Lesson 6: Using Tree Diagrams to Represent a Sample Space and to Calculate Probabilities

Student Outcomes

- Given a description of a chance experiment that can be thought of as being performed in two or more stages, students use tree diagrams to organize and represent the outcomes in the sample space.
- Students calculate probabilities of compound events.

Suppose a girl attends a preschool where the students are studying primary colors. To help teach calendar skills, the teacher has each student maintain a calendar in his or her cubby. For each of the four days that the students are covering primary colors in class, each student gets to place a colored dot on his/her calendar: blue, yellow, or red. When the four days of the school week have passed (Monday–Thursday), what might the young girl's calendar look like?

One outcome would be four blue dots if the student chose blue each day. But consider that the first day (Monday) could be blue, and the next day (Tuesday) could be yellow, and Wednesday could be blue, and Thursday could be red. Or, maybe Monday and Tuesday could be yellow, Wednesday could be blue, and Thursday could be red. Or, maybe Monday, Tuesday, and Wednesday could be blue, and Thursday could be red …

As hard to follow as this seems now, we have only mentioned of the possible outcomes in terms of the four days of colors! Listing the other outcomes would take several pages! Rather than listing outcomes in the manner described above (particularly when the situation has multiple stages, such as the multiple days in the case above), we often use a *tree diagram* to display all possible outcomes visually. Additionally, when the outcomes of each stage are the result of a chance experiment, tree diagrams are helpful for computing probabilities.

Classwork

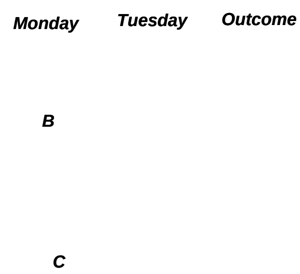
Example 1 (10 minutes): Two Nights of Games

- The tree diagram is an important way of organizing and visualizing outcomes.
- The tree diagram is a particularly useful device when the experiment can be thought of as occurring in stages.
- When the information about probabilities associated with each branch is included, the tree diagram facilitates the computation of the probabilities of the different possible outcomes.

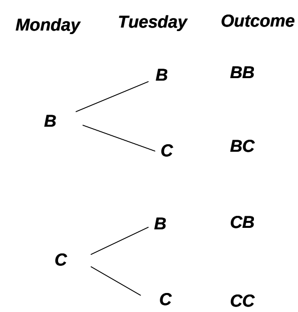
Example 1: Two Nights of Games

Imagine that a family decides to play a game each night. They all agree to use a tetrahedral die (i.e., a four-sided pyramidal die where each of four possible outcomes is equally likely—see image on page 9) each night to randomly determine if they will play a board game ( ) or a card game ( ). The tree diagram mapping the possible overall outcomes over two consecutive nights will be developed below.

To make a tree diagram, first present all possibilities for the first stage. (In this case, Monday.)



Then, from *each* branch of the first stage, attach all possibilities for the second stage (Tuesday).



Note: If the situation has more than two stages, this process would be repeated until all stages have been presented.

a. **If " " represents two straight nights of board games, what does " " represent?**

" " would represent a card game on the first night and a board game on the second night.

b. **List the outcomes where exactly one board game is played over two days. How many outcomes were there?**

and —there are two outcomes.

Example 2 (10 minutes): Two Nights of Games (with Probabilities)

Now include probabilities on the tree diagram from Example 1. Explain that the probability for each "branch of the tree" can be found by multiplying the probabilities of the outcomes from each stage.

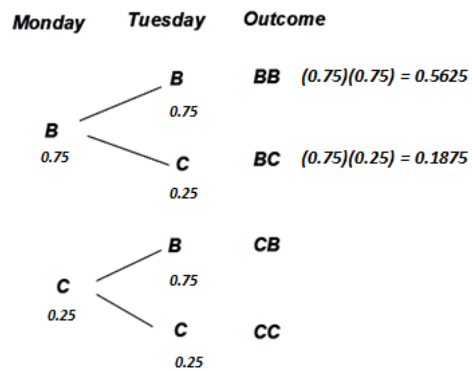
Pose each question in the example to the class. Give students a moment to think about the problem.

Example 2: Two Nights of Games (with Probabilities)

In the example above, each night's outcome is the result of a chance experiment (rolling the tetrahedral die). Thus, there is a probability associated with each night's outcome.

