



## Lesson 9: Comparing Estimated Probabilities to Probabilities Predicted by a Model

### Student Outcomes

- Students compare estimated probabilities to those predicted by a probability model.

### Classwork

This lesson continues students' exploration in collecting data from chance experiments to estimate probabilities. Students will be presented a scenario of a pretend quiz show (Picking Blue!). A summary of the game is provided to students and should be read as a group before the activity of this lesson begins. The game is simple, but it is important that students understand its rules.

Prepare bags of chips, labeled as Bag A and Bag B, for students to use in this lesson. As a suggestion, make Bag A the bag with the same number of blue and red chips (e.g., five red and five blue), and Bag B the bag with a different number of blue to red (e.g., three blue and nine red). Students will ultimately decide which of two bags gives them a better chance of picking a blue chip. When the game begins, a chip is picked from one of the bags. If a blue chip is picked, students who selected that bag win a Blue Token. The chip is then placed back in the bag, and another chip is selected. If this chip is blue, they win another Blue Token, and the process continues; but, if the chip is red, the game moves to the other bag. If a chip picked from that bag is blue, the students who picked that bag win a Blue Token. The process continues until a red chip is selected that ends the game.

Students are provided a 5-minute research time. It is the research that allows students to estimate the probability of picking a blue chip and to use their estimated probability to decide which bag to select when they play the game. They are faced with two bags. They know the probability of picking a blue chip is  $\frac{1}{2}$  in one of the bags, although they do not know which bag it is. Nor do they know the composition of red and blue chips in the other bag. The process of pulling out a chip, recording its color, putting it back in the bag, and then repeating this process with each bag provides data to estimate the probability. Students examine the relative frequencies of picking blue from each bag and continue to make selections until they are confident that they have a good estimate of the probability of picking blue based on the relative frequencies.

The exploration is presented to students in stages. Questions are posed at each stage that offer opportunities to help students think about the process and the goals of this game.

## Exploratory Challenge (35 minutes)

## Game Show—Picking Blue!

Imagine, for a moment, the following situation: You and your classmates are contestants on a quiz show called *Picking Blue!* There are two bags in front of you, Bag A and Bag B. Each bag contains red and blue chips. You are told that one of the bags has exactly the same number of blue chips as red chips. But you are told nothing about the number of blue and red chips in the second bag.

Each student in your class will be asked to select either Bag A or Bag B. Starting with Bag A, a chip is randomly selected from the bag. If a blue chip is drawn, all of the students in your class who selected Bag A win a Blue Token. The chip is put back in the bag. After mixing up the chips in the bag, another chip is randomly selected from the bag. If the chip is blue, the students who picked Bag A win another Blue Token. After the chip is placed back into the bag, the process continues until a red chip is picked. When a red chip is picked, the game moves to Bag B. A chip from the Bag B is then randomly selected. If it is blue, all of the students who selected Bag B win a Blue Token. But if the chip is red, the game is over. Just like for Bag A, if the chip is blue, the process repeats until a red chip is picked from the bag. When the game is over, the students with the greatest number of Blue Tokens are considered the winning team.

Without any information about the bags, you would probably select a bag simply by guessing. But surprisingly, the show's producers are going to allow you to do some research before you select a bag. For the next minutes, you can pull a chip from either one of the two bags, look at the chip, and then put the chip back in the bag. You can repeat this process as many times as you want within the minutes. At the end of minutes, you must make your final decision and select which of the bags you want to use in the game.

## Getting Started

Assume that the producers of the show do not want give away a lot of their Blue Tokens. As a result, if one bag has the same number of red and blue chips, do you think the other bag would have more, or fewer, blue chips than red chips? Explain your answer.

*Anticipate answers that indicate the producers would likely want the second bag to have fewer blue chips. If a participant selects that bag, it would mean the participant is more likely to lose this game.*

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The question posed in this stage asks students to think about the number of blue and red chips in the bag that does not have the same number of blue and red chips. During the discussion of this question, ask students to consider their options. What if there were more blue chips than red chips? What if there was the same number of blue and red chips in this bag also? What if there were fewer blue chips than red chips? Is it likely the producers of the game would create a second bag in which the probability of picking a blue chip is less than ? The purpose of this question, however, is to have students think about what they will find from their research.

