



## Lesson 7: Calculating Probabilities of Compound Events

### Student Outcomes

- Students will calculate probabilities of compound events.

### Lesson Overview

The content of this lesson is deliberately similar to the material in Lesson 6. For most questions, an extra stage has been added to the corresponding original Lesson 6 question.

Students are likely to notice this relation to previous material. When they do, consider reinforcing the idea that tree diagrams can be used to effectively map out a sample space, and to compute probabilities, no matter how many stages or outcomes a situation has.

A previous lesson introduced *tree diagrams* as an effective method of displaying the possible outcomes of certain multistage chance experiments. Additionally, in such situations, tree diagrams were shown to be helpful for computing probabilities.

In those previous examples, diagrams primarily focused on cases with two stages. However, the basic principles of tree diagrams can apply to situations with more than two stages.

### Classwork

#### Example 1 (4–6 minutes): Three Nights of Games

Discuss examples in which a tree diagram could organize and visualize outcomes. Pose the following discussion problems to students, and have them sketch trees as possible ways to organize outcomes simply. Emphasize the ways in which the trees help to organize the outcomes in these two examples.

- Decorating a House

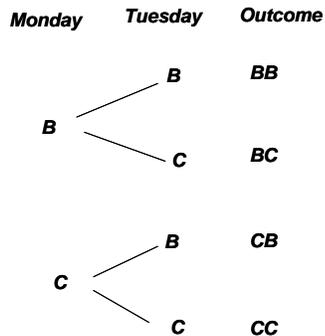
A designer has three different paints to use for a client's house—red, white, and blue. The designer is responsible for decorating two rooms of the house—the kitchen and a home office. If each room is expected to be a different color, how could a tree diagram be used to organize the possible outcomes of what color each room is painted? (Organize the tree by setting up kitchen to office. Under each room, use a tree to organize the possible paint colors that could be used for each room.) What are the different ways the two rooms could be painted?

- Organizing a School Schedule

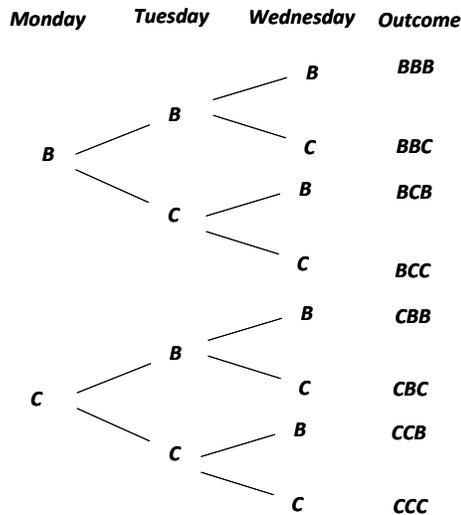
Consider the following possible scheduling problem. Ralph needs to select courses for two hours of his school day. Possible courses he can take are music, science, and mathematics. Each course is offered during the first and second hours of a school day. How could a tree diagram be used to organize the possible outcomes of Ralph's schedule for the two hours? (Organize the tree by setting up Hour 1 to Hour 2. Under each period, use a tree to organize the possible courses Ralph could take.) What are the different ways Ralph's schedule could look?

**Example 1: Three Nights of Games**

Recall a previous example where a family decides to play a game each night, and they all agree to use a four-sided die in the shape of a pyramid (where each of four possible outcomes is equally likely) each night to randomly determine if the game will be a board ( ) or a card ( ) game. The tree diagram mapping the possible overall outcomes over two consecutive nights was as follows:



But how would the diagram change if you were interested in mapping the possible overall outcomes over three consecutive nights? To accommodate this additional “third stage,” you would take steps similar to what you did before. You would attach all possibilities for the third stage (Wednesday) to each branch of the previous stage (Tuesday).



Read through the example in the student lesson as a class. Convey the following important points about the tree diagram for this example:

**MP.**

- The tree diagram is an important way of organizing and visualizing outcomes.
- The tree diagram is particularly useful when the experiment can be thought of as occurring in stages.
- When the information about probabilities associated with each branch is included, the tree diagram facilitates the computation of the probabilities of possible outcomes.
- The basic principles of tree diagrams can apply to situations with more than two stages.