By multiplying the probabilities of the outcomes from each stage, we can obtain the probability for each "branch of the tree." In this case, we can figure out the probability of each of our four outcomes: , , , and .

For this family, a card game will be played if the die lands showing a value of and a board game will be played if the die lands showing a value of , , or . This makes the probability of a board game ( ) on a given night .



a. **The probabilities for two of the four outcomes are shown. Now, compute the probabilities for the two remaining outcomes.**

:

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b. **What is the probability that there will be exactly one night of board games over the two nights?**

The two outcomes which contain exactly one night of board games are and (see Example 2). The probability of exactly one night of board games would be the sum of the probabilities of these outcomes (since the outcomes are disjoint). .

Exercises 1–3 (15 minutes): Two Children

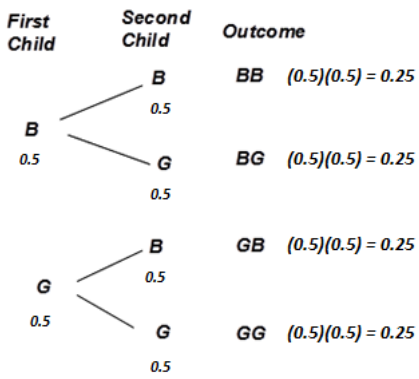
MP.

After developing the tree diagram, pose the questions to students one at a time. Allow for more than one student to offer an answer for each question, encouraging a brief (2 minute) discussion.

Exercises 1–3: Two Children

Two friends meet at a grocery store and remark that a neighboring family just welcomed their second child. It turns out that both children in this family are girls, and they are not twins. One of the friends is curious about what the chances are of having girls in a family's first 2 births. Suppose that for each birth the probability of a "boy" birth is and the probability of a "girl" birth is also .

1. **Draw a tree diagram demonstrating the four possible birth outcomes for a family with children (no twins). Use the symbol " " for the outcome of "boy" and "G" for the outcome of "girl." Consider the first birth to be the "first stage." (Refer to Example 1 if you need help getting started.)**



MP.

2. **Write in the probabilities of each stage's outcome to the tree diagram you developed above, and determine the probabilities for each of the possible birth outcomes for a family with children (no twins).**

   In this case, since the probability of a boy is and the probability of a girl is , all four outcomes will have a probability of , or probability of occurring.

3. **What is the probability of a family having girls in this situation? Is that greater than or less than the probability of having exactly girl in births?**

   The probability of a family having girls is . This is less than the probability of having exactly girl in births, which is (the sum of the probabilities of and).

Closing (5 minutes)

Consider posing the following question; discuss with students:

- Can you think of any situations where the first stage of a tree diagram might have two possibilities but the second stage might have more than two possibilities attached to each first-stage "branch"?
  - *Answers will vary, but an example will be shown in Lesson 7 where males and females are then split into Democrat, Republican, and Other.*

Lesson Summary

Tree diagrams can be used to organize outcomes in the sample space for chance experiments that can be thought of as being performed in multiple stages. Tree diagrams are also useful for computing probabilities of events with more than one outcome.

Exit Ticket (5 minutes)

Name ____ Date____________________

Lesson 6: Using Tree Diagrams to Represent a Sample Space and to Calculate Probabilities

Exit Ticket

In a laboratory experiment, two mice will be placed in a simple maze with one decision point where a mouse can turn either left ( ) or right ( ). When the first mouse arrives at the decision point, the direction it chooses is recorded. Then, the process is repeated for the second mouse.

1. Draw a tree diagram where the first stage represents the decision made by the first mouse, and the second stage represents the decision made by the second mouse. Determine all four possible decision outcomes for the two mice.

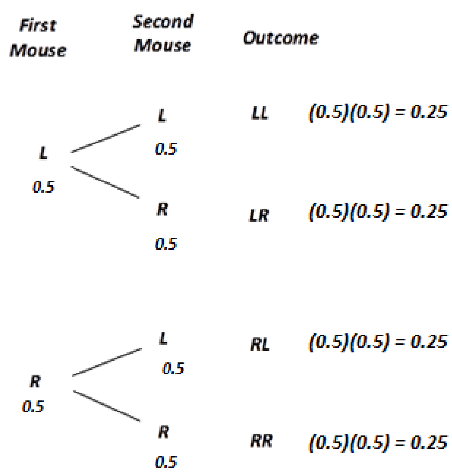
2. If the probability of turning left is , and the probability of turning right is for each mouse, what is the probability that only one of the two mice will turn left?

