# Lesson 3: Chance Experiments with Equally Likely Outcomes

## Student Outcomes

- Students determine the possible outcomes for simple chance experiments.
- Given a description of a simple chance experiment, students determine the sample space for the experiment.
- Given a description of a chance experiment and an event, students determine for which outcomes in the sample space the event will occur.
- Students distinguish between chance experiments with equally likely outcomes and chance experiments for which the outcomes are not equally likely.

## Lesson Overview

This lesson continues to build students' understanding of probability. This lesson begins to formalize concepts explored in the first two lessons of the module. Students are presented with several descriptions of simple chance experiments such as spinning a spinner and drawing objects from a bag. Note that students will be working on a chance experiment in class that utilizes paper cups. Teachers will need to provide cups for the experiment.

## Classwork

### Example 1 (5 minutes)

Begin this example by showing the students a paper cup and asking them how it might land when tossed. Explain that each possibility is called an *outcome* of the experiment. Then list the three outcomes on the board and explain that the three outcomes form the *sample space* of the experiment.

Provide other examples of an experiment, and ask students to describe the sample space:

- Flipping a coin: heads, tails
- Drawing a colored cube from a bag that has red, blue, yellow, and green: red, blue, yellow, and green
- Picking a letter from the word "classroom": c, l, a, s, r, o, m

Note: Some students will want to list all of the letters but explain that, when listing the sample space, you only need to list the possibilities, not how many times the letter appears.

**Example 1**

**Jamal, a 7th grader, wants to design a game that involves tossing paper cups. Jamal tosses a paper cup five times and records the outcome of each toss. An outcome is the result of a single trial of an experiment.**

**Here are the results of each toss:**

**Jamal noted that the paper cup could land in one of three ways: on its side, right side up, or upside down. The collection of these three outcomes is called the *sample space* of the experiment. The sample space of an experiment is the set of all possible outcomes of that experiment.**

**For example, the sample space when flipping a coin is Heads, Tails.**

**The sample space when drawing a colored cube from a bag that has red, blue, yellow, and green cubes is red, blue, yellow, green.**

## Exercises 1–6 (15 minutes)

Allow students to work with a partner on Exercises 1–6. Then discuss and confirm as a class.

**Exercise 1–6**

**For each of the following chance experiments, list the sample space (i.e., all the possible outcomes).**

1. **Drawing a colored cube from a bag with green, red, blue, and black.**

   ***Green, Red, Blue, Black.***

2. **Tossing an empty soup can to see how it lands.**

   ***Right side up, upside down, on its side.***

3. **Shooting a free-throw in a basketball game.**

   ***Made shot, missed shot.***

MP.2

4. **Rolling a number cube with the numbers – on its faces.**

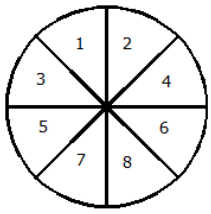
   ***or .***

5. **Selecting a letter from the word: probability**

   ***p, r, o, b, a, i, l, t, y.***

6. **Spinning the spinner:**

   ***or .***



## Example 2 (15 minutes): Equally Likely Outcomes

In this example, students will carry out the experiment of tossing a paper cup. Small Dixie cups work well. Styrofoam cups tend not to work as well. You will need one cup for each pair of students. Before the students begin this experiment ask:

- Do you think each of the possible outcomes of tossing the paper cup has the same chance of occurring?

Explain to the students that if the outcomes have the same chance of occurring, then they are *equally likely* to occur.

The students should toss the paper cup times, and record the results of each toss.

**Example 2: Equally Likely Outcomes**

**The sample space for the paper cup toss was on its side, right side up, and upside down. Do you think each of these outcomes has the same chance of occurring? If they do, then they are equally likely to occur.**

**The outcomes of an experiment are equally likely to occur when the probability of each outcome is equal.**

**You and your partner toss the paper cup times and record in a table the results of each toss.**

| Toss | Outcome |
|---|---|
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### Exercises 7–12 (10 minutes)

Allow students to work with a partner on Exercises 7–12. Then, discuss and confirm as a class.

