



Lesson 4: Writing Products as Sums and Sums as Products

Student Outcomes

- Students use an area model to write products as sums and sums as products.
- Students use the fact that the opposite of a number is the same as multiplying by -1 to write the opposite of a sum in standard form.
- Students recognize that rewriting an expression in a different form can shed light on the problem and how the quantities in it are related.

Classwork

Example 1 (4 minutes)

Give students two minutes to write equivalent expressions using the distributive property for the first four problems. Then, ask students to try to factor out a common factor and write equivalent expressions for the last four problems.

Example 1

a. $2(x + 5)$	$2x + 10$
b. $3(x + 4)$	$3x + 12$
c. $6(x + 1)$	$6x + 6$
d. $7(x - 3)$	$7x - 21$
e. $5(x + 6)$	$5x + 30$
f. $8(x + 1)$	$8x + 8$
g. $3(x - 4)$	$3x - 12$
h. $5(3x + 4)$	$15x + 20$

- What is happening when you factor and write equivalent expressions for parts (e), (f), (g), and (h)?
 - In the same way that dividing is the opposite or inverse operation of multiplying, factoring is the opposite of expanding.*
- What are the terms being divided by?
 - They are being divided by a common factor.*

Have students write an expression that is equivalent to $8x + 4$.

- Would it be incorrect to factor out a 2 instead of a 4?
 - *It would not be incorrect, but in order to factor completely, we would need to factor out another 2.*
- | | |
|----------------|------------------------------|
| $8x + 4$ | <i>Commutative property</i> |
| $4(2x) + 4(1)$ | <i>Equivalent expression</i> |
| $4(2x + 1)$ | <i>Distributive property</i> |

Exercise 1 (3 minutes)

Students work independently and share their answers with a partner. Discuss together as a class.

Exercise 1

Rewrite the expressions as a product of two factors.

a. $72t + 8$ $8(9t + 1)$	c. $36z + 72$ $36(z + 2)$	e. $3r + 3s$ $3(r + s)$
b. $55a + 11$ $11(5a + 1)$	d. $144q - 15$ $3(48q - 5)$	

Example 2 (5 minutes)

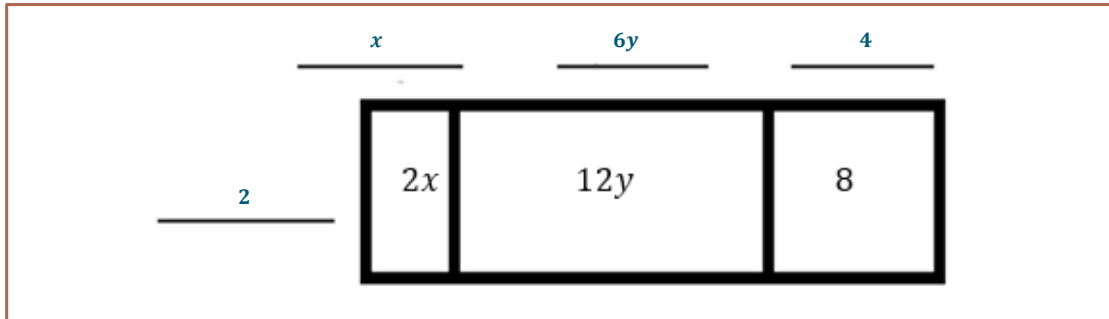
In this example, let the variables x and y stand for positive integers, and let $2x$, $12y$, and 8 represent the area of three rectangles. The goal is to find the lengths of each individual region with areas of $2x$, $12y$, and 8 , so that all three regions have the same width. Let students explore different possibilities.

Example 2

Let the variables x and y stand for positive integers, and let $2x$, $12y$, and 8 represent the area of three regions in the array. Determine the length and width of each rectangle if the width is the same for each rectangle.