3. If the researchers add food in the simple maze such that the probability of each mouse turning left is now , what is the probability that only one of the two mice will turn left?

Exit Ticket Sample Solutions

In a laboratory experiment, two mice will be placed in a simple maze with one decision point where a mouse can turn either left ( ) or right ( ). When the first mouse arrives at the decision point, the direction it chooses is recorded. Then, the process is repeated for the second mouse

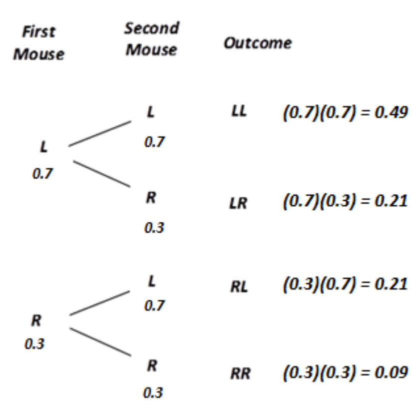
1. **Draw a tree diagram where the first stage represents the decision made by the first mouse, and the second stage represents the decision made by the second mouse. Determine all four possible decision outcomes for the two mice.**



2. **If the probability of turning left is , and the probability of turning right is for each mouse, what is the probability that only one of the two mice will turn left?**

   There are two outcomes that have exactly one mouse turning left: and . Each has a probability of , so the probability of only one of the two mice turning left is .

3. **If the researchers add food in the simple maze such that the probability of each mouse turning left is now , what is the probability that only one of the two mice will turn left?**

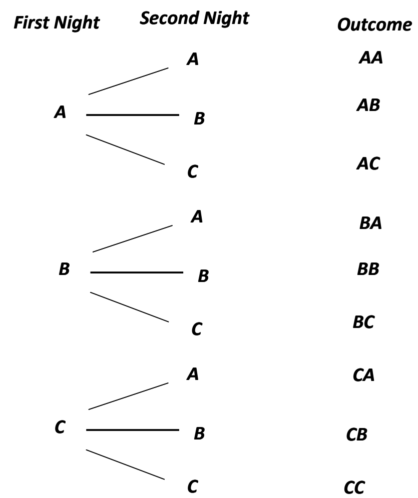


   As in Question 2, there are two outcomes that have exactly one mouse turning left: and . However, with the adjustment made by the researcher, each of these outcomes now has a probability of . So now, the probability of only one of the two mice turning left is .

Problem Set Sample Solutions

1. Imagine that a family of three (Alice, Bill, and Chester) plays bingo at home every night. Each night, the chance that any one of the three players will win is –.

   a. Using " " for Alice wins, " " for Bill wins, and " " for Chester wins, develop a tree diagram that shows the nine possible outcomes for two consecutive nights of play.



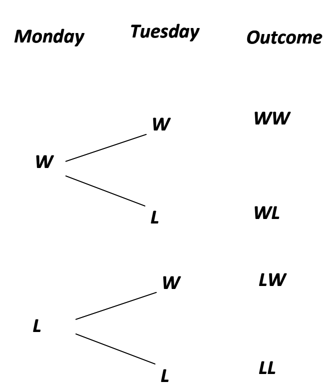
   b. Is the probability that "Bill wins both nights" the same as the probability that "Alice wins the first night and Chester wins the second night"? Explain.

   Yes. The probability of Bill winning both nights is – – –, which is the same as the probability of Alice winning the first night and Chester winning the second night (– – –).

2. According to the Washington, DC Lottery's website for its "Cherry Blossom Doubler" instant scratch game, the chance of winning a prize on a given ticket is about . Imagine that a person stops at a convenience store on the way home from work every Monday and Tuesday to buy a "scratcher" ticket to play the game.

   (Source: http://dclottery.com/games/scratchers/1223/cherry-blossom-doubler.aspx accessed May 27, 2013).

   a. Develop a tree diagram showing the four possible outcomes of playing over these two days. Call stage 1 "Monday", and use the symbols " " for a winning ticket and " for a non-winning ticket.



b. **What is the chance that the player will not win on Monday but will win on Tuesday?**

outcome:

c. **What is the chance that the player will win at least once during the two-day period?**

"Winning at least once" would include all outcomes except (which has a probability). The probabilities of these outcomes would sum to .

Image of Tetrahedral Die

Source: http://commons.wikimedia.org/wiki/File:4-sided_dice_250.jpg