Lesson 8: Using If-Then Moves in Solving Equations

Student Outcomes

- Students understand and use the addition, subtraction, multiplication, division, and substitution properties of equality to solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$ where p, q, and r are specific rational numbers.
- Students understand that any equation with rational coefficients can be written as an equation with expressions that involve only integer coefficients by multiplying both sides by the least common multiple of all the rational number terms.

Lesson Notes

The intent of this lesson is for students to make the transition from an arithmetic approach of solving a word problem to an algebraic approach of solving the same problem. Recall from Module 2 that the process for solving linear equations is to isolate the variable by making 0s and 1s. In this module, the emphasis will be for students to rewrite an equation using if-then moves into a form where the solution is easily recognizable. The main issue is that, in later grades, equations are rarely solved by "isolating the variable" (e.g., How do you isolate the variable for $3x^2 - 8 = -2x$?). Instead, students learn how to rewrite equations into different *forms* where the solutions are easy to recognize.

- Examples of Grade 7 forms: The equation $\frac{2}{3}x + 27 = 31$ is put into the form $x = 6$, where it is easy to recognize that the solution is 6.
- Example of an Algebra I form: The equation $3x^2 - 8 = -2x$ is put into factored form $(3x - 4)(x + 2) = 0$, where it is easy to recognize that the solutions are $\left\{\frac{4}{3}, -2\right\}$.
- Example of a Algebra II and Precalculus form: The equation $\sin^3 x + \sin x \cos^2 x = \cos x \sin^2 x + \cos^3 x$ is simplified to $\tan x = 1$, where it is easy to recognize that the solutions are $\left\{\frac{\pi}{4} + k\pi \,\middle|\, k \text{ integer}\right\}$.

Regardless of the type of equation students are studying, the if-then moves play an essential role in rewriting equations into different useful forms for solving, graphing, etc.

The FAQs on solving equations below are designed to help teachers understand the structure of the next set of lessons. Before reading the FAQ, it may be helpful to review the properties of operations and the properties of equality listed in Table 3 and Table 4 of the Common Core Learning Standards (CCLS).

What are the "if-then moves"? Recall the following *if-then moves* from Lesson 21 of Module 2:

1. Addition property of equality: If $a = b$, then $a + c = b + c$.
2. Subtraction property of equality: If $a = b$, then $a - c = b - c$.
3. Multiplication property of equality: If $a = b$, then $a \times c = b \times c$.
4. Division property of equality: If $a = b$ and $c \neq 0$, then $a \div c = b \div c$.

All eight properties of equality listed in Table 4 of the CCLS are if-then statements used in solving equations, but these four properties are separated out and collectively called the if-then moves.

What points should I try to communicate to my students about solving equations? The goal is to make three important points about solving equations.

- The technique for solving equations of the form $px + q = r$ and $p(x + q) = r$ is to rewrite them into the form

  $$x = \text{"a number,"}$$

  using the properties of operations (Lessons 1–6) and the properties of equality (i.e., the if-then moves) to make 0s and 1s. This technique is sometimes called "isolating the variable," but that name really only applies to *linear equations*. You might mention that students will learn other techniques for other types of equations in later grades.
- The properties of operations are used to modify *one* side of an equation at a time by changing the expression on a side into another equivalent expression.
- The if-then moves are used to modify *both* sides of an equation simultaneously in a controlled way. The two expressions in the new equation are different than the two expressions in the old equation, but the solutions are the same.

How do if-then statements show up when solving equations? We will continue to use the normal convention of writing a sequence of equations underneath each other, linked together by if-then moves and/or properties of operations. For example, the sequence of equations and reasons for solving $3x = 3$ is as follows:

$\frac{1}{3}(3x) = \frac{1}{3}(3)$	If-then move: multiply both sides by $\frac{1}{3}$.
$\left(\frac{1}{3} \cdot 3\right) x = \frac{1}{3}(3)$	Associative property
$1 \cdot x = 1$	Multiplicative inverse
$x = 1.$	Multiplicative identity

This is a welcomed abbreviation for the if-then statements used in Lesson 21 of Module 2:

If $3x = 3$, then $\frac{1}{3}(3x) = \frac{1}{3}(3)$ by the if-then move of multiplying both sides by $\frac{1}{3}$.
If $\frac{1}{3}(3x) = \frac{1}{3}(3)$, then $\left(\frac{1}{3} \cdot 3\right) x = \frac{1}{3}(3)$ by the associative property.
If $\left(\frac{1}{3} \cdot 3\right) x = \frac{1}{3}(3)$, then $1 \cdot x = 1$ by the multiplicative inverse property.
If $1 \cdot x = 1$, then $x = 1$ by the multiplicative identity property.

The abbreviated form is visually much easier for students to understand *provided that you explain to your students* that each pair of equations is part of an if-then statement.