- What does $2x$ represent in the first region of the array?
 - *The region has an area of $2x$ or can be covered by $2x$ square units.*

- What does the entire array represent?
 - *The entire array represents $2x + 12y + 8$ square units.*
- What is the common factor of $2x$, $12y$, and 8 ?
 - *The common factor of $2x$, $12y$, and 8 is 2 .*
- What are the missing values, and how do you know?
 - *The missing values are x , $6y$, and 4 . If the products are given in the area of the regions, divide the regions by 2 to determine the missing values.*



- Write the expression as a sum and then as a product of two factors.
 - $2x + 12y + 8 = 2(x + 6y + 4)$
- How does this exercise differ from the exercises we did during the previous lesson? How is this exercise similar to the ones we did during the previous lesson?
 - *We are doing the inverse of writing products as sums. Before, we wrote a product as a sum using the distributive property. Now, we are writing a sum as a product using the distributive property.*

Exercise 2 (3 minutes)

Have students work on the following exercise individually and discuss the results as a class.

Exercise 2

a. Write the product and sum of the expressions being represented in the rectangular array.

	$12d$	$4e$	3
2	$24d$	$8e$	6

$2(12d + 4e + 3)$, $24d + 8e + 6$

b. Factor $48j + 60k + 24$ by finding the greatest common factor of the terms.

$12(4j + 5k + 2)$

Exercise 3 (4 minutes)

Exercise 3

For each expression, write each sum as a product of two factors. Emphasize the importance of the distributive property. Use various equivalent expressions to justify equivalency.

a. $2 \cdot 3 + 5 \cdot 3$

Both have a common factor of 3, so the two factors would be $3(2 + 5)$. Demonstrate that $3(7)$ is equivalent to $6 + 15 = 21$.

b. $(2 + 5) + (2 + 5) + (2 + 5)$

This expression is 3 groups of $(2 + 5)$ or $3(2) + 3(5)$, which is $3(2 + 5)$.

c. $2 \cdot 2 + (5 + 2) + (5 \cdot 2)$

Rewrite the expression as $2 \cdot 2 + (5 \cdot 2) + (2 + 5)$, so $2(2 + 5) + (2 + 5)$, which equals to $3(2 + 5)$.

d. $x \cdot 3 + 5 \cdot 3$

The greatest common factor is 3, so factor out the 3: $3(x + 5)$.

e. $(x + 5) + (x + 5) + (x + 5)$

Similar to part (b), this is 3 groups of $(x + 5)$, so $3(x + 5)$.

f. $2x + (5 + x) + 5 \cdot 2$

*Combine like terms and then identify the common factor. $3x + 15$, where 3 is the common factor: $3(x + 5)$. Or, $2x + 2 \cdot 5 + (x + 5)$, so that $2(x + 5) + (x + 5) = 3(x + 5)$. Or, use the associative property and write:
 $2x + (5 \cdot 2) + (5 + x)$
 $2(x + 5) + (5 + x)$
 $3(x + 5)$.*

g. $x \cdot 3 + y \cdot 3$

The greatest common factor is 3, so $3(x + y)$.

h. $(x + y) + (x + y) + (x + y)$

There are 3 groups of $(x + y)$, so $3(x + y)$.

i. $2x + (y + x) + 2y$

*Combine like terms and then identify the common factor. $3x + 3y$, where 3 is the common factor. $3(x + y)$. Or, $2x + 2y + (x + y)$, so that $2(x + y) + (x + y) = 3(x + y)$. Or, use the associative property and write:
 $2x + 2y + (y + x)$
 $2(x + y)$
 $3(x + y)$.*

Example 3 (4 minutes)

Allow students to read the problem and address the task individually. Share student responses as a class.

Example 3

A new miniature golf and arcade opened up in town. For convenient ordering, a play package is available to purchase. It includes two rounds of golf and 20 arcade tokens, plus \$3.00 off the regular price. There is a group of six friends purchasing this package. Let g represent the cost of a round of golf, and let t represent the cost of a token. Write two different expressions that represent the total amount this group spent. Explain how each expression describes the situation in a different way.



- What two equivalent expressions could be used to represent the situation?
 - $6(2g + 20t - 3)$; Each person will pay for two rounds of golf and 20 tokens and will be discounted \$3.00. This expression is six times the quantity of each friend's cost.
 - $12g + 120t - 18$; The total cost is equal to 12 games of golf plus 120 tokens, minus \$18.00 off the entire bill.