**Planning the Research**

Your teacher will provide you with two bags labeled A and B. You have \_\_\_\_\_ minutes to experiment with pulling chips one at a time from the bags. After you examine a chip, you must put it back in the bag. Remember, no fair peeking in the bags as that will disqualify you from the game. You can pick chips from just one bag, or you can pick chips from one bag and then the other bag.

Use the results from \_\_\_\_\_ minutes of research to determine which bag you will choose for the game.

Provide a description outlining how you will carry out your research:

Organize the class in small groups. Before class, prepare Bag A and Bag B for each group so that they can carry out their research. As previously suggested, make Bag A the bag with same number of blue and red chips (e.g., five red and five blue), and Bag B the bag with a different number of blue to red (e.g., three blue and nine red). These two bag compositions were tested, and at about \_\_\_\_\_ picks from each bag, the estimated probabilities were able to provide a good indication of the probability of picking a blue chip. (Remember, however, there is still a possibility that you or the students may run into a situation that is not clear after \_\_\_\_\_ picks!)

Students are asked to present their plans for developing this research. They are expected to outline the steps they will implement during the next \_\_\_\_\_ minutes to estimate the probability of picking blue for each bag. They are expected to carry out their plans as a small group. The plans provide you with an indication of whether or not students understand the game.

Students should organize the results of the data collection on their own. Most students will likely summarize the outcomes in a table. Provide students a ruler to assist in making a table. A table was intentionally not provided in the student lesson so that you can see if they can organize the data and calculate the relative frequencies. For students who struggle with this process, however, you are encouraged to review the tables used in the previous lesson or to provide them with a blank table similar to the table used in Lesson 8.

**Carrying Out the Research**

Share your plan with your teacher. Your teacher will verify whether your plan is within the rules of the quiz show.

Approving your plan does not mean, however, that your teacher is indicating that your research method offers the most accurate way to determine which bag to select. If your teacher approves your research, carry out your plan as outlined.

Record the results from your research, as directed by your teacher.

Students implement their plan in small groups. Ask students what the data indicate about the probability of picking a blue chip. If time permits, have students produce a brief report of their research that summarizes their findings. Their report should include what choice they are making and why. Allow students to explain to you their reports, or encourage students to briefly summarize their findings to the class.

**Playing the Game**

After the research has been conducted, the competition begins. First, your teacher will shake up Bag A. A chip is selected. If the chip is blue, all students who selected Bag A win an imaginary Blue Token. The chip is put back in the bag, and the process continues. When a red chip is picked from Bag A, students selecting Bag A have completed the competition. Your teacher will now shake up Bag B. A chip is selected. If it is blue, all students who selected Bag B win an imaginary Blue Token. The process continues until a red chip is picked. At that point, the game is over.

How many Blue Tokens did you win?

After each group has obtained an estimated probability of picking blue from each bag, play the game. Divide the class into two groups based on which bag they will pick when the game starts. It is very possible the entire class will pick the same bag. If that happens, then for the purposes of a wrap-up discussion, you select the other bag and tell students that when you pick a chip from that bag, you will get the Blue Token if a blue chip is selected.

Play the game a couple of times by picking a chip and determining if a Blue Token is awarded. It is possible no tokens were given out if the first chip picked from each bag was red. Play the game several times, so that students can discuss the possible outcomes of winning.

**Examining Your Results**

At the end of the game, your teacher will open the bags and reveal how many blue and red chips were in each bag. Answer the following questions. (See: *Closing Questions*.) After you have answered these questions, discuss them with your class.

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After playing the game several times, open the bags and show students the contents. Discuss whether the estimated probabilities provided an indication of the contents of the bags. Expect students to comment that they were looking for an estimated probability of  $\frac{1}{2}$  and that the relative frequencies provided them with an indication of the bag with the same number of red and blue chips.

Direct students to read and answer the five questions provided in the student lesson as a summary of the process. After 5 minutes of reading and writing their responses, discuss the questions as a group. Use their answers as an indication of student outcomes. These questions serve as the Exit Ticket for this lesson.

**Closing (10 minutes)**

1. Before you played the game, what were you trying to learn about the bags from your research?

*Students should indicate they were trying to learn the estimated probability of picking a blue chip, without knowing the theoretical probability.*

2. What did you expect to happen when you pulled chips from the bag with the same number of blue and red chips? Did the bag that you thought had the same number of blue and red chips yield the results you expected?