In the unabbreviated if-then statements above, it looks like the properties of operations are also if-then statements. Are they? No. The properties of operations are not if-then statements themselves; most of them (associative, commutative, distributive, etc.) are statements about equivalent expressions. However, they are often used with combinations of the *transitive and substitution properties of equality*, which are if-then statements. For example, the transitive property states in this situation that if two expressions are equivalent, and if one of the expressions is substituted for the other in a true equation, then the resulting equation is also true (if $a = b$ and $b = c$, then $a = c$).

Thus, the sentence above, "If $\frac{1}{3}(3x) = \frac{1}{3}(3)$, then $\left(\frac{1}{3}\cdot 3\right)x = \frac{1}{3}(3)$ by the associative property," can be expanded as follows:

1. In solving the equation $\frac{1}{3}(3x) = \frac{1}{3}(3)$, we assume x is a number that makes this equation true.
2. $\left(\frac{1}{3}\cdot 3\right)x = \frac{1}{3}(3x)$ is true by the associative property.
3. Therefore, we can replace the expression $\frac{1}{3}(3x)$ in the equation $\frac{1}{3}(3x) = \frac{1}{3}(3)$ with the equivalent expression $\left(\frac{1}{3}\cdot 3\right)x$ by the transitive property of equality.

(You might check that this fits the form of the transitive property described in the CCLS: If $a = b$ and $b = c$, then $a = c$.)

Teachers do not necessarily need to drill down to this level of detail when solving equations with students. Carefully monitor students for understanding and drill down to this level as needed.

Should I show every step in solving an equation? Yes and no: Please use your best judgment given the needs of your students. We generally do not write *every* step on the board when solving an equation. Otherwise, we would need to include discussions like the one above about the transitive property, which can throw off the lesson pace and detract from understanding. Here are general guidelines to follow when solving an equation with a class, which should work well with how these lessons are designed:

1. It is almost always better to initially include more steps than less. A good rule of thumb is to double the number of steps you would personally need to solve an equation. As adults, we do a lot more calculating in our heads than we realize. Doubling the number of steps slows down the pace of the lesson, which can be enormously beneficial to your students.
2. As students catch on, look for ways to shorten the number of steps (for example, using any order/any grouping to collect all like terms at once rather than showing each associative/commutative property). Regardless, it is still important to verbally describe or ask for the properties being used in each step.
3. Write the reason (on the board) if it is one of the main concepts being learned in a lesson. For example, the next few lessons focus on if-then moves. Writing the if-then moves on the board calls them out to your students and helps them focus on the main concept. As students become comfortable using the language of if-then moves, further reduce what you write on the board but verbally describe (or ask students to describe) the properties being used in each step.

We end with a quote from the *High School, Algebra Progressions* that encapsulates this entire FAQ:

> "In the process of learning to solve equations, students learn certain 'if-then' moves, for example, 'if $x = y$, then $x + 2 = y + 2$.' The danger in learning algebra is that students emerge with nothing but the moves, which may make it difficult to detect incorrect or made-up moves later on. Thus, the first requirement in the standards in this domain is that students understand that solving equations is a process of reasoning. This does not necessarily mean that they always write out the full text; part of the advantage of algebraic notation is its compactness. Once students know what the code stands for, they can start writing in code."

Classwork

Opening Exercise (5 minutes)

Opening Exercise

Recall and summarize the if-then moves.

If a number is added or subtracted to both sides of a true equation, then the resulting equation is also true:

If $a = b$, then $a + c = b + c$.

If $a = b$, then $a - c = b - c$.

If a number is multiplied or divided to each side of a true equation, then the resulting equation is also true:

If $a = b$, then $ac = bc$.

If $a = b$ and $c \neq 0$, then $a \div c = b \div c$.

Write $3 + 5 = 8$ in as many true equations as you can using the if-then moves. Identify which if-then move you used.

Answers will vary, but some examples are as follows:

If $3 + 5 = 8$, then $3 + 5 + 4 = 8 + 4$. *Add 4 to both sides.*

If $3 + 5 = 8$, then $3 + 5 - 4 = 8 - 4$. *Subtract 4 from both sides.*

If $3 + 5 = 8$, then $4(3 + 5) = 4(8)$. *Multiply both sides by 4.*

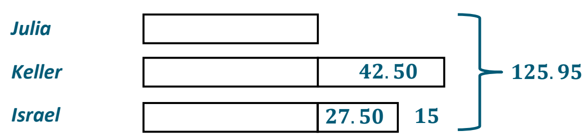
If $3 + 5 = 8$, then $(3 + 5) \div 4 = 8 \div 4$. *Divide both sides by 4.*

Example 1 (10 minutes)

Example 1

Julia, Keller, and Israel are volunteer firefighters. On Saturday, the volunteer fire department held its annual coin drop fundraiser at a streetlight. After one hour, Keller had collected $\$42.50$ more than Julia, and Israel had collected $\$15$ less than Keller. The three firefighters collected $\$125.95$ in total. How much did each person collect?