Example 4 (3 minutes)

- What does it mean to take the opposite of a number?
 - You can determine the additive inverse of a number or a multiplicative inverse.
- What is the opposite of 2?
 - -2
- What is $(-1)(2)$?
 - -2
- What is $(-1)(n)$?
 - $-n$
- What are two mathematical expressions that represent the opposite of $(2a + 3b)$?
 - $(-1)(2a + 3b)$ or $-(2a + 3b)$
- Use the distributive property to write $(-1)(2a + 3b)$ as an equivalent expression.
 - $-2a - 3b$ or $-2a + (-3b)$
- To go from $-2a - 3b$ to $-(2a + 3b)$, what process occurs?
 - The terms $-2a$ and $-3b$ are written as $(-1)(2a)$ and $(-1)(3b)$, and the -1 is factored out of their sum.

Exercise 4 (3 minutes)

Exercise 4

- a. What is the opposite of $(-6v + 1)$?

$$-(-6v + 1)$$

- b. Using the distributive property, write an equivalent expression for part (a).

$$6v - 1$$



Example 5 (3 minutes)

With the class, rewrite $5a - (a - 3b)$ applying the rules for subtracting and Example 2.

Example 5

Rewrite $5a - (a - 3b)$ in standard form. Justify each step, applying the rules for subtracting and the distributive property.

$5a + (-(a + -3b))$	<i>Subtraction as adding the inverse</i>
$5a + (-1)(a + -3b)$	<i>Opposite of a number is same as multiplying by -1</i>
$5a + (-1)(a) + (-1)(-3b)$	<i>Distributive property</i>
$5a + -a + 3b$	<i>Multiplying by -1 is the same as the opposite of the number.</i>
$4a + 3b$	<i>Collect like terms</i>

Exercise 5 (7 minutes)

Encourage students to work with partners to expand each expression and collect like terms while applying the rules of subtracting and the distributive property.

Exercise 5

Expand each expression and collect like terms.

a.	$-3(2p - 3q)$	
	$-3(2p + (-3q))$	<i>Subtraction as adding the inverse</i>
	$-3 \cdot 2p + (-3) \cdot (-3q)$	<i>Distributive property</i>
	$-6p + 9q$	<i>Apply integer rules</i>
b.	$-a - (a - b)$	
	$-a + (-(a + -b))$	<i>Subtraction as adding the inverse</i>
	$-1a + (-1)(a + -1b)$	<i>Opposite of a number is same as multiplying by -1</i>
	$-1a + (-1a) + 1b$	<i>Distributive property</i>
	$-2a + b$	<i>Apply integer addition rules</i>

Closing (2 minutes)

- In writing products as sums, what is happening when you take the opposite of a term or factor?
 - *The term or factor is multiplied by -1 . When using the distributive property, every term inside the parentheses is multiplied by -1 .*
- Describe the process you used to write an expression in the form of the sum of terms as an equivalent expression in the form of a product of factors.
 - *Writing sums as products is the opposite of writing products as sums; so, instead of distributing and multiplying, the product is being factored.*

Exit Ticket (4 minutes)



Name _____

Date _____

Lesson 4: Writing Products as Sums and Sums as Products

Exit Ticket

1. Write the expression below in standard form.

$$3h - 2(1 + 4h)$$

2. Write the expression below as a product of two factors.

$$6m + 8n + 4$$



Exit Ticket Sample Solutions

1. Write the expression below in standard form.

$$3h - 2(1 + 4h)$$

$$3h + (-2(1 + 4h))$$

Subtraction as adding the inverse

$$3h + (-2 \cdot 1) + (-2h \cdot 4)$$

Distributive property

$$3h + (-2) + (-8h)$$

Apply integer rules

$$-5h - 2$$

Collect like terms

2. Write the expression below as a product of two factors.

$$6m + 8n + 4$$

The GCF for the terms is 2. Therefore, the factors are $2(3m + 4n + 2)$.

