Lesson 11: Using Simulation to Estimate a Probability

Student Outcomes

- Students design their own simulations.
- Students learn to use two more devices in simulations: colored disks and a random number table.

Classwork

Example 1 (5 minutes): Simulation

> **Example 1: Simulation**
>
> **In the last lesson, we used coins, number cubes, and cards to carry out simulations. Another option is putting identical pieces of paper or colored disks into a container, mixing them thoroughly, and then choosing one.**
>
> **For example, if a basketball player typically makes five out of eight foul shots, then a colored disk could be used to simulate a foul shot. A green disk could represent a made shot, and a red disk could represent a miss. You could put five green and three red disks in a container, mix them, and then choose one to represent a foul shot. If the color of the disk is green, then the shot is made. If the color of the disk is red, then the shot is missed. This procedure simulates one foul shot.**

Ask your students what device they would use to simulate problems in which the probability of winning in a single outcome is –. A coin or number cube will not work. A deck of eight cards (with five of the cards designated as winners) would work but shuffling cards between draws can be time-consuming and difficult for many students. Suggest a new device: colored disks in which five green disks could represent a win and three red disks could represent a miss. Put the eight disks in a bag, shake the bag, and choose a disk. Do this as many times as are needed to comprise a trial, and then do as many trials as needed to carry out the simulation. Students could also create their own spinner with eight sections, with three sections colored one color, and five sections a different color, to represent the two different outcomes.

Exercises 1–2 (3–5 minutes)

Let students work on Exercises 1 and 2 independently. Then discuss and confirm as a class.

> **Exercises 1–2**
>
> 1. **Using colored disks, describe how one at-bat could be simulated for a baseball player who has a batting average of . Note that a batting average of means the player gets a hit (on average) three times out of every ten times at bat. Be sure to state clearly what a color represents.**
>
>   *Put ten disks in a bag, three of which are green (representing a hit), and seven are red (representing a non-hit).*
>
> 2. **Using colored disks, describe how one at-bat could be simulated for a player who has a batting average of . Note that a batting average of means that on average, the player gets hits out of at-bats.**
>
>   *Put disks in a bag, green ones (hit), and red ones (non-hit).*

Example 2 (5–7 minutes): Using Random Number Tables

Example 2: Using Random Number Tables

Why is using colored disks not practical for the situation described in Exercise 2? Another way to carry out a simulation is to use a random-number table, or a random-number generator. In a random-number table, the digits , , , , , , , , , and occur equally often in the long run. Pages and pages of random numbers can be found online.

For example, here are three lines of random numbers. The space after every five digits is only for ease of reading. Ignore the spaces when using the table.

To use the random-number table to simulate an at-bat for the hitter in Exercise 2, you could use a three-digit number to represent one at bat. The three-digit numbers from – could represent a hit, and the three-digit numbers from – could represent a non-hit. Using the random numbers above, and starting at the beginning of the first line, the first three-digit random number is , which is between and , so that simulated at-bat is a hit. The next three-digit random number is , which is a non-hit.

Begin the example by posing the question to the class. Allow for multiple responses.

- Why would using colored disks to simulate a probability of for a win not be practical?

Explain random-number tables and how they work. Be sure to clarify the following to students:

- Note that in a random-number table, the gaps between every five digits do not mean anything. They are there to help you keep track of where you are. So, ignore the gaps when generating numbers as though they are not present.
- Continue to the next line as though the lines were contiguous. Also, in a large page of random digits, start by closing your eyes and placing your finger on the page. Then, begin generating random numbers from there.
- To help students interpret these numbers, try starting with just single-digit outcomes. For example, ask students how they could use the digits to simulate a fair coin toss. Use a set of random numbers to illustrate that an even digit could be “heads,” and an odd digit could be “tails.” Another example might be how to simulate the toss of a -sided number cube. Digits to would represent the outcome of a toss, while any other digits are ignored. If necessary, develop further with students who are still unclear with this process, or create a few sample coin or number-cube questions.

