



Lesson 23: Using Sample Data to Decide if Two Population Means Are Different

Student Outcomes

- Students use data from random samples to draw informal inferences about the difference in population means.

Lesson Notes

Statistics is about making decisions concerning populations. In many settings, this involves deciding whether two population means are similar or different. More formal procedures are presented at the high school level and beyond. This lesson builds on the previous lesson to present an informal inferential procedure. The bottom line is that if two sample means are separated by at least k MADs, then it is reasonable to conclude that there is a difference between the two corresponding population means. Recall that in Lesson 21, a difference between two sample means was called “meaningful” if it is greater than what would have been expected just due to sampling variability. This is an indication that the corresponding population means are not equal.

In the previous lesson, students calculated how many MADs separated two sample means. This lesson extends that procedure to informal inference by introducing a way to decide if the two sample means are different enough to be confident in deciding that the population means differ.

Classwork

In the previous lesson, you described how far apart the means of two data sets are in terms of the MAD (mean absolute deviation), a measure of variability. In this lesson, you will extend that idea to informally determine when two sample means computed from random samples are far enough apart from each other so to imply that the population means also differ in a “meaningful” way. Recall that a “meaningful” difference between two means is a difference that is greater than would have been expected just due to sampling variability.

Example 1 (3 minutes): Texting

The purpose of this example is to take students through the informal inferential process focusing on the concept and not on calculation. Introduce the scenario presented in the example. The statistical question is whether there is a difference on average in recalling *real* words or *fake* words. Linda randomly selected n students from her district’s middle school students and assigned $\frac{n}{2}$ of them (chosen at random from the n) a list of $\frac{n}{2}$ real words and assigned the other half a list of $\frac{n}{2}$ fake words.

Example 1: Texting

With texting becoming so popular, Linda wanted to determine if middle school students memorize *real* words more or less easily than *fake* words. For example, real words are “food,” “car,” “study,” “swim;” whereas fake words are “stk,” “fonw,” “cqr,” “ttensp.” She randomly selected n students from all middle school students in her district and gave half of them a list of $\frac{n}{2}$ real words and the other half a list of $\frac{n}{2}$ fake words.

Exercises 1–3 (5–7 minutes)

Give students a moment to think about the questions and then discuss as a class.

Exercises 1–3

- How do you think Linda might have randomly selected students from all middle school students in her district?

Random selection is done in an attempt to obtain students to represent all middle school students in the district. Linda would need to number all middle school students and use a random device to generate numbers. One device to generate integers is <http://www.rossmanchance.com/applets/RandomGen/GenRandom01.htm>. Note that if there are duplicates, additional random numbers need to be generated. A second way to generate the random selections is using the random-number table from previous lessons.

- Why do you think Linda selected the students for her study randomly? Explain.

Linda randomly assigned her chosen students to two groups of each. Random assignment is done to help assure that groups are similar to each other.

- She gave the selected students one minute to memorize their list after which they were to turn the list over and after two minutes write down all the words that they could remember. Afterwards, they calculated the number of correct “words” that they were able to write down. Do you think a penalty should be given for an incorrect “word” written down? Explain your reasoning.

Answers will vary. Either position is acceptable. The purpose is to get students to take a position and argue for it.

MP.3

Exercises 4–7 (8–10 Minutes)

Let students work independently, and compare answers with a neighbor. Then discuss Exercises 6 and 7 as a class.

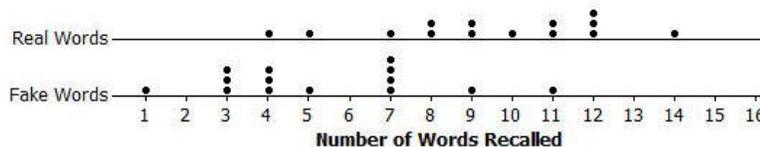
Exercises 4–7

Suppose the data (number of correct words recalled) she collected were:

For students given the real words list:

For students given the fake words list:

- On the same scale, draw dot plots for the two data sets.



- From looking at the dot plots, write a few sentences comparing the distribution of the number of correctly recalled real words with the distribution of number of correctly recalled fake words. In particular, comment on which type of word, if either, that students recall better. Explain.