**Exercises 7–12**

7. **Using the results of your experiment, what is your estimate for the probability of a paper cup landing on its side?**

   *Answers will vary. Students should write their answer as a fraction with a denominator of and a numerator of the number of times cup landed on its side.*

8. **Using the results of your experiment, what is your estimate for the probability of a paper cup landing upside down?**

   *Answers will vary. Students should write their answer as a fraction with a denominator of and a numerator of the number of times cup landed upside down.*

9. **Using the results of your experiment, what is your estimate for the probability of a paper cup landing right side up?**

   *Answers will vary. Students should write their answer as a fraction with a denominator of and a numerator of the number of times cup landed right side up.*

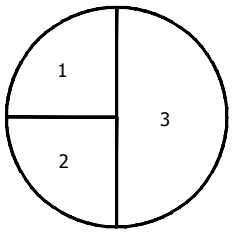
MP.6

10. **Based on your results, do you think the three outcomes are equally likely to occur?**

    *Answers will vary but, generally, the results are not equally likely.*

Based on their results of tossing the cup times, ask students to predict how many times the cup will land on its side, right side up, or upside down for approximately tosses. If time permits, allow students to carry out the experiment for a total of tosses, or combine results of students to examine the number of outcomes for approximately tosses. Compare the predicted numbers and the actual numbers. It is likely the results from approximately tosses will not match the predicted numbers. Discuss with students why they generally do not agree.

11. **Using the spinner below, answer the following questions.**




   a. **Are the events spinning and landing on or a equally likely?**

      *Yes, the area of sections and are equal.*

   b. **Are the events spinning and landing on or equally likely?**

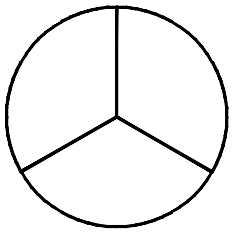
      *No, the area of sections and are not equal.*

   c. **How many times do you predict the spinner to land on each section after spins?**

      *Based on the area of the sections, approximately times each for sections and , and times for section .*

12. Draw a spinner that has sections that are equally likely to occur when the spinner is spun. How many times do you think the spinner will land on each section after spins?

*The three sectors should be equal in area. Expect the spinner to land on each section approximately times ( – times).*



## Closing (5 minutes)

Remind students of the new vocabulary: *outcome, sample space, and equally likely.*

Use the following example to summarize the main ideas of the lesson:

Suppose a bag contains Green, Red, yellow, Orange, and Purple crayons. If one crayon is selected from the bag and the color is noted, the *outcome* is the color that will will be chosen. The *sample space* will be the colors: green, red, yellow, orange, and purple. Each color is *equally likely* to be selected because each color has the same chance of being chosen.

**Lesson Summary**

**An outcome is the result of a single observation of an experiment.**

**The sample space of an experiment is the set of all possible outcomes of that experiment.**

**The outcomes of an experiment are equally likely to occur when the probability of each outcome is equal.**

**Suppose a bag of crayons contains green, red, yellow, orange, and purple pieces of crayons. If one crayon is selected from the bag and the color is noted, the *outcome* is the color that will be chosen. The *sample space* will be the colors: green, red, yellow, orange, and purple. Each color is *equally likely* to be selected because each color has the same chance of being chosen.**

## Exit Ticket (10 minutes)

Name _____ Date_____

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## Exit Ticket

The numbers from – are written on note cards and placed in a bag. One card will be drawn from the bag at random.

1. List the sample space for this experiment.

2. Are the events selecting an even number and selecting an odd number equally likely? Explain your answer.

3. Are the events selecting a number divisible by and selecting a number divisible by equally likely? Explain your answer.

## Exit Ticket Sample Solutions

The numbers from – are written on note cards and placed in a bag. One card will be drawn from the bag at random.

1. List the sample space for this experiment.

2. Are the events selecting an even number and selecting an odd number equally likely? Explain your answer.

   *Yes, each has the same chance of occurring. There are even and odd numbers in the bag.*

3. Are the events selecting a number divisible by and selecting a number divisible by equally likely? Explain your answer.