Find the solution using a tape diagram.



$3 \text{ units} + 42.50 + 27.50 = 125.95$ $\quad$ $42.50 + 27.50 = 70$

$3 \text{ units} + 70 = 125.95$ $\quad$ $125.95 - 70 = 55.95$

$3 \text{ units} = 55.95$ $\quad$ $55.95 \div 3 = 18.65$

$1 \text{ unit} = 18.65$

What were the operations we used to get our answer?

First, we added 42.50 and 27.50 to get 70. Next, we subtracted 70 from 125.95. Finally, we divided 55.95 by 3 to get 18.65.

The amount of money Julia collected is *j* dollars. Write an expression to represent the amount of money Keller collected in dollars.

$j + 42.50$

Using the expressions for Julia and Keller, write an expression to represent the amount of money Israel collected in dollars.

$j + 42.50 - 15$

or

$j + 27.50$

Using the expressions written above, write an equation in terms of *j* that can be used to find the amount each person collected.

$j + (j + 42.50) + (j + 27.50) = 125.95$

Solve the equation written above to determine the amount of money each person collected, and describe any if-then moves used.

$j + (j + 42.50) + (j + 27.50) = 125.95$

$3j + 70 = 125.95$	*Any order, any grouping*
$(3j + 70) - 70 = 125.95 - 70$	*If-then move: Subtract 70 from both sides (to make a 0).*
$3j + 0 = 55.95$	*Any grouping, additive inverse*
$3j = 55.95$	*Additive identity*
$\left(\frac{1}{3}\right)(3j) = (55.95)\left(\frac{1}{3}\right)$	*If-then move: Multiply both sides by $\frac{1}{3}$ (to make a 1).*
$\left(\frac{1}{3} \cdot 3\right) j = 18.65$	*Associative property*
$1 \cdot j = 18.65$	*Multiplicative inverse*
$j = 18.65$	*Multiplicative identity*

If Julia collected $\$18.65$, then Keller collected $18.65 + 42.50 = \$61.15$, and Israel collected $61.15 - 15 = \$46.15$.

Scaffolding:

Teachers may need to review the process of solving an equation algebraically from Module 2, Lessons 17, 22, and 23.

Discussion (5 minutes)

Have students present the models they created based upon the given relationships and then have the class compare different correct models and/or discuss why the incorrect models were incorrect. Some possible questions from the different models are as follows:

- How does the tape diagram translate into the initial equation?
  - *Each unknown unit represents how much Julia collected: j dollars.*
- The initial step to solve the equation algebraically is to collect all like terms on the left hand side of the equation using the any order, any grouping property.

The goal is to rewrite the equation into the form $x =$ "a number" by making zeros and ones.

- How can we make a zero or one?
  - *Zeros are made with addition, and ones are made through multiplication and division.*

- *We can make a* 0 *by subtracting* 70 *from both sides, or we can make a* 1 *by multiplying both sides by* $\frac{1}{3}$*. (Both are correct if-then moves, but point out to students that making a* 1 *will result in extra calculations.)*
- *Let us subtract* 70 *from both sides. The if-then move of subtracting* 70 *from both sides will change both expressions of the equation (left and right sides) to new nonequivalent expressions, but the new expression will have the same solution as the old one did.*

- In subtracting 70 from both sides, what do a, b, and c represent in the if-then move, "If $a = b$, then $a - c = b - c$"?
  - *In this specific example,* a *represents the left side of the equation,* $3j + 70$*,* b *represents the right side of the equation,* 125.95*, and* c *is* 70*.*
- Continue to simplify the new equation using the properties of operations until reaching the equation $3j = 55.95$. Can we make a zero or a one?
  - *Yes, we can make a one by multiplying both sides by* $\frac{1}{3}$*. Since we are assuming that* j *is a number that makes the equation* $3j = 55.95$ *true, we can apply the if-then move of multiplying both sides by* $\frac{1}{3}$*. The resulting equation will also be true.*
- How is the arithmetic approach (the tape diagram with arithmetic) similar to the algebraic approach (solving an equation)?
  - *The operations performed in solving the equation algebraically are the same operations done arithmetically.*
- How can the equation $3j + 70 = 125.95$ be written so that the equation will contain only integers? What would the new equation be?
  - *You can multiply each term by* 100*. The equivalent equation would be* $300j + 7000 = 12595$*.*

Show students that solving this problem also leads to $j = 18.65$.

- What if, instead, we used the amount Keller collected: k dollars. Would that be okay? How would the money collected by the other people then be defined?
  - *Yes, that would be okay. Since Keller has* $\$42.50$ *more than Julia, then Julia would have* $\$42.50$ *less than Keller. Julia's money would be* $k - 42.50$*. Since Israel's money is* $\$15.00$ *less than Keller, his money is* $k - 15$*.*

MP.2

- The expressions defining each person's amount differ depending on who we choose to represent the other two people. Complete the chart to show how the statements vary when x changes.

