Lesson 4: Calculating Probabilities for Chance Experiments with Equally Likely Outcomes

Student Outcomes

- Students will calculate probabilities of events for chance experiments that have equally likely outcomes.

Classwork

Example 1 (8 minutes): Theoretical Probability

This example is a chance experiment similar to those conducted in Lesson 2. The experiment requires a brown paper bag that contains yellow, green, red, and blue cubes. Unifix cubes work well for this experiment. In the experiment, cubes are drawn at random and *with replacement.* After each cube is drawn, have students record the outcome in the table.

Before starting the experiment ask students:

- What does it mean to draw a cube out at random?
  - *Random means that cube has an equal chance in being selected.*
- What does it mean to draw a cube with replacement?
  - *The cube is put back before you pick again.*

Example 1: Theoretical Probability

In a previous lesson, you saw that to find an estimate of the probability of an event for a chance experiment you divide:

_____

Your teacher has a bag with some cubes colored yellow, green, blue, and red. The cubes are identical except for their color. Your teacher will conduct a chance experiment by randomly drawing a cube with replacement from the bag. Record the outcome of each draw in the table below.

Trial	Outcome

Exercises 1–6 (20 minutes)

Allow students to work with their partners on Exercises 1–3. After students have completed the three questions, discuss the answers.

Exercises 1–6

1. Based on the trials, estimate for the probability of

a. choosing a yellow cube.

Answers vary – but should be approximately — or –.

b. choosing a green cube.

Answers vary – but should be approximately — or –.

c. choosing a red cube.

Answers vary – but should be approximately — or –.

d. choosing a blue cube.

Answers vary – but should be approximately — or –.

2. If there are cubes in the bag, how many cubes of each color are in the bag? Explain.

Answers will vary. Because the estimated probabilities are about the same for each color, we can predict that there are approximately the same number of each color of cubes in the bag. Since an equal number of each color is estimated, approximately of each color are predicted.

3. **If your teacher were to randomly draw another cubes one at a time and with replacement from the bag, would you see exactly the same results? Explain.**

No, this is an example of a chance experiment, so the results will vary.

Now tell the students what is in the bag (each of yellow, green, red, and blue cubes). Allow students to work with a partner on Exercise 4. Then discuss and confirm the answers.

4. **Find the fraction of each color of cubes in the bag.**

Yellow — *or* –

Green — *or* –

Red — *or* –

Blue — *or* –

Present the formal definition of the *theoretical probability* of an outcome when outcomes are equally likely. Then ask:

- Why is the numerator of the fraction just ?

Define the word *event* as "a collection of outcomes." Then, present that definition to students and ask:

- Why is the numerator of the fraction not always ?

Use the cube example to explain the difference between an *outcome* and an *event*. Explain that each cube is equally likely to be chosen (an outcome) while the probability of drawing a blue cube (an event) is —.

Each fraction is the theoretical probability of choosing a particular color of cube when a cube is randomly drawn from the bag.

When all the possible outcomes of an experiment are equally likely, the probability of each outcome is

_____

An event is a collection of outcomes, and when the outcomes are equally likely, the theoretical probability of an event can be expressed as

_____

The theoretical probability of drawing a blue cube is

_____ —

Allow students to work with a partner to answer Exercises 5 and 6. Then discuss and confirm the answers.

5. Is each color equally likely to be chosen? Explain your answer.

Yes, there are the same numbers of cubes for each color.

6. How do the theoretical probabilities of choosing each color from Exercise 4 compare to the experimental probabilities you found in Exercise 1?

Answers will vary.

Example 2 (10 minutes)

This example connects the concept of sample space from Lesson 3 to finding probability. Present the example of flipping a nickel and then a dime. List the sample space representing the outcomes of a head or tail on the nickel and a head or tail on the dime (HH, HT, TH, and TT). Discuss how each outcome is equally likely to occur. Then ask students:

- What is the probability of getting two heads?
  - *Probability is – or or*
- What is the probability of getting exactly one head of either the nickel or the dime? (This is an example of an event with two outcomes.)
  - *Probability of the outcomes of HT and TH, or – or – or or*

Example 2

An experiment consisted of flipping a nickel and a dime. The first step in finding the theoretical probability of obtaining a head on the nickel and a head on the dime is to list the sample space. For this experiment, the sample space is shown below.