   *No. There are 3 numbers divisible by ( , , and ), but only numbers divisible by ( and ). So the chance of selecting a number divisible by is slightly greater than selecting a number divisible by .*

## Problem Set Sample Solutions

1. For each of the following chance experiments, list the sample space (all the possible outcomes).

   a. Rolling a -sided die with the numbers – on the faces of the die.

      *or .*

   b. Selecting a letter from the word: mathematics.

      *m, a, t, h, e, i, c, or s.*

   c. Selecting a marble from a bag containing black marbles and orange marbles.

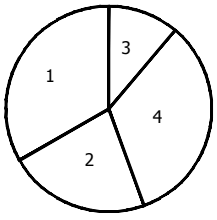
      *Black or Orange.*

   d. Selecting a number from the even numbers from – , inclusive.

      *or .*

   e. Spinning the spinner below:

      *or .*



2. For each of the following decide if the two outcomes listed are equally likely to occur. Give a reason for your answer.

   a. Rolling a or a when a -sided number cube with the numbers – on the faces of the cube is rolled.

   *Yes, each has the same chance of occurring.*

   b. Selecting the letter *a* or *k* from the word: take.

   *Yes, each has the same chance of occurring.*

   c. Selecting a black or an orange marble from a bag containing black and orange marbles.

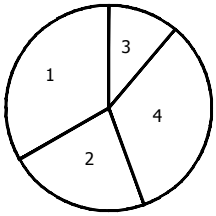
   *No, Black has a slightly greater chance of being chosen.*

   d. Selecting a or an from the even numbers from – , inclusive.

   *Yes, each has the same chance of being chosen.*

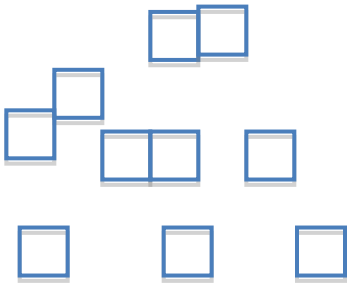
   e. Landing on a or when spinning the spinner below.

   *No, has a larger area, so it has a greater chance of occurring.*



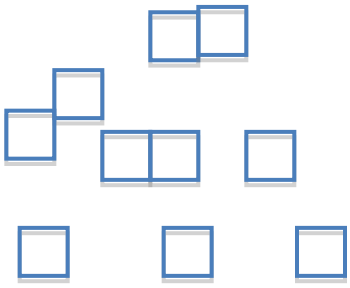

3. Color the cubes below so that it would be equally likely to choose a blue or yellow cube.

   *Answers will vary, but the students should have colored blue and colored yellow.*

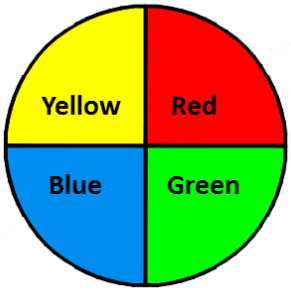


4. **Color the cubes below so that it would be more likely to choose a blue than a yellow cube.**

*Answers will vary. Students should have more cubes colored blue than yellow.*



5. **You are playing a game using the spinner below. The game requires that you spin the spinner twice. For example, one outcome could be Yellow on 1st spin and Red on 2nd spin. List the sample space (all the possible outcomes) for the two spins.**




*There are possibilities:*

| *1st spin* | *2nd spin* |
|---|---|
| *Y* | *Y* |
| *Y* | *R* |
| *Y* | *G* |
| *Y* | *B* |
| *R* | *Y* |
| *R* | *R* |
| *R* | *G* |
| *R* | *B* |
| *G* | *Y* |
| *G* | *R* |
| *G* | *G* |
| *G* | *B* |
| *B* | *Y* |
| *B* | *R* |
| *B* | *G* |
| *B* | *B* |

6. **List the sample space for the chance experiment of flipping a coin twice.**

*There are four possibilities:*

| *1st toss* | *2nd toss* |
|---|---|
| *H* | *H* |
| *H* | *T* |
| *T* | *H* |
| *T* | *T* |