In terms of	Julia	Israel	Keller
Julia's amount (j)	j	$j + 27.50$	$j + 42$
Israel's amount (i)	$i - 27.50$	i	$i + 15$
Keller's amount (k)	$k - 42.50$	$k - 15$	k

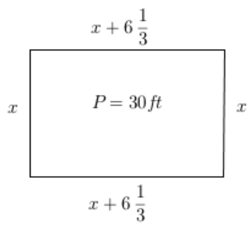
  - *If time, set up and solve the equation in terms of* k*. Show students that the equation and solution are different than the equation based upon Julia's amount, but that the solution,* $\$61.15$*, matches how much Keller collected.*

Example 2 (10 minutes)

Example 2

You are designing a rectangular pet pen for your new baby puppy. You have 30 feet of fence barrier. You decide that you would like the length to be $6\frac{1}{3}$ feet longer than the width.

Draw and label a diagram to represent the pet pen. Write expressions to represent the width and length of the pet pen.



Width of the pet pen: x ft.

Then, $\left(x+6\frac{1}{3}\right)$ ft. represents the length of the pet pen.

Find the dimensions of the pet pen.

Arithmetic

$$\left(30-6\frac{1}{3}-6\frac{1}{3}\right)\div 4$$
$$17\frac{1}{3}\div 4$$
$$4\frac{1}{3}$$

The width is $4\frac{1}{3}$ ft.

The length is $4\frac{1}{3}+6\frac{1}{3}=10\frac{2}{3}$ ft.

Algebraic

$$x+\left(x+6\frac{1}{3}\right)+x+\left(x+6\frac{1}{3}\right)=30$$
$$4x+12\frac{2}{3}=30$$
$$4x+12\frac{2}{3}-12\frac{2}{3}=30-12\frac{2}{3}$$
If-then move: Subtract $12\frac{2}{3}$ from both sides.
$$4x=17\frac{1}{3}$$
$$\left(\frac{1}{4}\right)(4x)=\left(17\frac{1}{3}\right)\left(\frac{1}{4}\right)$$
If-then move: Multiply both sides by $\frac{1}{4}$.
$$x=4\frac{1}{3}$$

If the perimeter of the pet pen is 30 ft. and the length of the pet pen is $6\frac{1}{3}$ ft. longer than the width, then the width would be $4\frac{1}{3}$ ft., and the length would be $4\frac{1}{3}+6\frac{1}{3}=10\frac{2}{3}$ ft.

If an arithmetic approach was used to determine the dimensions, write an equation that can be used to find the dimensions. Encourage students to verbalize their strategy and the if-then moves used to rewrite the equation with the same solution.

Example 3 (5 minutes)

Example 3

Nancy's morning routine involves getting dressed, eating breakfast, making her bed, and driving to work. Nancy spends $\frac{1}{3}$ of the total time in the morning getting dressed, 10 minutes eating breakfast, 5 minutes making her bed, and the remaining time driving to work. If Nancy spends $35\frac{1}{2}$ minutes getting dressed, eating breakfast, and making her bed, how long is her drive to work?

Write and solve this problem using an equation. Identify the if-then moves used when solving the equation.

Total time of routine: x minutes

$$\frac{1}{3}x + 10 + 5 = 35\frac{1}{2}$$

$$\frac{1}{3}x + 15 = 35\frac{1}{2}$$

$$\frac{1}{3}x + 15 - 15 = 35\frac{1}{2} - 15$$ *If-then move: Subtract 15 from both sides.*

$$\frac{1}{3}x + 0 = 20\frac{1}{2}$$

$$3\left(\frac{1}{3}x\right) = 3\left(20\frac{1}{2}\right)$$ *If-then move: Multiply both sides by 3.*

$$x = 61\frac{1}{2}$$

$$61\frac{1}{2} - 35\frac{1}{2} = 26$$

It takes Nancy 26 minutes to drive to work.

Is your answer reasonable? Explain.

