



Lesson 22: Using Sample Data to Decide if Two Population Means Are Different

Student Outcomes

- Students express the difference in sample means as a multiple of a measure of variability.
- Students understand that a difference in sample means provides evidence that the population means are different if the difference is larger than what would be expected as a result of sampling variability alone.

Lesson Notes

In previous lessons, seventh graders have been challenged to learn the very important roles that random sampling and sampling variability play in estimating a population mean. This lesson provides the groundwork for Lesson 23, in which students are asked to informally decide if two population means differ from each other using data from a random sample taken from each population. Implicit in the inference is sampling variability. Explicit in the inference is calculating sample means and the process of comparing them.

This lesson develops a measure for describing how far apart two sample means are from each other in terms of a multiple of some measure of variability. In this lesson, the MAD (mean absolute deviation) is used as the measure of variability. The next lesson builds on this work, introducing the idea of making informal inferences about the difference between two population means.

Classwork

In previous lessons, you worked with one population. Many statistical questions involve comparing two populations. For example:

- On average, do boys and girls differ on quantitative reasoning?
- Do students learn basic arithmetic skills better with or without calculators?
- Which of two medications is more effective in treating migraine headaches?
- Does one type of car get better mileage per gallon of gasoline than another type?
- Does one type of fabric decay in landfills faster than another type?
- Do people with diabetes heal more slowly than people who do not have diabetes?

In this lesson, you will begin to explore how big of a difference there needs to be in sample means in order for the difference to be considered meaningful. The next lesson will extend that understanding to making informal inferences about population differences.

Example 1 (2–3 Minutes)

Briefly summarize the data in the example. Note that KenKen is not important for this problem, any type of puzzle would do. It was mentioned only to possibly spark student interest.

Example 1

Tamika’s mathematics project is to see whether boys or girls are faster in solving a KenKen-type puzzle. She creates a puzzle and records the following times that it took to solve the puzzle (in seconds) for a random sample of boys from her school and a random sample of girls from her school:

																			Mean	MAD
Boys																				
Girls																				

Exercises 1–3 (12–15 Minutes)

Work through the exercises one at a time as a class.

Exercises 1–3

- On the same scale, draw dot plots for the boys’ data and for the girls’ data. Comment on the amount of overlap between the two dot plots. How are the dot plots the same, and how are they different?
The dot plots appear to have a considerable amount of overlap. The boys’ data may be slightly skewed to the left, whereas the girls’ are relatively symmetric.

- Compare the variability in the two data sets using the MAD (mean absolute deviation). Is the variability in each sample about the same? Interpret the MAD in the context of the problem.
The variability in each data set is about the same as measured by the mean absolute deviation (around sec.). For boys and girls, a typical deviation from their respective mean times (for boys and for girls) is about sec.
- In the previous lesson, you learned that a difference between two sample means is considered to be meaningful if the difference is more than what you would expect to see just based on sampling variability. The difference in the sample means of the boys’ times and the girls’ times is seconds (seconds seconds). This difference is approximately MAD.

Verify that the students understand the difference between the sample means is approximately MAD. Boys and girls mean times differ by sec. The number of MADs that separate the sample means is — , about MAD.

a. If sec. is used to approximate the values of MAD (mean absolute deviation) for both boys and for girls, what is the interval of times that are within MAD of the sample mean for boys?
sec. equals sec., and sec. equals sec. The interval of times that are within MAD of the boys’ mean time is approximately sec. to sec.

b. Of the sample means for boys, how many of them are within that interval?
Six of the sample means for boys are within the interval.

c. Of the sample means for girls, how many of them are within the interval you calculated in part (a)?
Five of the sample means for girls are within the interval.

d. Based on the dot plots, do you think that the difference between the two sample means is a meaningful difference? That is, are you convinced that the mean time for all girls at the school (not just this sample of girls) is different from the mean time for all boys at the school? Explain your choice based on the dot plots.
Answers will vary. Sample answer: I don't think that the difference is meaningful. The dot plots overlap a lot and there is a lot of variability in the times for boys and the times for girls.

Point out to students that here they are being asked to use just their own judgment to decide if the difference is meaningful, but later in this lesson a more objective criteria will be introduced.

Note: If some of your students prefer to use box plots rather than dot plots, then the more appropriate measure of center is the median rather than the mean, and the more appropriate measure of variability to use for measuring the separation of the medians would be the interquartile range (IQR), or the difference of quartile 3 and quartile 1.