Additional considerations:

- Note that to have three-digit numbers, needs to be included. You need to decide what to do with . It’s perhaps more natural to use – to represent numbers for a hit, while putting with – to represent a non-hit. The example suggests using the contiguous numbers – to represent a hit and – for a non-hit. Either way is fine; choose whichever way your students feel more comfortable using.

- The zero difficulty is eliminated if you have a calculator that has a random number key on it. This key typically allows you to input the bounds between which you want a random number, e.g., rand( ). It also allows you to specify how many random numbers you want to generate, e.g., if you would use rand( ), which is very convenient. Use of the calculator might be considered as an extension of the following exercises.

If you think your students might have difficulty jumping into Exercise 3, consider providing more practice with the random-number table by investigating the situation in which two-digit numbers would be used. For example, if a player has a batting average of , you could generate two-digit numbers from the random table, specifically – , where a selection of – would be considered a hit and – as a non-hit. (Point out to students that – would not work, because is a three-digit number.) When students are comfortable using two-digit numbers, then you can move to Exercise 3, where they use three-digit numbers to carry out the simulation.

Exercise 3 (5 minutes)

Give students an opportunity to try using a random-number table on their own. Let students work in pairs to identify the outcomes for the next six at-bats. Then confirm as a class.

Exercise 3

3. **Continuing on the first line of the random numbers above, what would the hit/non-hit outcomes be for the next six at-bats? Be sure to state the random number and whether it simulates a hit or non-hit.**

    The numbers are (non-hit), (non-hit), (non-hit), (hit), (non-hit), and (hit).

Example 3 (10–12 minutes): Baseball Player

Work through this example as a class. Students use the random-number table to simulate the probability that a hitter with a batting average gets three or four hits in four at-bats. Their simulation should be based on trials.

Pose each question one at a time, and allow for multiple responses.

Example 3: Baseball Player

A batter typically gets to bat four times in a ballgame. Consider the hitter of the previous example. Use the following steps (and the random numbers shown above) to estimate that player's probability of getting at least three hits (three or four) in four times at-bat.

a. **Describe what one trial is for this problem.**

    A trial consists of four three-digit numbers. For the first trial, , , , constitute one trial.

MP.

b. **Describe when a trial is called a success and when it is called a failure.**

    A success is getting or hits per game; a failure is getting , , or hits. For the first trial, the hitter got only hit, so it would be a failure.

c. **Simulate trials. (Continue to work as a class, or let students work with a partner.)**

MP.

d. Use the results of the simulation to estimate the probability that a hitter gets three or four hits in four times at-bat. Compare your estimate with other groups.

As a side note, the theoretical probability is calculated by considering the possible outcomes for four at bats that have either hits (HHHH) or hits (HHHM, HHMH, HMHH, and MHHH). The outcome that consists of hits has probability , and each of the outcomes with hits has probability . The theoretical probability is approximately .

Example 4 (5 minutes): Birth Month

Example 4: Birth Month

In a group of more than people, is it likely that at least two people, maybe more, will have the same birth-month? Why? Try it in your class.

Now suppose that the same question is asked for a group of only seven people. Are you likely to find some groups of seven people in which there is a match, but other groups in which all seven people have different birth-months? In the following exercise, you will estimate the probability that at least two people in a group of seven, were born in the same month.

Begin this example by posing the questions in the text to the class:

- If there are more than people in a group, will at least two people have the same birth month?
  - *Yes*
- Why?
  - *There are months and there are more than people in the group. So even if the first people in the group have different birth months, the th member will have to share with someone else.*
- What is the probability this will occur?
  - *The probability is .*

Introduce the next scenario where there are only seven people in the group. Pose the following question to the class:

- Is it clear that there could be some groups of seven people in which there is a match, but other groups in which no one shares the same birth-month?
  - *Yes, it is possible for all seven people to be born in different months.*

Note to teacher: There is a famous birthday problem that asks: "What is the probability of finding (at least one) birthday match in a group of people?" The surprising result is that there is a - chance of finding at least one birthday match in as few as people. Simulating birthdays is a bit time-consuming, so this problem simulates birth-months.