**Exercises 1–3 (6 minutes)**

Let students work independently, and then discuss and confirm answers as a class.

**Exercises 1–3**

- If “ ” represents three straight nights of board games, what does “ ” represent?  
 “ ” would represent a card game on the first night and a board game on each of the second and third nights.
- List all outcomes where exactly two board games were played over three days. How many outcomes were there?  
 , , and —there are outcomes.
- There are eight possible outcomes representing the three nights. Are the eight outcomes representing the three nights equally likely? Why or why not?  
 As in the exercises of the previous lesson, the probability of and are not the same. As a result, the probability of the outcome (all three nights involve card games) is not the same as (all three nights playing board games). The probability of playing cards was only – in the previous lesson.

**Example 2 (5 minutes): Three Nights of Games (with Probabilities)**

**Example 2: Three Nights of Games (with Probabilities)**

In the example above, each night's outcome is the result of a chance experiment (rolling the four-sided die). Thus, there is a probability associated with each night's outcome.

By multiplying the probabilities of the outcomes from each stage, you can obtain the probability for each “branch of the tree.” In this case, you can figure out the probability of each of our eight outcomes.

For this family, a card game will be played if the die lands showing a value of , and a board game will be played if the die lands showing a value of , , or . This makes the probability of a board game ( ) on a given night .

Let's use a tree to examine the probabilities of the outcomes for the three days.

Monday	Tuesday	Wednesday	Outcome		
B 0.75	B 0.75	B	BBB	$(0.75)(0.75)(0.75) = 0.421875$	
		C 0.25	BBC		
	C 0.25	B 0.75	BCB		
		C 0.25	BCC		
	C 0.25	B 0.75	B 0.75	CBB	
			C 0.25	CBC	$(0.25)(0.75)(0.25) = 0.046875$
C 0.25		B 0.75	CCB		
		C 0.25	CCC		

MP.

Read through the example with students and explain how to calculate the probabilities for the outcomes and as shown on the tree diagram. Then ask students:

- What does “ ” represent?
  - “ ” represents the sequence of card game the first night, board game the second night, and card came the third night.
- The probability of “ ” is approximately . What does this probability mean?
  - The probability of “ ” is very small. This means that the outcome of is not expected to happen very often.

**Exercises 4–6 (5 minutes)**

Let students work independently on Exercises 4–5, and confirm their answers with a neighbor.

**Exercises 4–6**

4. Probabilities for two of the eight outcomes are shown. Calculate the approximate probabilities for the remaining six outcomes.

:

:

:

:

:

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5. What is the probability that there will be exactly two nights of board games over the three nights?

*The three outcomes that contain exactly two nights of board games are , and . The probability of exactly two nights of board games would be the sum of the probabilities of these outcomes, or:*

Point out to students that the sum of probabilities of these three outcomes is not  $\frac{1}{8}$ , or . Why? Students previously discussed that outcomes for the three nights are not equally likely.

6. What is the probability that the family will play at least one night of card games?

*This “at least” question could be answered by subtracting the probability of no card games ( ) from . Discuss with students that when you remove the from the list of eight outcomes, the remaining outcomes include at least one night of card games. The probability of three nights of board games ( ) is . Therefore, the probability of at least one night of card games would be the probability of all outcomes (or ) minus the probability of all board games, or:*

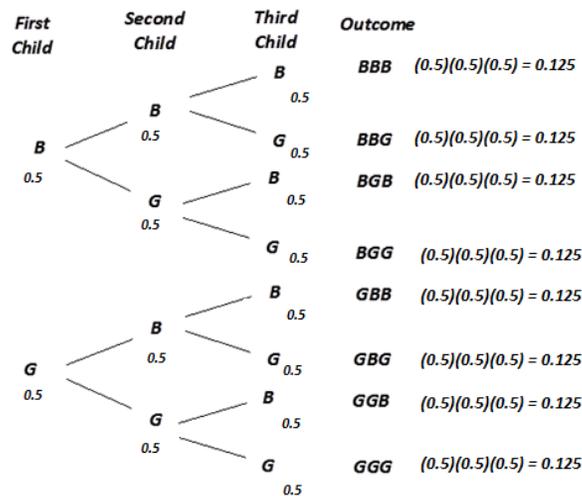
**Exercises 7–10 (15 minutes): Three Children**

After developing the tree diagram, pose the questions to students one at a time. Allow for more than one student to offer an answer for each question, encouraging a brief (2 minute) discussion.