Example 2 (5–7 Minutes)

Read through the directions with students. Each pair of students will need a stopwatch to record time. Record the times for each group on the board.

An online stopwatch may be found at <http://www.online-stopwatch.com>. Start the following activity by asking students, “How many of you would be better at guessing the length of a minute when it is quiet? How many of you would be better at guessing a minute when people are talking?”

Example 2

How good are you at estimating a minute? Work in pairs. Flip a coin to determine which person in the pair will go first. One of you puts your head down and raises your hand. When your partner says “start,” keep your head down and your hand raised. When you think a minute is up, put your hand down. Your partner will record how much time has passed. Note that the room needs to be quiet. Switch roles except this time you talk with your partner during the period when the person with his or her head down is indicating when they think a minute is up. Note that the room will not be quiet.

Exercises 4–7 (10–12 minutes)

Let students work with their partner on Exercises 4–7. Then discuss and confirm as a class.

Suppose that the following data were collected:

Exercises 4–7

Group	Estimate for a minute												
Quiet													
Talking													

Use your class data to complete the following.

4. Calculate the mean minute time for each group. Then find the difference between the “quiet” mean and the “talking” mean.

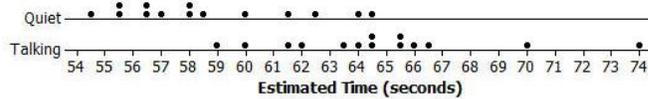
The mean of the “quiet” estimates is sec.

The mean of the “talking” estimates is sec.

The difference between the two means is sec.

5. On the same scale, draw dot plots of the two data distributions and discuss the similarities and differences in the two distributions.

The dot plots have quite a bit of overlap. The quiet group distribution is fairly symmetric; the talking group distribution is skewed somewhat to the right. The variability in each is about the same. The quiet group appears to be centered around sec. and the talking group appears to be centered around sec.



6. Calculate the mean absolute deviation (MAD) for each data set. Based on the MADs, compare the variability in each sample. Is the variability about the same? Interpret the MADs in the context of the problem.

The MAD for the quiet distribution is sec.

The MAD for the talking distribution is sec.

The MAD measurements are about the same indicating that the variability in each data set is similar. In both groups, a typical deviation of students’ minute estimates from their respective means is about sec.

7. Based on your calculations, is the difference in mean time estimates meaningful? Part of your reasoning should involve the number of MADs that separate the two sample means. Note that if the MADs differ, use the larger one in determining how many MADs separate the two means.

The number of MADs that separate the two sample means is . There is a meaningful difference between the means.

MP.6

Note: In the next lesson, it is suggested that if the number of MADs that separate two means is 3 or more, then the separation is meaningful. (i.e., is due to more than sampling variability). The number of MADs separating two means is one way to informally gauge whether or not there is a meaningful difference in the population means. The suggestion of MADs is not a hard and fast rule that provides a definitive estimate about the differences, but it represents a reasonable gauge based on both the variability observed in the distributions and the dot plots. Although a different number of MADs separating the means could have been used, providing a specific value (such as 3 MADs) is a reasonable starting point for discussing the differences in means. It also addresses the suggested measure as outlined in the standards (namely, 7.SP.B.3). It is interesting to note in this example that although the dot plots of the sample data overlap by quite a bit, it is still reasonable to think that the population means are different.

Lesson Summary

Variability is a natural occurrence in data distributions. Two data distributions can be compared by describing how far apart their sample means are. The amount of separation can be measured in terms of how many MADs separate the means. (Note that if the two sample MADs differ, the larger of the two is used to make this calculation.)

Exit Ticket (8–10 minutes)

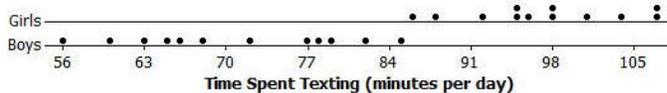
Exit Ticket Sample Solutions

Suppose that Brett randomly sampled tenth grade girls and boys in his school district and asked them for the number of minutes per day that they text. The data and summary measures follow.

Gender	Number of Minutes of Texting										mean	MAD	
Girls													
Boys													

1. Draw dot plots for the two data sets using the same numerical scales. Discuss the amount of overlap between the two dot plots that you drew and what it may mean in the context of the problem.

There is no overlap between the two data sets. This indicates that the sample means probably differ, with girls texting more than boys on average. The girls' data set is a little more compact than the boys, indicating that their measure of variability is smaller.