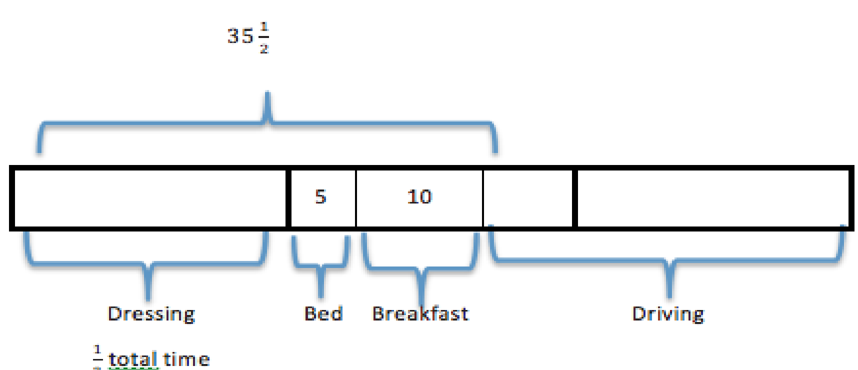
Yes, the answer is reasonable because some of the morning activities take $35\frac{1}{2}$ minutes, so the total amount of time for everything will be more than $35\frac{1}{2}$ minutes. Also, when checking the total time for all of the morning routine, the total sum is equal to total time found. However, to find the time for driving to work, a specific activity in the morning, it is necessary to find the difference from the total time and all the other activities.

Encourage students to verbalize their strategy of solving the problem by identifying what the unknown represents and then using if-then moves to make 0 and 1.

- What does the variable x represent in the equation?
  - *x represents the total amount of time of Nancy's entire morning routine.*
- Explain how to use the answer for x to determine the time that Nancy spends driving to work.
  - *Since x represents the total amont of time in the morning, and the problem gives the amount of time spent on all other activities besides driving, the total time spent driving is the difference of the two amounts. Therefore, you must subtract the total time and the time doing the other activities.*

MP.4

Discuss how the arithmetic approach can be modeled with a bar model.



MP.4

- Getting dressed represents $\frac{1}{3}$ of the total time as modeled.

  We know part of the other morning activities takes a total of 15 minutes; therefore, part of a bar is drawn to model the 15 minutes.

  We know that the bar that represents the time getting dressed and the other activities of 15 minutes equals a total of $35\frac{1}{2}$ minutes. Therefore, the getting dressed bar is equal to $35\frac{1}{2} - 15 = 20\frac{1}{2}$.

  The remaining bars that represent a third of the total time also equal $20\frac{1}{2}$. Therefore, the total time is

  $$20\frac{1}{2} + 20\frac{1}{2} + 20\frac{1}{2} = 61\frac{1}{2}.$$

  The time spent driving would be equal to the total time less the time spent doing all other activities,

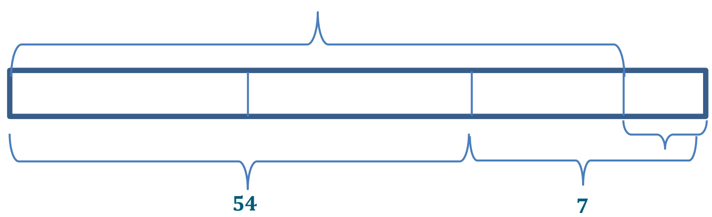
  $$61\frac{1}{2} - 35\frac{1}{2} = 26.$$

Example 4 (5 minutes)

Example 4

The total number of participants who went on the seventh-grade field trip to the Natural Science Museum consisted of all of the seventh-grade students and 7 adult chaperones. Two-thirds of the total participants rode a large bus, and the rest rode a smaller bus. If 54 of them rode the large bus, how many students went on the field trip?

Arithmetic Approach:



Total on both buses: $(54 \div 2) \times 3 = 81$ $\quad$ $\frac{2}{3}$ ***on large bus*** $\quad$ $\frac{1}{3}$ ***on smaller bus***

Total number of students: $81 - 7 = 74$***; 74 students went on the field trip.***

Algebraic Approach: Challenge students to build the equation and solve it on their own first. Then, go through the steps with them, pointing out how we are "making zeros" and "making ones." Point out that, in this problem, it is advantageous to make a 1 first. (This example is an equation of the form $p(x+q) = r$***.)***

Number of students: s

Total number of participants: $s + 7$

$$\frac{2}{3}(s+7) = 54$$

$$\frac{3}{2}\left(\frac{2}{3}(s+7)\right) = \frac{3}{2}(54)$$ ***If-then move: Multiply both sides by*** $\frac{3}{2}$ ***(to make a 1).***

$$\left(\frac{3}{2}\cdot\frac{2}{3}\right)(s+7) = 81$$

$$1(s+7) = 81$$

$$s + 7 = 81$$

$$(s+7) - 7 = 81 - 7$$ ***If-then move: Subtract 7 from both sides (to make a 0).***

$$s + 0 = 74$$

$$s = 74$$

74 students went on the field trip.

- How can the model be used to write an equation?
  - *By replacing the question mark with* s*, we see that the total number of participants is* $s+7$*. Since the diagram shows that* $\frac{2}{3}$ *of the total is* 54*, we can write* $\frac{2}{3}(s+7)=54$*.*
- How is the calculation $(54 \div 2) \times 3$ in the arithmetic approach similar to making a 1 in the algebraic approach?
  - *Dividing by* 2 *and multiplying by* 3 *is the same as multiplying by* $\frac{3}{2}$*.*
- Which approach did you prefer? Why?
  - *Answers will vary, but try to bring out: The tape diagram in this problem was harder to construct than usual, while the equation seemed to make more sense.*

Closing (3 minutes)

- Describe how if-then moves are applied to solving a word problem algebraically.
- Compare the algebraic and arithmetic approaches. Name the similarities between them. Which approach do you prefer? Why?
- How can equations be rewritten so the equation contains only integer coefficients and constants?