**Exercises 7–10**

A neighboring family just welcomed their third child. It turns out that all 3 of the children in this family are girls, and they are not twins. Suppose that for each birth the probability of a “boy” birth is  $\frac{1}{2}$ , and the probability of a “girl” birth is also  $\frac{1}{2}$ . What are the chances of having 3 girls in a family’s first 3 births?

7. Draw a tree diagram showing the eight possible birth outcomes for a family with 3 children (no twins). Use the symbol “B” for the outcome of “boy” and “G” for the outcome of “girl.” Consider the first birth to be the “first stage.” (Refer to Example 1 if you need help getting started.)



8. Write in the probabilities of each stage's outcomes in the tree diagram you developed above, and determine the probabilities for each of the eight possible birth outcomes for a family with 3 children (no twins).

*In this case, since the probability of a boy is  $\frac{1}{2}$ , and the probability of a girl is  $\frac{1}{2}$ , all eight outcomes will have a probability of  $\frac{1}{8}$  of occurring.*

9. What is the probability of a family having 3 girls in this situation? Is that greater than or less than the probability of having exactly 2 girls in 3 births?

*The probability of a family having 3 girls is  $\frac{1}{8}$ . This is less than the probability of having exactly 2 girls in 3 births, which is  $\frac{3}{8}$  (the sum of the probabilities of  $\frac{1}{8}$ ,  $\frac{1}{8}$ , and  $\frac{1}{8}$ ). Another way of explaining this to students is to point out that the probability of each outcome is the same, or  $\frac{1}{8}$ . Therefore, the probability of 3 girls in 3 births is the probability of just one possible outcome. The probability of having exactly 2 girls in 3 births is sum of the probabilities of three outcomes. Therefore, the probability of one of the outcomes is less than the probability of the sum of three outcomes (and again, emphasize that the probability of each outcome is the same).*

10. What is the probability of a family of 3 children having at least 1 girl?

*The probability of having at least 1 girl would be found by subtracting the probability of no girls (or all boys,  $\frac{1}{8}$ ) from 1, or  $\frac{7}{8}$ .*

Ask students if they know any families with exactly 3 children. Consider organizing the possible examples students provide (e.g., 3 boys, 2 boys and 1 girl, etc.). Compare and discuss the relative frequencies of the examples to the above probabilities.

**Closing (5 minutes)**

Consider posing the following questions, and allow a few student responses for each:

- If you knew that a certain game where the only outcomes are “win” or “lose” offered a probability of winning each time of  $\frac{1}{2}$ , given what has been discussed so far, can you calculate the probability of losing such a game five consecutive times?
  - *If the chance of winning a game is  $\frac{1}{2}$ , the chance of losing a game is  $\frac{1}{2}$ . So the chance of losing five times would be  $\frac{1}{2^5}$  or approximately  $\frac{1}{32}$ .*
- Did you need to draw a tree diagram for that question? Why or why not?
  - *No, it is not necessary for this particular outcome. When the outcome represents the same event occurring several times, the probability of that outcome is the probability of the event multiplied by itself for each time the event occurred. In this case, it was (  $\frac{1}{2}$  ) times itself 5 times.*
- What are the chances of winning at least once in those five games?
  - *If you win at least once, the only outcome you avoided would be “losing five times.” So all other outcomes would be valid, and the sum of all of their probabilities would be  $1 - \frac{1}{2^5} = \frac{31}{32}$ , so there’s actually a good chance you would win at least once over five games.*

**Lesson Summary**

The use of tree diagrams is not limited to cases of just two stages. For more complicated experiments, tree diagrams are used to organize outcomes and to assign probabilities. The tree diagram is a visual representation of outcomes that involve more than one event.