There is a considerable amount of overlap between data from the two random samples. The distribution of number of real words recalled is somewhat skewed to the left; the distribution of number of fake words recalled is fairly symmetric. The real words distribution appears to be centered around 8 or 9, whereas the fake words distribution appears to be centered around 7. Whether the separation between 8 and 9 is meaningful remains to be seen. If it is meaningful, then the mean number of real words recalled is greater than the mean number of fake words recalled.

6. Linda made the following calculations for the two data sets:

	Mean	MAD
Real words recalled		
Fake words recalled		

In the previous lesson, you calculated the number of MADs that separated two sample means. You used the larger MAD to make this calculation if the two MADs were not the same. How many MADs separate the mean number of real words recalled and the mean number of fake words recalled for the students in the study?

The difference between the two means is _____ . The larger of the two MADs is _____. The number of MADs that separate the two means is _____ .

7. In the last lesson, our work suggested that if the number of MADs that separate the two sample means is 3 or more, then it is reasonable to conclude that not only do the means differ in the samples, but that the means differ in the populations as well. If the number of MADs is less than 3, then you can conclude that the difference in the sample means might just be sampling variability and that there may not be a meaningful difference in the population means. Using these criteria, what can Linda conclude about the difference in population means based on the sample data that she collected? Be sure to express your conclusion in the context of this problem.

Since _____ is below the suggested _____ MADs, Linda would conclude that the average number of real words that all middle school students in her district would recall might be the same as the average number of fake words that they would recall.

Ask your students if they are surprised by her conclusion. Some may say that they are surprised because _____ is bigger than _____. Be sure that those students understand the concept of sampling variability. Their response might be a mathematical one in comparing two numbers. Other students may not be surprised, as they can see from the dot plots that there is quite a bit of overlap.

Example 2 (2 minutes)

This example has a population database in Excel consisting of four numerical variables: Texting is in column _____, the number of minutes per day students text (whole number); ReactTime is in column _____, the number of seconds that it takes students to respond to a computer screen stimulus (two decimal places); Homework is in column _____, the total number of hours per week that students spend on doing homework both in school and at home (one decimal place); and Sleep is in column _____, the number of hours per night that students sleep (one decimal place). Display the data file on an overhead projector and be sure students understand the data located in each column.

Example 2

Ken, an eighth grade student, was interested in doing a statistics study involving sixth grade and eleventh grade students in his school district. He conducted a survey on four numerical variables and two categorical variables (grade level and gender). His Excel population database for the _____ sixth graders and _____ eleventh graders in his district has the following description:

Column	Name	Description
	ID	ID numbers are from _____ through _____ Sixth grade females Sixth grade males Eleventh grade females Eleventh grade males
	Texting	Number of minutes per day text (whole number)
	ReactTime	Time in seconds to respond to a computer screen stimulus (two decimal places)
	Homework	Total number of hours per week spend on doing homework (one decimal place)
	Sleep	Number of hours per night sleep (one decimal place)



11. From looking at the dot plots, list some observations comparing the number of hours per week that sixth graders spend on doing homework and the number of hours per week that eleventh graders spend on doing homework.

There is some overlap between the data for the two random samples. The sixth grade distribution may be slightly skewed to the left. The eleventh grade distribution is fairly symmetric. The mean number of homework hours for sixth graders appears to be around hours, whereas that for the eleventh graders is around .

12. Calculate the mean and MAD for each of the data sets. How many MADs separate the two sample means? (Use the larger MAD to make this calculation if the sample MADs are not the same.)

	Mean (hr.)	MAD (hr.)
Sixth Grade		
Eleventh Grade		

The number of MADs that separate the two means is _____ .

13. Ken recalled Linda suggesting that if the number of MADs is greater than or equal to then it would be reasonable to think that the population of all sixth grade students in his district and the population of all eleventh grade students in his district have different means. What should Ken conclude based on his homework study?

Since is greater than , it is reasonable to conclude that on average eleventh graders spend more time doing homework per week than do sixth graders.

Note: Some students may ask why they have to do the sampling when they have the population database. Since they have all the information, why not just calculate the two population means and be done with it? Good question. Perhaps the best answer is that they are learning a procedure. In practice, populations are often very large and observations for the subjects are not known. If you think that your students are bothered by having a population database, then you may decide not to give them access to it, and provide them just the data corresponding to the given random numbers. A minor rewrite of the following problem set would be needed in that case.

Closing (4 minutes)

Discuss the Lesson Summary with students.