*Students should indicate that they were looking for an estimated probability that was close to  $\frac{1}{2}$ . They should connect that estimated probability of picking blue with the bag that had the same number of red and blue chips.*

3. How confident were you in predicting which bag had the same number of blue and red chips? Explain.

*Answers will vary. Students' confidence will be based on the data collected. The more data collected, the closer the estimates are likely to be to the actual probabilities.*

4. What bag did you select to use in the competition and why?

*Answers will vary. It is anticipated that they would pick the bag with the larger estimated probability of picking a blue chip based on the many chips they selected during the research stage. Ask students to provide evidence of their choice. For example, "I picked chips from Bag A, and were blue and were red. The results are very close to probability of picking each color. I think this indicates that there were likely an equal number of each color in this bag."*

5. If you were the show's producers, how would make up the second bag? (Remember, one bag has the same number of red and blue chips.)

*Answers will vary. Allow students to speculate on a number of possible scenarios. Most students should indicate a second bag that had fewer blue chips, but some may speculate on a second bag that is nearly the same (to make the game more of a guessing game), or a bag with very few blue, (thus providing a clearer indication which bag had the same number of red and blue chips).*

6. If you picked a chip from Bag B times and found that you picked each color exactly times, would you know for sure that that bag was the one with equal numbers of each color?

*Students indicate that they are quite confident they have the right bag, as getting that result is likely to occur from the bag with equal numbers of each colored chips (or close to equal numbers). However, students should understand that even a bag with a different number of colored chips could have that outcome. But the probability of that happening from a bag with noticeably different numbers of chips is low but not impossible.*

#### Lesson Summary

- The long-run relative frequencies can be used as estimated probabilities of events.
- Collecting data on a game or chance experiment is one way to estimate the probability of an outcome.
- The more data collected on the outcomes from a game or chance experiment, the closer the estimates of the probabilities are likely to be the actual probabilities.

### Exit Ticket

As this was an exploratory lesson, the exit problem is incorporated into the closing questions of Examining Your Results.

Problem Set Sample Solutions

Jerry and Michael played a game similar to *Picking Blue!*. The following results are from their research using the same two bags:

Jerry's research:

	Number of Red chips picked	Number of Blue chips picked
Bag A		
Bag B		

Michael's research:

	Number of Red chips picked	Number of Blue chips picked
Bag A		
Bag B		

1. If all you knew about the bags were the results of Jerry's research, which bag would you select for the game? Explain your answer.

*Using only Jerry's research, the greater relative frequency of picking a blue chip would be Bag A, or . There were selections. Eight of the selections resulted in picking blue.*

2. If all you knew about the bags were the results of Michael's research, which bag would you select for the game? Explain your answer.

*Using Michael's research, the greater relative frequency of picking a blue chip would be Bag B, or . There were selections. Eighteen of the selections resulted in picking blue.*

3. Does Jerry's research or Michael's research give you a better indication of the make-up of the blue and red chips in each bag? Explain why you selected this research.

*Michael's research would provide a better indication of the probability of picking a blue chip as it was carried out times compared to times for Jerry's research. The more outcomes that are carried out, the closer the relative frequencies approach the estimated probability of picking a blue chip.*

4. Assume there are chips in each bag. Use either Jerry's or Michael's research to estimate the number of red and blue chips in each bag. Then explain how you made your estimates.

Bag A

Number of red chips:

Number of blue chips:

Bag B

Number of red chips:

Number of blue chips:

*Answers will vary. Anticipate that students see Bag B as the bag with nearly the same number of blue and red chips, and Bag A as possibly having a third of the chips blue. Answers provided by students should be based on the relative frequencies. A sample answer based on this reasoning is*

*Bag A: blue chips and red chips.*

*Bag B: blue chips and red chips.*

5. In a different game of *Picking Blue!*, two bags each contain red, blue, green, and yellow chips. One bag contains the same number of red, blue, green, and yellow chips. In the second bag, half the chips are blue. Describe a plan for determining which bag has more blue chips than any of the other colors.

*Students should describe a plan similar to the plan implemented in the lesson. Students would collect data by selecting chips from each bag. After several selections, the estimated probabilities of selecting blue from each bag would suggest which bag has more blue chips than chips of the other colors.*