w + 40 \geq 265.49$.
  - *The* 38 *represents the amount of money saved each week. As the weeks increase, the amount of money increases. The* $\$40$ *was the initial amount of money saved, not the amount saved every week.*

Example 1 (13 minutes): Evaluating Inequalities—Finding a Solution

Example 1: Evaluating Inequalities—Finding a Solution

The sum of two consecutive odd integers is more than -12. Write several true numerical inequality expressions.

$5 + 7 > -12$	$3 + 5 > -12$	$1 + 3 > -12$	$-1 + 1 > -12$	$-3 + -1 > -12$
$12 > -12$	$8 > -12$	$4 > -12$	$0 > -12$	$-4 > -12$

The sum of two consecutive odd integers is more than -12. What is the smallest value that will make this true?

a. **Write an inequality that can be used to find the smallest value that will make the statement true.**

x: an integer

$2x + 1$: *odd integer*

$2x + 3$: *next consecutive odd integer*

$2x + 1 + 2x + 3 > -12$

b. **Use if-then moves to solve the inequality written in part (a). Identify where the 0s and 1s were made using the if-then moves.**

$4x + 4 > -12$

$4x + 4 - 4 > -12 - 4$ — *If $a > b$, then $a - 4 > b - 4$.*

$4x + 0 > -16$ — *0 was the result.*

$\left(\frac{1}{4}\right)(4x) > \left(\frac{1}{4}\right)(-16)$ — *If $a > b$, then $a\left(\frac{1}{4}\right) > b\left(\frac{1}{4}\right)$.*

$x > -4$ — *1 was the result.*

The values that will make this true are all consecutive odd integers -3 and larger.

> *Scaffolding:*
>
> To ensure that the integers will be odd and not even, the first odd integer is one unit greater than or less than an even integer. If x is an integer, then $2x$ would ensure an even integer, and $2x + 1$ would be an odd integer since it is one unit greater than an even integer.

Questions to discuss leading to writing the inequality:

- What is the difference between consecutive integers and consecutive even or odd integers?
  - *Consecutive even/odd integers increase or decrease by 2 units compared to consecutive integers that increase by 1 unit.*
- What inequality symbol represents "is more than"? Why?
  - *$>$, because a number that is more than another number is bigger than the original number.*

Questions leading to finding a solution:

- What is a solution set of an inequality?
  - *A solution set contains more than one number that makes the inequality a true statement.*
- Is -3 a solution?
  - *Yes, because when the value of -3 is substituted into the inequality, the resulting statement is true.*
- Could -4 be a solution?
  - *Substituting -4 results in a true statement; however, the solution must be an odd integer, and -4 is not an odd integer. Therefore, -4 is NOT a solution.*

MP.2

- We have found that -3 is a solution to the problem where -4 and -5 are not. What is meant by the minimum value? Have we found the minimum value? Explain.
  - *The minimum value is the smallest odd integer that makes the statement true. Since -3 is a solution to the problem and it is an odd integer, it is the minimum value. All odd integers smaller than -5, such as -7, -9, etc., are not solutions; -3 is a solution, but it is greater than -5.*
- How is solving an inequality similar to solving an equation? How is it different?
  - *Solving an equation and an inequality are similar in the sequencing of steps taken to solve for the variable. The same if-then moves are used to solve for the variable.*
  - *They are different because in an equation, you get one solution, but in an inequality, there are an infinite number of solutions.*
- Discuss the steps to solving the inequality algebraically.
  - *First collect like terms on each side of the inequality. To isolate the variable, subtract 4 from both sides. Subtracting a value, 4, from each side of the inequality does not change the solution of the inequality. Continue to isolate the variable by multiplying both sides by $\frac{1}{4}$. Multiplying a positive value, $\frac{1}{4}$, to both sides of the inequality does not change the solution of the inequality.*

Exercise 1 (8 minutes)

Exercises

1. Connor went to the county fair with a $\$22.50$ in his pocket. He bought a hot dog and drink for $\$3.75$, and then wanted to spend the rest of his money on ride tickets, which cost $\$1.25$ each.

a. Write an inequality to represent the total spent where r is the number of tickets purchased.

$$1.25r + 3.75 \le 22.50$$

b. Connor wants to use this inequality to determine whether he can purchase 10 tickets. Use substitution to show whether he will have enough money.

$$\begin{aligned} 1.25r + 3.75 &\le 22.50 \\ 1.25(10) + 3.75 &\le 22.50 \\ 12.5 + 3.75 &\le 22.50 \\ 16.25 &\le 22.50 \end{aligned}$$

True.

He will have enough money since a purchase of 10 tickets brings his total spending to $\$16.25$.

c. What is the total maximum number of tickets he can buy based upon the given information?

$$\begin{aligned} 1.25r + 3.75 &\le 22.50 \\ 1.25r + 3.75 - 3.75 &\le 22.50 - 3.75 \\ 1.25r + 0 &\le 18.75 \\ \left(\frac{1}{1.25}\right)(1.25r) &\le \left(\frac{1}{1.25}\right)(18.75) \\ r &\le 15 \end{aligned}$$

The maximum number of tickets he can buy is 15.

Exercise 2 (4 minutes)

2. Write and solve an inequality statement to represent the following problem:

On a particular airline, checked bags can weigh no more than 50 pounds. Sally packed 32 pounds of clothes and five identical gifts in a suitcase that weighs 8 pounds. Write an inequality to represent this situation.

x: weight of one gift

$$5x + 8 + 32 \leq 50$$
$$5x + 40 \leq 50$$
$$5x + 40 - 40 \leq 50 - 40$$
$$5x \leq 10$$
$$\left(\frac{1}{5}\right)(5x) \leq \left(\frac{1}{5}\right)(10)$$
$$x \leq 2$$

Each of the 5 gifts can weigh 2 pounds or less.