Problem Set Sample Solutions

1. Write each expression as the product of two factors.

a. $1 \cdot 3 + 7 \cdot 3$

$$3(1 + 7)$$

b. $(1 + 7) + (1 + 7) + (1 + 7)$

$$3(1 + 7)$$

c. $2 \cdot 1 + (1 + 7) + (7 \cdot 2)$

$$3(1 + 7)$$

d. $h \cdot 3 + 6 \cdot 3$

$$3(h + 6)$$

e. $(h + 6) + (h + 6) + (h + 6)$

$$3(h + 6)$$

f. $2h + (6 + h) + 6 \cdot 2$

$$3(h + 6)$$

g. $j \cdot 3 + k \cdot 3$

$$3(j + k)$$

h. $(j + k) + (j + k) + (j + k)$

$$3(j + k)$$

i. $2j + (k + j) + 2k$

$$3(j + k)$$

2. Write each sum as a product of two factors.

a. $6 \cdot 7 + 3 \cdot 7$

$$7(6 + 3)$$

b. $(8 + 9) + (8 + 9) + (8 + 9)$

$$3(8 + 9)$$

c. $4 + (12 + 4) + (5 \cdot 4)$

$$4(1 + 4 + 5)$$

d. $2y \cdot 3 + 4 \cdot 3$

$$3(2y + 4)$$

e. $(x + 5) + (x + 5)$

$$2(x + 5)$$

f. $3x + (2 + x) + 5 \cdot 2$

$$4(x + 3)$$

g. $f \cdot 6 + g \cdot 6$

$$6(f + g)$$

h. $(c + d) + (c + d) + (c + d) + (c + d)$

$$4(c + d)$$

i. $2r + r + s + 2s$

$$3(r + s)$$

3. Use the following rectangular array to answer the questions below.

	?	?	?
?	15f	5g	45

a. Fill in the missing information.

	$3f$	g	9
	?	?	?
5 ?	15f	5g	45

b. Write the sum represented in the rectangular array.

$$15f + 5g + 45$$

c. Use the missing information from part (a) to write the sum from part (b) as a product of two factors.

$$5(3f + g + 9)$$

4. Write the sum as a product of two factors.

a. $81w + 48$

$$3(27w + 16)$$

b. $10 - 25t$

$$5(2 - 5t)$$

c. $12a + 16b + 8$

$$4(3a + 4b + 2)$$

5. Xander goes to the movies with his family. Each family member buys a ticket and two boxes of popcorn. If there are five members of his family, let t represent the cost of a ticket and p represent the cost of a box of popcorn. Write two different expressions that represent the total amount his family spent. Explain how each expression describes the situation in a different way.

$$5(t + 2p)$$

Five people each buy a ticket and two boxes of popcorn, so the cost is five times the quantity of a ticket and two boxes of popcorn.

$$5t + 10p$$

There are five tickets and 10 boxes of popcorn total. The total cost will be five times the cost of the tickets, plus 10 times the cost of the popcorn.

6. Write each expression in standard form.

a. $-3(1 - 8m - 2n)$

$$-3(1 + (-8m) + (-2n))$$

$$-3 + 24m + 6n$$

b. $5 - 7(-4q + 5)$

$$5 + -7(-4q + 5)$$

$$5 + 28q + -35$$

$$28q - 35 + 5$$

$$28q - 30$$

c. $-(2h - 9) - 4h$

$$-(2h + (-9)) + (-4h)$$

$$-2h + 9 + (-4h)$$

$$-6h + 9 \text{ or } 9 - 6h$$

d. $6(-5r - 4) - 2(r - 7s - 3)$

$$6(-5r + -4) + -2(r - 7s + -3)$$

$$-30r + -24 + -2r + 14s + 6$$

$$-30r + -2r + 14s + -24 + 6$$

$$-32r + 14s - 18$$

7. Combine like terms to write each expression in standard form.

a. $(r - s) + (s - r)$

$$0$$

b. $(-r + s) + (s - r)$

$$-2r + 2s$$

c. $(-r - s) - (-s - r)$

$$0$$

d. $(r - s) + (s - t) + (t - r)$

$$0$$

e. $(r - s) - (s - t) - (t - r)$

$$2r - 2s$$