Nickel	Dime
H	H
H	T
T	H
T	T

If the counts are fair, these outcomes are equally likely, so the probability of each outcome is –.

Nickel	Dime	Probability
H	H	–
H	T	–
T	H	–
T	T	–

The probability of two heads is – or –.

Exercises 7–10 (10 minutes)

Allow students to work with a partner on Exercises 7–10.

Exercises 7–10

7. Consider a chance experiment of rolling a number cube.

a. What is the sample space? List the probability of each outcome in the sample space.

Sample Space: and

Probability of each outcome is –.

b. What is the probability of rolling an odd number?

– or –

c. What is the probability of rolling a number less than ?

– or –

8. Consider an experiment of randomly selecting a letter from the word: number.

a. What is the sample space? List the probability of each outcome in the sample space.

Sample space: n, u, m, b, e, and r.

Probability of each outcome is –

MP. 2 &

b. What is the probability of selecting a vowel?

– or –

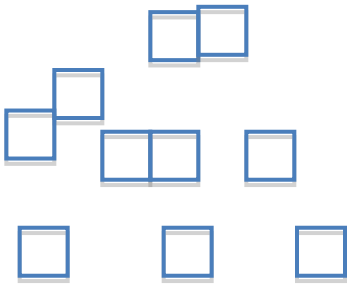
c. What is the probability of selecting the letter z?

– or

9. Consider an experiment of randomly selecting a cube from a bag of cubes.

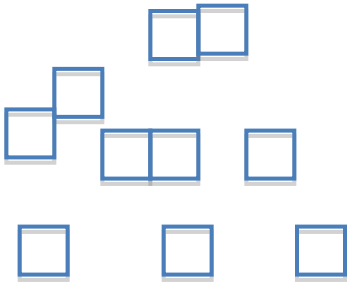
a. Color the cubes below so that the probability of selecting a blue cube is –.

Answers will vary; of the cubes should be colored blue.



b. Color the cubes below so that the probability of selecting a blue cube is –.

Answers will vary; of the cubes will be colored blue.

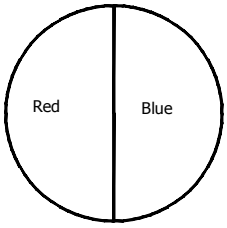


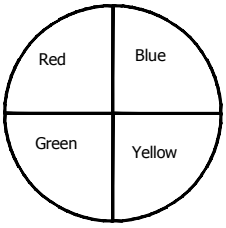
10. Students are playing a game that requires spinning the two spinners shown below. A student wins the game if both spins land on Red. What is the probability of winning the game? Remember to first list the sample space and the probability of each outcome in the sample space. There are eight possible outcomes to this chance experiment.

Sample Space: R1 R2, R1 B2, R1 G2, R1 Y2, B1 R2, B1 B2, B1 G2, and B1 Y2.

Each outcome has a probability of –.

Probability of a win (both red) is –.





Closing (5 minutes)

Summarize the two formal definitions of theoretical probability. The first is the probability of an outcome when all of the possible outcomes are equally likely, and the second is the probability of an event when the possible outcomes are equally likely. Remind students that an event is a collection of outcomes.

For example, in the experiment of rolling two number cubes, obtaining a sum of is an event.

Lesson Summary

When all the possible outcomes of an experiment are equally likely, the probability of each outcome is

_____

An event is a collection of outcomes, and when all outcomes are equally likely, the theoretical probability of an event can be expressed as

_____

Exit Ticket (10 minutes)

Name _____ Date_____

Lesson 4: Calculating Probabilities for Chance Experiments with Equally Likely Outcomes

Exit Ticket

An experiment consists of randomly drawing a cube from a bag containing three red and two blue cubes.