Closing (3 minutes)

- How do you know when you need to use an inequality instead of an equation to model a given situation?
- Is it possible for an inequality to have exactly one solution? Exactly two solutions? Why or why not?

Exit Ticket (5 minutes)

Name ____ Date____________________

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Exit Ticket

Shaggy earned $7.55 per hour plus an additional $100 in tips waiting tables on Saturday. He earned at least $160 in all. Write an inequality and find the minimum number of hours, to the nearest hour, that Shaggy worked on Saturday.

Exit Ticket Sample Solutions

Shaggy earned $\$7.55$ per hour plus an additional $\$100$ in tips waiting tables on Saturday. He earned at least $\$160$ in all. Write an inequality and find the minimum number of hours, to the nearest hour, that Shaggy worked on Saturday.

Let h represent the number of hours worked.

$$7.55h + 100 \geq 160$$
$$7.55h + 100 - 100 \geq 160 - 100$$
$$7.55h \geq 60$$
$$\left(\frac{1}{7.55}\right)(7.55h) \geq \left(\frac{1}{7.55}\right)(60)$$
$$h \geq 7.9$$

If Shaggy earned at least $\$160$, he would have worked at least 8 hours.

Note: The solution shown above is rounded to the nearest tenth. The overall solution, though, is rounded to the nearest hour since that is what the question asks for.

Problem Set Sample Solutions

1. **Match each problem to the inequality that models it. One choice will be used twice.**

___*c*___	The sum of three times a number and -4 is greater than 17.	a.	$3x + -4 \geq 17$
___*b*___	The sum of three times a number and -4 is less than 17.	b.	$3x + -4 < 17$
___*d*___	The sum of three times a number and -4 is at most 17.	c.	$3x + -4 > 17$
___*d*___	The sum of three times a number and -4 is no more than 17.	d.	$3x + -4 \leq 17$
___*a*___	The sum of three times a number and -4 is at least 17.		

2. **If x represents a positive integer, find the solutions to the following inequalities.**

   a. $x < 7$

   $x < 7$

   b. $x - 15 < 20$

   $x < 35$

   c. $x + 3 \leq 15$

   $x \leq 12$

   d. $-x > 2$

   $x < -2$

   e. $10 - x > 2$

   $x < 8$

   f. $-x \geq 2$

   $x \leq -2$

   g. $\frac{x}{3} < 2$

   $x < 6$

   h. $-\frac{x}{3} > 2$

   $x < -6$

   i. $3 - \frac{x}{4} > 2$

   $x < 4$

3. Recall that the symbol $\neq$ means "not equal to." If x represents a positive integer, state whether each of the following statements is always true, sometimes true, or false.

a. $x > 0$

Always true

b. $x < 0$

False

c. $x > -5$

Always true

d. $x > 1$

Sometimes true

e. $x \geq 1$

Always true

f. $x \neq 0$

Always true

g. $x \neq -1$

Always true

h. $x \neq 5$

Sometimes true

4. Twice the smaller of two consecutive integers increased by the larger integer is at least 25.

Model the problem with an inequality, and determine which of the given values 7, 8, and/or 9 are solutions. Then, find the smallest number that will make the inequality true.

$2x + x + 1 \geq 25$

$x = 7$	$x = 8$	$x = 9$
$2x + x + 1 \geq 25$	$2x + x + 1 \geq 25$	$2x + x + 1 \geq 25$
$2(7) + 7 + 1 \geq 25$	$2(8) + 8 + 1 \geq 25$	$2(9) + 9 + 1 \geq 25$
$14 + 7 + 1 \geq 25$	$16 + 8 + 1 \geq 25$	$18 + 9 + 1 \geq 25$
$22 \geq 25$	$25 \geq 25$	$28 \geq 25$
False	*True*	*True*

The smallest integer would be 8.

5.

a. The length of a rectangular fenced enclosure is 12 feet more than the width. If Farmer Dan has 100 feet of fencing, write an inequality to find the dimensions of the rectangle with the largest perimeter that can be created using 100 feet of fencing.

Let w represent the width of the fenced enclosure.

$w + 12$: *length of the fenced enclosure*

$$w + w + w + 12 + w + 12 \leq 100$$
$$4w + 24 \leq 100$$

b. What are the dimensions of the rectangle with the largest perimeter? What is the area enclosed by this rectangle?

$$4w + 24 \le 100$$
$$4w + 24 - 24 \le 100 - 24$$
$$4w + 0 \le 76$$
$$\left(\frac{1}{4}\right)(4w) \le \left(\frac{1}{4}\right)(76)$$
$$w \le 19$$

maximum width is 19 feet

maximum length is 31 feet

maximum area: $A = lw$

$A = (19)(31)$

$A = 589\ sq.ft.$

6. At most, Kyle can spend $\$50$ on sandwiches and chips for a picnic. He already bought chips for $\$6$ and will buy sandwiches that cost $\$4.50$ each. Write and solve an inequality to show how many sandwiches he can buy. Show your work and interpret your solution.

Let s represent the number of sandwiches.

$$4.50s + 6 \le 50$$
$$4.50s + 6 - 6 \le 50 - 6$$
$$4.50s \le 44$$
$$\left(\frac{1}{4.50}\right)(4.50s) \le \left(\frac{1}{4.50}\right)(44)$$
$$s \le 9\frac{7}{9}$$

At most, Kyle can buy 9 sandwiches with $\$50$.