Lesson Summary

Algebraic Approach: To "solve an equation" algebraically means to use the properties of operations and if-then moves to simplify the equation into a form where the solution is easily recognizable. For the equations we are studying this year (called linear equations), that form is an equation that looks like $x=$ "a number," where the number is the solution.

If-Then Moves: If x is a solution to an equation, it will continue to be a solution to the new equation formed by adding or subtracting a number from both sides of the equation. It will also continue to be a solution when both sides of the equation are multiplied by or divided by a non-zero number. We use these if-then moves to make zeros and ones in ways that simplify the original equation.

Useful First Step: If one is faced with the task of finding a solution to an equation, a useful first step is to collect like terms on each side of the equation.

Exit Ticket (5 minutes)

Name ____ Date____________________

Lesson 8: Using If-Then Moves in Solving Equations

Exit Ticket

Mrs. Canale's class is selling frozen pizzas to earn money for a field trip. For every pizza sold, the class makes $5.35. They have already earned $182.90 toward their $750 goal. How many more pizzas must they sell to earn $750? Solve this problem first by using an arithmetic approach, then by using an algebraic approach. Compare the calculations you made using each approach.

Exit Ticket Sample Solutions

Mrs. Canale's class is selling frozen pizzas to earn money for a field trip. For every pizza sold, the class makes $\$5.35$. They have already earned $\$182.90$, but they need $\$750$. How many more pizzas must they sell to earn $\$750$? Solve this problem first by using an arithmetic approach, then by using an algebraic approach. Compare the calculations you made using each approach.

Arithmetic Approach:

Amount of money needed: $750 - 182.90 = 567.10$
Number of pizzas needed: $567.10 \div 5.35 = 106$

If the class wants to earn a total of $\$750$*, then they must sell* 106 *more pizzas.*

Algebraic Approach:

Let x *represent the number of additional pizzas they need to sell.*

$$5.35x + 182.90 = 750$$
$$5.35x + 182.90 - 182.90 = 750 - 182.90$$
$$5.35x + 0 = 567.10$$
$$\left(\frac{1}{5.35}\right)(5.35x) = \left(\frac{1}{5.35}\right)(567.10)$$
$$x = 106$$

OR

$$5.35x + 182.90 = 750$$
$$100(5.35x + 182.90) = 100(750)$$
$$535x + 18290 = 75000$$
$$535x + 18290 - 18290 = 75000 - 18290$$
$$\left(\frac{1}{535}\right)(535x) = \left(\frac{1}{535}\right)(56710)$$
$$x = 106$$

If the class wants to earn $\$750$*, then they must sell* 106 *more pizzas.*

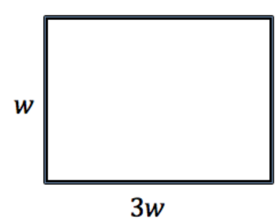
Both approaches subtract 182.90 *from* 750 *to get* 567.10*. Dividing by* 5.35 *is the same as multiplying by* $\frac{1}{5.35}$*. Both result in* 106 *more pizzas that the class needs to sell.*

Problem Set Sample Solutions

Write and solve an equation for each problem.

1. **The perimeter of a rectangle is 30 inches. If its length is three times its width, find the dimensions.**

The width of the rectangle: w *inches*
The length of the rectangle: $3w$ *inches*



$$\text{Perimeter} = 2(\text{length} + \text{width})$$

$$2(w + 3w) = 30$$
$$2(4w) = 30$$
$$8w = 30$$
$$\left(\frac{1}{8}\right)(8w) = \left(\frac{1}{8}\right)(30)$$
$$w = 3\frac{3}{4}$$

OR

$$2(w + 3w) = 30$$
$$(w + 3w) = 15$$
$$4w = 15$$
$$w = 3\frac{3}{4}$$

The width is $3\frac{3}{4}$ *inches.*

The length is $(3)\left(3\frac{3}{4}\right) = (3)\left(\frac{15}{4}\right) = 11\frac{1}{4}$ *inches.*

2. A cell phone company has a basic monthly plan of $\$40$ plus $\$0.45$ for any minutes used over 700. Before receiving his statement, John saw he was charged a total of $\$48.10$. Write and solve an equation to determine how many minutes he must have used during the month. Write an equation without decimals.