1. What is the sample space of this experiment?

2. List the probability of each outcome in the sample space.

3. Is the probability of selecting a red cube equal to the probability of selecting a blue cube? Explain.

Exit Ticket Sample Solutions

An experiment consists of randomly drawing a cube from a bag containing three red and two blue cubes.

1. **What is the sample space of this experiment?**

   R, R, R, B, and B are representing the five cubes.

2. **List the probability of each outcome in the sample space.**

   Each outcome has a probability of –.

3. **Is the probability of selecting a red cube equal to the probability of selecting a blue cube? Explain.**

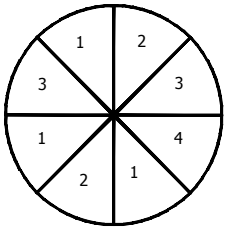
   No, there are more red cubes than blue cubes, so red has a greater probability of being chosen.

Problem Set Sample Solutions

1. **In a seventh grade class of students, there are girls and boys. If one student is randomly chosen to win a prize, what is the probability that a girl is chosen?**

   — *or* –

2. **An experiment consists of spinning the spinner once.**



   a. **Find the probability of landing on a .**

      – *or* –

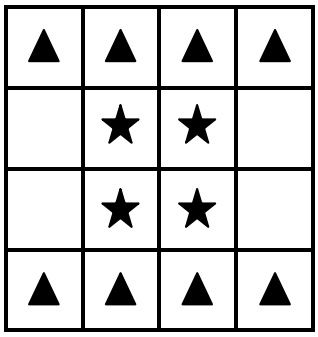
   b. **Find the probability of landing on a .**

      –

   c. **Is landing in each section of the spinner equally likely to occur? Explain.**

      Yes, each section is the same size.

3. An experiment consists of randomly picking a square section from the board shown below.



a. Find the probability of choosing a triangle.

— *or* –

b. Find the probability of choosing a star.

— *or* –

c. Find the probability of choosing an empty square.

— *or* –

d. Find the probability of choosing a circle.

— *or*

4. Seventh graders are playing a game where they randomly select two integers from – , inclusive, to form a two-digit number. The same integer might be selected twice.

a. List the sample space for this chance experiment. List the probability of each outcome in the sample space.

Sample Space: numbers from – . Probability of each outcome is .

b. What is the probability that the number formed is between and , inclusive?

—— *or* ——

c. What is the probability that the number formed is evenly divisible by ?

—— *or* –

d. What is the probability that the number formed is a factor of ?

—— *(Factors of are , , , , , , and .)*

5. A chance experiment consists of flipping a coin and rolling a number cube with the numbers – on the faces of the cube.

a. List the sample space of this chance experiment. List the probability of each outcome in the sample space.

and . The probability of each outcome is —.

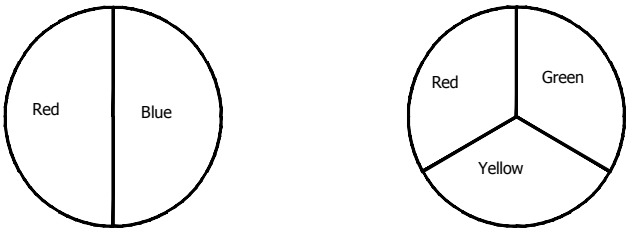
b. What is the probability of getting a head on the coin and the number 3 on the number cube?

—

c. What is the probability of getting a tail on the coin and an even number on the number cube?

— *or* –

6. A chance experiment consists of spinning the two spinners below.



a. List the sample space and the probability of each outcome.

Sample Space: R1 R2, R1 G2, R1 Y2, B1 R2, B1 G2, and B1 Y2. Each outcome has a probability of —.

b. Find the probability of the event of getting a red on the first spinner and a red on the second spinner.

—

c. Find the probability of a red on at least one of the spinners.

— *or* —