Exercises 4–7 (10–15 minutes)

This exercise asks students to estimate the probability of finding at least two people with the same birth month in a group of people but does not have students actually carry out the simulation. If time allows, you could have each student carry out the simulation times, and then either compare or pool their results.

Let students work on the exercises in small groups. Then, let each group share its design.

Exercises 4–7

4. **What might be a good way to generate outcomes for the birth-month problem—using coins, number cubes, cards, spinners, colored disks, or random numbers?**

Answers will vary; keep in mind that the first thing to do is specify how a birth-month for one person is going to be simulated. For example, a dodecahedron is a -sided solid. Each of its sides would represent one month.

The following will not work: coin (only two outcomes), number cube (only six outcomes and need .)

The following devices will work: cards (could label twelve cards January through December), spinners (could make a spinner with twelve equal sectors), colored disks (would need different colors and then remember which color represents which month), disks would work if you could write the name of a month on them, random number table (two-digit numbers , , ..., would work, but , , through would have to discarded, very laborious).

5. **How would you simulate one trial of seven birth-months?**

Answers will vary; suppose students decide to use disks with the names of the months printed on them. To generate a trial, put the disks in a bag. Then, shake the bag and choose a disk to represent the first person's birthday. Then, replace the disk and do the process six more times. The list of seven birth-months generates a trial.

6. **How is a success determined for your simulation?**

A success would be if there is at least one match in the seven.

7. **How is the simulated estimate determined for the probability that a least two in a group of seven people were born in the same month?**

Repeat this times, count the number of successes, and divide it by to get the estimated probability of having at least one birth-month match in a group of seven people.

Closing (5 minutes)

Discuss with students the Lesson Summary.

Lesson Summary

In the previous lesson, you carried out simulations to estimate a probability. In this lesson, you had to provide parts of a simulation design. You also learned how random numbers could be used to carry out a simulation.

To design a simulation:

- **Identify the possible outcomes and decide how to simulate them, using coins, number cubes, cards, spinners, colored disks, or random numbers.**
- **Specify what a trial for the simulation will look like and what a success and a failure would mean.**
- **Make sure you carry out enough trials to ensure that the estimated probability gets closer to the actual probability as you do more trials. There is no need for a specific number of trials at this time; however, you want to make sure to carry out enough trials so that the relative frequencies level off.**

Exit Ticket (5–7 minutes)

Name ____ Date____________________

Lesson 11: Using a Simulation to Estimate a Probability

Exit Ticket

Liang wants to form a chess club. His principal says that he can do that if Liang can find six players, including him. How would you conduct a simulated model that estimates the probability that Liang will find at least five other players to join the club if he asks eight players who have a chance of agreeing to join the club? Suggest a simulation model for Liang by describing how you would do the following parts.

a. Specify the device you want to use to simulate one person being asked.

b. What outcome(s) of the device would represent the person agreeing to be a member?

c. What constitutes a trial using your device in this problem?

d. What constitutes a success using your device in this problem?

e. Based on trials, using the method you have suggested, how would you calculate the estimate for the probability that Liang will be able to form a chess club?

Exit Ticket Sample Solutions

Liang wants to form a chess club. His principal says that he can do that if Liang can find six players, including him. How would you conduct a simulated model that estimates the probability that Liang will find at least five other players willing to join the club if he asks eight players who have a chance of agreeing to join the club? Suggest a simulation model for Liang by describing how you would do the following parts:

a. Specify the device you want to use to simulate one person being asked.

Answers will vary; you may want to discuss what devices can be used to simulate a chance. Using single digits in a random number table would probably be the quickest and most efficient device. – could represent "yes," and could represent a "no" response.

b. What outcome(s) of the device would represent the person agreeing to be a member?

Answers will vary based on device from part (a).

c. What constitutes a trial using your device in this problem?

Using a random-number table, a trial would consist of eight random digits.

d. What constitutes a success using your device in this problem?