The number of minutes over 700: m minutes

$$40 + 0.45m = 48.10$$
$$0.45m + 40 - 40 = 48.10 - 40$$
$$0.45m = 8.10$$
$$\left(\frac{1}{0.45}\right)(0.45m) = 810\left(\frac{1}{0.45}\right)$$
$$m = 18$$

$$4000 + 45m = 4810$$
$$45m + 4000 - 4000 = 4810 - 4000$$
$$45m = 810$$
$$\left(\frac{1}{45}\right)(45m) = 810\left(\frac{1}{45}\right)$$
$$m = 18$$

John used 18 minutes over 700 for the month. He used a total of 718 minutes.

3. A volleyball coach plans her daily practices to include 10 minutes of stretching, $\frac{2}{3}$ of the entire practice scrimmaging, and the remaining practice time working on drills of specific skills. On Wednesday, the coach planned 100 minutes of stretching and scrimmaging. How long, in hours, is the entire practice?

The duration of the entire practice: x hours

$$\frac{2}{3}x + \frac{10}{60} = \frac{100}{60}$$
$$\frac{2}{3}x + \frac{1}{6} = \frac{5}{3}$$
$$\frac{2}{3}x + \frac{1}{6} - \frac{1}{6} = \frac{5}{3} - \frac{1}{6}$$
$$\frac{2}{3}x = \frac{9}{6}$$
$$\left(\frac{3}{2}\right)\left(\frac{2}{3}x\right) = \frac{3}{2}\left(\frac{9}{6}\right)$$
$$x = \frac{27}{12} = 2\frac{1}{4}$$

The entire practice is a length of $2\frac{1}{4} = 2.25$ hours.

4. The sum of two consecutive even numbers is 54. Find the numbers.

First consecutive even integer: x

Second consecutive even integer: $x + 2$

$$x + (x + 2) = 54$$
$$2x + 2 = 54$$
$$2x + 2 - 2 = 54 - 2$$
$$2x + 0 = 52$$
$$\left(\frac{1}{2}\right)(2x) = \left(\frac{1}{2}\right)(52)$$
$$x = 26$$

The consecutive even integers are 26 and 28.

5. **Justin has $\$7.50$ more than Eva and Emma has $\$12$ less than Justin. Together, they have a total of $\$63.00$. How much money does each person?**

The amount of money Eva has: x dollars

The amount of money Justin has: $(x+7.50)$ dollars

The amount of money Emma has: $\big((x+7.50)-12\big)$ dollars, or $(x-4.50)$ dollars

$$x+(x+7.50)+(x-4.50)=63$$
$$3x+3=63$$
$$3x+3-3=63-3$$
$$3x+0=60$$
$$\left(\frac{1}{3}\right)3x=\left(\frac{1}{3}\right)60$$
$$x=20$$

If the total amount of money all three people have is $\$63$, then Eva has $\$20$, Justin has $\$27.50$, and Emma has $\$15.50$.

6. **Barry's mountain bike weighs 6 pounds more than Andy's. If their bikes weigh 42 pounds all together, how much does Barry's bike weigh? Identify the if-then moves in your solution.**

If I let a represent the weight in pounds of Andy's bike, then $a+6$ represents the weight in pounds of Barry's bike.

$$a+(a+6)=42$$
$$(a+a)+6=42$$
$$2a+6=42$$
$$2a+6-6=42-6$$
$$2a+0=36$$
$$2a=36$$
$$\frac{1}{2}\cdot 2a=\frac{1}{2}\cdot 36$$
$$1\cdot a=18$$
$$a=18$$

Barry's Bike: $a+6$ pounds

$$(18)+6=24$$

Barry's bike weighs 24 pounds.

7. **Trevor and Marissa together have 26 t-shirts to sell. If Marissa has 6 fewer t-shirts than Trevor, find how many t-shirts Trevor has. Identify the if-then moves in your solution.**

Let t represent the number of t-shirts that Trevor has, and let $t-6$ represent the number of t-shirts that Marissa has.

$$t+(t-6)=26$$
$$(t+t)+(-6)=26$$
$$2t+(-6)=26$$
$$2t+(-6)+6=26+6$$ *If-then move: Addition property of equality*
$$2t+0=32$$
$$2t=32$$
$$\frac{1}{2}\cdot 2t=\frac{1}{2}\cdot 32$$ *If-then move: Multiplication property of equality*
$$1\cdot t=16$$
$$t=16$$

Trevor has 16 t-shirts to sell, and Marissa has 10 t-shirts to sell.

8. **A number is $\frac{1}{7}$ of another number. The difference of the numbers is 18. (Assume that you are subtracting the smaller number from the larger number.) Find the numbers.**

If we let n represent a number, then $\frac{1}{7}n$ represents the other number.

$$n-\left(\frac{1}{7}n\right)=18$$
$$\frac{7}{7}n-\frac{1}{7}n=18$$
$$\frac{6}{7}n=18$$
$$\frac{7}{6}\cdot\frac{6}{7}n=\frac{7}{6}\cdot 18$$
$$1n=7\cdot 3$$
$$n=21$$

The numbers are 21 and 3.