2. Compare the variability in the two data sets using the MAD. Interpret the result in the context of the problem.

The MAD for the boys number of texting minutes is min., which is higher than that for the girls, min. This is not surprising as seen in the dot plots. The typical deviation from the mean of is about min. for boys. The typical deviation from the mean of is about min. for girls.

3. From 1 and 2, does the difference in the two means appear to be meaningful?

The difference in means is min. Using the larger MAD of min., the means are separated by — MADs. Looking at the dot plots, it certainly seems as though a separation of more than MADs is meaningful.

Problem Set Sample Solutions

Use this space to describe any specific details about the problem set for teacher reference.

1. A school is trying to decide which reading program to purchase.
 - a. How many MADs separate the mean reading comprehension score for a standard program (,) and an activity-based program (, ,)?

The number of MADs that separate the sample mean reading comprehension score for a standard program and an activity-based program is — , about half a MAD.

- b. What recommendation would you make based on this result?

The number of MADs that separate the programs is not large enough to indicate that one program is better than the other program based on mean scores. There is no noticeable difference in the two programs.

2. Does a football filled with helium go farther than one filled with air? Two identical footballs were used, one filled with helium, and one filled with air to the same pressure. Matt was chosen from your team to do the kicking. You did not tell Matt which ball he was kicking. The data (yards) follow.

Air																			
Helium																			

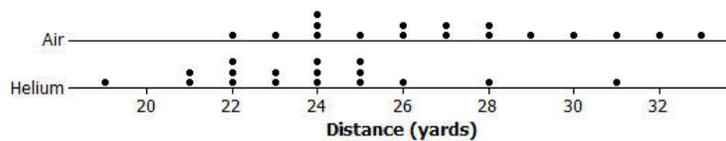
	Mean	MAD
Air		
Helium		

- a. Calculate the difference between the sample mean distance for the football filled with air, and for the one filled with helium.

The air-filled balls had a mean of _____ yds. compared to _____ yds. for the helium-filled balls, a difference of _____ yds.

- b. On the same scale, draw dot plots of the two distributions and discuss the variability in each distribution.

Based on the dot plots, it looks like the variability in the two distributions is about the same.



- c. Calculate the mean absolute deviations (MAD) for each distribution. Based on the MADs, compare the variability in each distribution. Is the variability about the same? Interpret the MADs in the context of the problem.

The MAD is _____ yds. for the air-filled balls and _____ yds. for the helium filled balls. The typical deviation from the mean of _____ is about _____ yds. for the air filled balls. The typical deviation from the mean of _____ is about _____ yds. for the helium filled balls. There is a slight difference in variability.

- d. Based on your calculations, is the difference in mean distance meaningful? Part of your reasoning should involve the number of MADs that separate the sample means. Note that if the MADs differ, use the larger one in determining how many MADs separate the two means.

There is a separation of _____ MADs. There is no meaningful distance between the means.

3. Suppose that your classmates were debating about whether going to college is really worth it. Based on the following data of annual salaries (rounded to the nearest thousands of dollars) for college graduates and high school graduates with no college experience, does it appear that going to college is indeed worth the effort? The data are from people in their second year of employment.

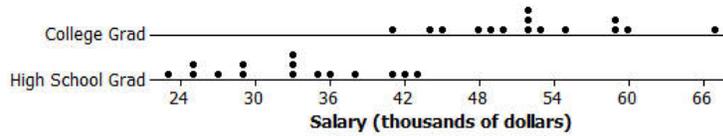
College Grad																			
High School Grad																			

- a. Calculate the difference between the sample mean salary for college graduates and for high school graduates.

The college graduates had a mean salary of _____, compared to _____ for the high school graduates, a difference of _____.

- b. On the same scale, draw dot plots of the two distributions and discuss the variability in each distribution.

Based on the dot plots, the variability of the two distributions appears to be about the same.



- c. Calculate the mean absolute deviations (MAD) for each distribution. Based on the MADs, compare the variability in each distribution. Is the variability about the same? Interpret the MADs in the context of the problem.

The MAD is _____ for college graduates, and _____ for high school graduates. The typical deviation from the mean of _____ is about _____ (or _____) for college graduates. The typical deviation from the mean of _____ is about _____ (_____) for high school graduates. The variability in the two distributions is nearly the same.

- d. Based on your calculations, is going to college worth the effort? Part of your reasoning should involve the number of MADs that separate the sample means.

There is a separation of _____ MADs. There is a meaningful difference between the population means. Going to college is worth the effort.