Answers will vary; based on the above, a success is at least five people agreeing to join and would be represented by any set of digits with at least five of the digits being from – . Note that a random string would represent of people agreeing to be a member, which is a success. The string would represent a failure.

e. Based on trials using the method you have suggested, how would you calculate the estimate for the probability that Liang will be able to form a chess club?

Based on such trials, the estimated probability that Liang will be able to form a chess club would be the number of successes divided by .

Problem Set Sample Solutions

Important Note: You will need to provide your students with a page of random numbers. For ease of grading, you may want students to generate their outcomes starting from the same place in the table.

1. A model airplane has two engines. It can fly if one engine fails but is in serious trouble if both engines fail. The engines function independently of one another. On any given flight, the probability of a failure is for each engine. Design a simulation to estimate the probability that the airplane will be in serious trouble the next time it goes up.

a. How would you simulate the status of an engine?

Answers will vary; it is possible to use a random number table. The failure status of an engine can be represented by the digit , while digits – represent an engine in good status.

b. **What constitutes a trial for this simulation?**

A trial for this problem would be a pair of random digits, one for each engine. The possible equally likely pairings would be (where stands for any digit from –). There are of them. represents both engines failing; represents the left engine failing, but the right engine is good; represents the right engine failing but the left engine is good; represents both engines are in good working order.

c. **What constitutes a success for this simulation?**

A success would be both engines failing, which is represented by .

d. **Carry out trials of your simulation, list your results, and calculate an estimate of the probability that the airplane will be in serious trouble the next time it goes up.**

Answers will vary; divide the number of successes by .

2. **In an effort to increase sales, a cereal manufacturer created a really neat toy that has six parts to it. One part is put into each box of cereal. Which part is in a box is not known until the box is opened. You can play with the toy without having all six parts, but it is better to have the complete set. If you are really lucky, you might only need to buy six boxes to get a complete set, but if you are very unlucky, you might need to buy many, many boxes before obtaining all six parts.**

a. **How would you represent the outcome of purchasing a box of cereal, keeping in mind that there are six different parts? There is one part in each box.**

Answers will vary; since there are six parts in a complete set, the ideal device to use in this problem is a number cube. Each number represents a different part.

b. **What constitutes a trial in this problem?**

Students are asked to estimate the probability that it takes or more boxes to get all six parts, so it is necessary to look at the outcomes of the first boxes. One roll of the number cube represents one box of cereal. A trial could be a string of digits from – , the results of rolling a number cube.

c. **What constitutes a success in a trial in this problem?**

A success would then be looking at the digits and seeing if at least one digit from – is missing. For example, the string: would count as a success since part 3 was not acquired, whereas would be considered a failure because it took fewer than boxes to get all six parts.

d. **Carry out trials, list your results, and compute an estimate of the probability that it takes the purchase of 10 or more boxes to get all six parts.**

Students are asked to generate such trials, count the number of successes in the trials, and divide the number by . The result is the estimated probability that it takes or more boxes to acquire all six parts.

3. **Suppose that a type A blood donor is needed for a certain surgery. Carry out a simulation to answer the following question: If of donors have type A blood, what is an estimate of the probability that it will take at least four donors to find one with type A blood?**

Note: this problem is taken from the Common Core Standards Grade 7.SP.8c.

a. **How would you simulate a blood donor having or not having type A?**

With taken as the probability that a donor has type A blood, a random digit would be a good device to use. For example, , , , could represent type A blood, and , , , , , could represent non-type A blood.

b. **What constitutes a trial for this simulation?**

The problem asks for the probability that it will take four or more donors to find one with type A blood. That implies that the first three donors do not have type A blood. So a trial is three random digits.

c. **What constitutes a success for this simulation?**

A success is none of the three digits are , , , or . For example, would be a success since none of the donors had type A blood. An example of a failure would be .

d. **Carry out trials, list your results, and compute an estimate for the probability that it takes at least four donors to find one with type A blood.**

Students are to generate such trials, count the number of successes, and divide by to calculate their estimated probability of needing four or more donors to get one with type A blood.