**Exit Ticket (5–7 minutes)**



Name \_\_\_\_\_

Date \_\_\_\_\_

## Lesson 7: Calculating Probabilities of Compound Events

### Exit Ticket

In a laboratory experiment, three mice will be placed in a simple maze that has just one decision point where a mouse can turn either left ( ) or right ( ). When the first mouse arrives at the decision point, the direction he chooses is recorded. The same is done for the second and the third mice.

1. Draw a tree diagram where the first stage represents the decision made by the first mouse, and the second stage represents the decision made by the second mouse, and so on. Determine all eight possible outcomes of the decisions for the three mice.

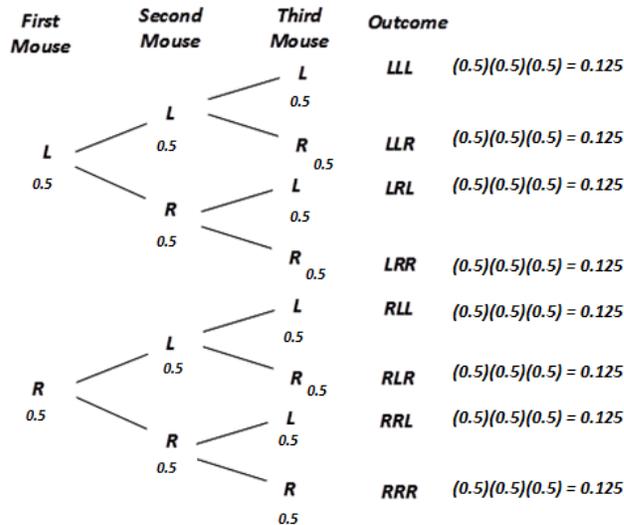


2. Use the tree diagram from Question 1 to help answer the following question. If, for each mouse, the probability of turning left is  $\frac{1}{2}$  and the probability of turning right is  $\frac{1}{2}$ , what is the probability that only one of the three mice will turn left?
3. If the researchers conducting the experiment add food in the simple maze such that the probability of each mouse turning left is now  $\frac{1}{3}$ , what is the probability that only one of the three mice will turn left? To answer question, use the tree diagram from Question 1.

Exit Ticket Sample Solutions

In a laboratory experiment, three mice will be placed in a simple maze that has just one decision point where a mouse can turn either left ( L ) or right ( R ). When the first mouse arrives at the decision point, the direction he chooses is recorded. The same is done for the second and the third mice.

1. Draw a tree diagram where the first stage represents the decision made by the first mouse, and the second stage represents the decision made by the second mouse, and so on. Determine all eight possible outcomes of the decisions for the three mice.

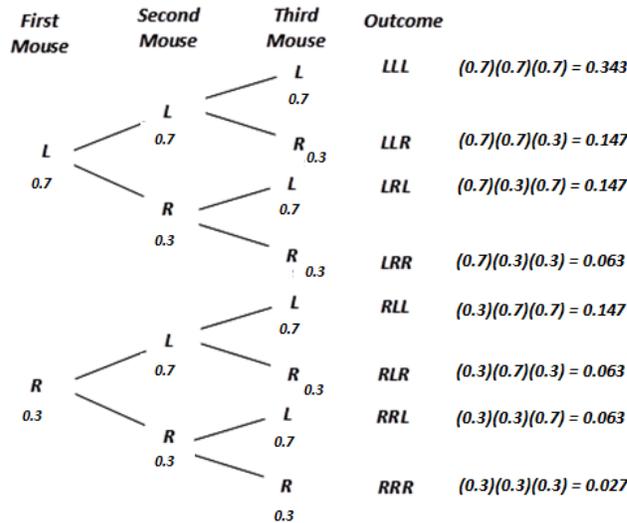


2. If the probability of turning left is  $\frac{1}{2}$  and the probability of turning right is  $\frac{1}{2}$  for each mouse, what is the probability that only one of the three mice will turn left?