Lesson Summary

To determine if the mean value of some numerical variable differs for two populations, take random samples from each population. It is very important that the samples be random samples. If the number of MADs that separate the two sample means is or more, then it is reasonable to think that the populations have different means. Otherwise, the population means are considered to be the same.

Exit Ticket (5 minutes)



Name _____

Date _____

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Exit Ticket

- Do eleventh grade males text more per day than eleventh grade females do? To answer this question, two randomly selected samples were obtained from the Excel data file used in this lesson. Indicate how many randomly selected eleventh grade females would be chosen for this study. Indicate how many randomly selected eleventh grade males would be chosen.
- Two randomly selected samples (one of eleventh grade females, and one of eleventh grade males) were obtained from the database. The results are indicated below:

	Mean number of minutes per day texting	MAD (minutes)
Eleventh grade females		
Eleventh grade males		

Is there a meaningful difference in the number of minutes per day that eleventh grade females and males text? Explain your answer.

Exit Ticket Sample Solutions

- Do eleventh grade males text more per day than eleventh grade females? To answer this question, two randomly selected samples were obtained from the Excel data file used in this lesson. Indicate how randomly selected eleventh grade females would be chosen for this study. Indicate how randomly selected eleventh grade males would be chosen.

To pick females, randomly selected numbers from — would be generated from a random-number generator or from a random-number table. Duplicates would be disregarded and a new number would be generated. To pick males, randomly selected numbers from — would be generated. Again, duplicates would be disregarded, and a new number would be generated.

- Two randomly selected samples (one of eleventh grade females, and one of eleventh grade males) were obtained from the database. The results are indicated below:

	Mean number of minutes per day texting	MAD (minutes)
Eleventh grade females		
Eleventh grade males		

Is there a meaningful difference in the number of minutes per day eleventh grade females and males text? Explain your answer.

The difference in the means is — min., or — min. (to the nearest hundredth of a minute).

Divide this by — minutes or the MAD for females (the larger of the two MADs): — min. to the nearest hundredth of a minute. This difference is less than — MADs, and therefore, the difference in the male and female number of minutes per day of texting is not a meaningful difference.

Problem Set Sample Solutions

- Based on Ken’s population database, compare the amount of sleep that sixth grade females get on average to the amount of sleep that eleventh grade females get on average.

Find the data for sixth grade females based on the following random ID numbers:

Find the data for eleventh grade females based on the following random ID numbers:

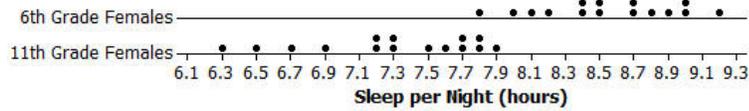
This problem compares the amount of sleep that sixth grade females get on average to the amount of sleep that eleventh grade females get on average.

Random numbers are provided for students. Provide students access to the data file (a printed copy or access to the file at the website), or if that is not possible, provide them the following values to use in the remaining questions:

Sixth grade females number of hours of sleep per night:

Eleventh grade females number of hours of sleep per night:

2. On the same scale, draw dot plots for the two sample data sets.



3. Looking at the dot plots, list some observations comparing the number of hours per week that sixth graders spend on doing homework and the number of hours per week that eleventh graders spend on doing homework.

There is a small amount of overlap between the data sets for the two random samples. The distribution of sixth grade hours of sleep is symmetric, whereas that for eleventh graders is skewed to the left. It appears that the mean number of hours of sleep for sixth grade females is around 8.5 and the mean number for eleventh grade females is around 7.5 or so. Whether or not the difference is meaningful depends on the amount of variability that separates them.

4. Calculate the mean and MAD for each of the data sets. How many MADs separate the two sample means? (Use the larger MAD to make this calculation if the sample MADs are not the same.)

	Mean (hr.)	MAD (hr.)
Sixth Grade Females		
Eleventh Grade Females		

The number of MADs that separate the two means is _____.

5. Recall that if the number of MADs in the difference of two sample means is greater than or equal to 2, then it would be reasonable to think that the population means are different. Using this guideline, what can you say about the average number of hours of sleep per night for all sixth grade females in the population compared to all eleventh grade females in the population?

Since 2.5 is well above the criteria of 2 MADs, it can be concluded that on average sixth grade females get more sleep per night than do eleventh grade females.