9. A number is 6 greater than $\frac{1}{2}$ another number. If the sum of the numbers is 21, find the numbers.

If we let n represent a number, then $\frac{1}{2}n + 6$ represents the first number.

$$n + \left(\frac{1}{2}n + 6\right) = 21$$
$$\left(n + \frac{1}{2}n\right) + 6 = 21$$
$$\left(\frac{2}{2}n + \frac{1}{2}n\right) + 6 = 21$$
$$\frac{3}{2}n + 6 = 21$$
$$\frac{3}{2}n + 6 - 6 = 21 - 6$$
$$\frac{3}{2}n + 0 = 15$$
$$\frac{3}{2}n = 15$$
$$\frac{2}{3} \cdot \frac{3}{2}n = \frac{2}{3} \cdot 15$$
$$1n = 2 \cdot 5$$
$$n = 10$$

Since the numbers sum to 21, they are 10 and 11.

10. Kevin is currently twice as old as his brother. If Kevin was 8 years old 2 years ago, how old is Kevin's brother now?

If we let b represent Kevin's brother's age in years, then Kevin's age in years is $2b$.

$$2b - 2 = 8$$
$$2b - 2 + 2 = 8 + 2$$
$$2b = 10$$
$$\left(\frac{1}{2}\right)(2b) = \left(\frac{1}{2}\right)(10)$$
$$b = 5$$

Kevin's brother is currently 5 years old.

11. The sum of two consecutive odd numbers is 156. What are the numbers?

If we let n represent one odd number, then $n + 2$ represents the next consecutive odd number.

$$n + (n + 2) = 156$$
$$2n + 2 - 2 = 156 - 2$$
$$2n = 154$$
$$\left(\frac{1}{2}\right)(2n) = \left(\frac{1}{2}\right)(154)$$
$$n = 77$$

The two numbers are 77 and 79.

12. If n represents an odd integer, write expressions in terms of n that represent the next three consecutive odd integers. If the four consecutive odd integers have a sum of 56, find the numbers.

If we let n represent an odd integer, then $n+2$, $n+4$, and $n+6$ represent the next three consecutive odd integers.

$$n+(n+2)+(n+4)+(n+6)=56$$
$$4n+12=56$$
$$4n+12-12=56-12$$
$$4n=44$$
$$n=11$$

The numbers are $11, 13, 15$, and 17.

13. The cost of admission to a history museum is $\$3.25$ per person over the age of 3; kids 3 and under get in for free. If the total cost of admission for the Warrick family, including their two 6-month old twins, is $\$19.50$, find how many family members are over 3 years old.

Let w represent the number of Warrick family members, then $w-2$ represents the number of family members over the age of 3 years.

$$3.25(w-2)=19.5$$
$$3.25w-6.5=19.5$$
$$3.25w-6.5+6.5=19.5+6.5$$
$$3.25w=26$$
$$w=8$$
$$w-2=6$$

There are 6 members of the Warrick family over the age of 3 years.

14. Six times the sum of three consecutive odd integers is -18. Find the integers.

If I let n represent the first odd integer, then $n+2$ and $n+4$ represent the next two consecutive odd integers.

$$6\big(n+(n+2)+(n+4)\big)=-18$$
$$6(3n+6)=-18$$
$$18n+36=-18$$
$$18n+36-36=-18-36$$
$$18n=-54$$
$$n=-3$$

$$n+2=-1$$
$$n+4=1$$

The integers are -3, -1, and 1.

15. I am thinking of a number. If you multiply my number by 4, add -4 to the product, and then take $\frac{1}{3}$ of the sum, the result is -6. Find my number.

Let n represent the given number.

$$\frac{1}{3}(4n + (-4)) = -6$$
$$\frac{4}{3}n - \frac{4}{3} = -6$$
$$\frac{4}{3}n - \frac{4}{3} + \frac{4}{3} = -6 + \frac{4}{3}$$
$$\frac{4}{3}n = \frac{-14}{3}$$
$$n = -3\frac{1}{2}$$

16. A vending machine has twice as many quarters in it as dollar bills. If the quarters and dollar bills have a combined value of $\$96.00$, how many quarters are in the machine?

If I let d represent the number of dollar bills in the machine, then $2d$ represents the number of quarters in the machine.

$$2d \cdot \left(\frac{1}{4}\right) + 1d \cdot (1) = 96$$
$$\frac{1}{2}d + 1d = 96$$
$$1\frac{1}{2}d = 96$$
$$\frac{3}{2}d = 96$$
$$\frac{2}{3}\left(\frac{3}{2}d\right) = \frac{2}{3}(96)$$
$$d = 64$$
$$2d = 128$$

There are 128 quarters in the machine.