*There are three outcomes that have exactly one mouse turning left: LRR, RLR, and RRL. Each has a probability of  $\frac{1}{8}$ , so the probability of having only one of the three mice turn left is  $\frac{3}{8}$ .*

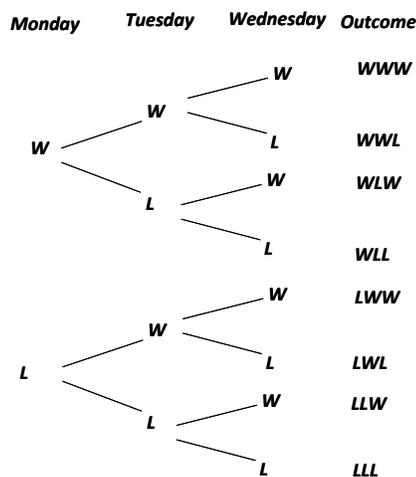
3. If the researchers conducting the experiment add food in the simple maze such that the probability of each mouse turning left is now  $\frac{1}{2}$ , what is the probability that only one of the three mice will turn left?

As in Question 2, there are three outcomes that have exactly one mouse turning left:  $LLR$ ,  $LRL$ , and  $RLL$ . However, with the adjustment made by the researcher, each of the three outcomes now has a probability of  $(0.5)(0.5)(0.5) = 0.125$ . So now, the probability of having only one of the three mice turn left is the sum of three equally likely outcomes of  $0.125 + 0.125 + 0.125 = 0.375$ . The tree provides a way to organize the outcomes and the probabilities.



### Problem Set Sample Solutions

1. According to the Washington, DC Lottery's website for its "Cherry Blossom Doubler" instant scratch game, the chance of winning a prize on a given ticket is about  $\frac{1}{10}$ . Imagine that a person stops at a convenience store on the way home from work every Monday, Tuesday, and Wednesday to buy a "scratcher" ticket and plays the game.  
(Source: <http://dclottery.com/games/scratchers/1223/cherry-blossom-doubler.aspx> accessed May 27, 2013)
- a. Develop a tree diagram showing the eight possible outcomes of playing over these three days. Call stage one "Monday," and use the symbols "W" for a winning ticket and "L" for a non-winning ticket.



b. What is the probability that the player will not win on Monday but will win on Tuesday and Wednesday?

*outcome:*

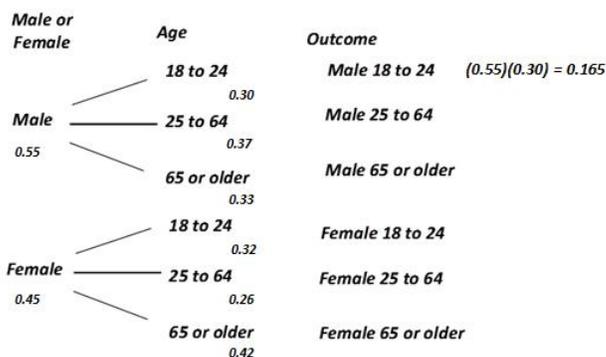
c. What is the probability that the player will win at least once during the -day period?

*“Winning at least once” would include all outcomes except (which has a probability). The probabilities of these outcomes would sum to about . This is also equal to - .*

2. A survey company is interested in conducting a statewide poll prior to an upcoming election. They are only interested in talking to registered voters.

Imagine that of the registered voters in the state are male, and are female. Also, consider that the distribution of ages may be different for each group. In this state, of male registered voters are age 18–24, are age 25–64, and are 65 or older. of female registered voters are age 18–24, are age 25–64, and are 65 or older.

The following tree diagram describes the distribution of registered voters. The probability of selecting a male registered voter age 18–24 is .



a. What is the chance that the polling company will select a registered female voter age 65 or older?

*Female 65 or Older:* .

b. What is the chance that the polling company will select any registered voter age 18–24?

*The probability of selecting any registered voter age 18–24 would be the sum of the probability of selecting a male registered voter age 18–24 and the probability of selecting a female registered voter age 18–24. Those values are and respectively. So the sum is*