

ALGEBRA II HONORS
 2014–2015 PRACTICE MATERIALS KEY
 UNIT 8.11-8.14
 SEMESTER 2



#	Question Type	Unit	Common Core State Standard(s)	DOK Level	Key
1	MC	8	F.TF.C.8, F.TF.C.9	1	B
2	MC	8	F.TF.C.8, F.TF.C.9	1	C
3	MC	8	F.TF.C.8, F.TF.C.9	2	B
4	MC	8	F.TF.C.8, F.TF.C.9	2	C
5	MC	8	F.TF.C.8, F.TF.C.9	3	A
6	MC	8	F.TF.C.8, F.TF.C.9	3	D
7	FR	8	F.TF.C.8, F.TF.C.9	1	-
8	FR	8	F.TF.C.8, F.TF.C.9	1	-
9	FR	8	F.TF.C.8, F.TF.C.9	3	-
10	FR	8	F.TF.C.9	1	-
11	FR	8	F.TF.C.9	2	-
12	FR	8	F.TF.C.9	1	-
13	MC	8	F.TF.B.7	3	D
14	MC	8	F.TF.B.7	3	A
15	FR	8	F.TF.B.7	2	-
16	FR	8	F.TF.B.7	2	-
17	FR	8	F.TF.B.7	1	-

7. (8.11) Verify that the following equation is an identity.

$$\frac{\cos x}{1 - \sin x} = \frac{1 + \sin x}{\cos x}$$

Work on the right side.

$$\begin{aligned} \frac{1 + \sin x}{\cos x} &= \frac{(1 + \sin x)(1 - \sin x)}{\cos x(1 - \sin x)} && \text{Multiply by 1 in the form } \frac{1 - \sin x}{1 - \sin x} \\ &= \frac{1 - \sin^2 x}{\cos x(1 - \sin x)} && (x + y)(x - y) = x^2 - y^2 \\ &= \frac{\cos^2 x}{\cos x(1 - \sin x)} && 1 - \sin^2 x = \cos^2 x \\ &= \frac{\cos x}{1 - \sin x} && \text{Lowest terms} \end{aligned}$$

8. (8.11) Use fundamental identities to simplify the expression.

$$\frac{\sin \beta \tan \beta}{\cos \beta}$$

$$\frac{\sin \beta \tan \beta}{\cos \beta}$$

$$\frac{\sin \beta}{\cos \beta} \cdot \frac{\tan \beta}{1}$$

$$\tan \beta \cdot \tan \beta$$

$$\tan^2 \beta$$

9. (8.11) Verify that the trigonometric equation is an identity.

$$\sin^4 \theta - \cos^4 \theta = 2 \sin^2 \theta - 1$$

$$\sin^4 \theta - \cos^4 \theta = 2 \sin^2 \theta - 1$$

original equation

$$(\sin^2 \theta + \cos^2 \theta)(\sin^2 \theta - \cos^2 \theta) = 2 \sin^2 \theta - 1$$

work the left side, factor into difference of squares

$$1(\sin^2 \theta - \cos^2 \theta) = 2 \sin^2 \theta - 1$$

substitute, $\sin^2 \theta + \cos^2 \theta = 1$

$$(\sin^2 \theta - (1 - \sin^2 \theta)) = 2 \sin^2 \theta - 1$$

substitute, $\cos^2 \theta = 1 - \sin^2 \theta$

$$2 \sin^2 \theta - 1 = 2 \sin^2 \theta - 1$$

10. (8.12) Find the exact value of $\sin 75^\circ$.

$$\sin 75^\circ = \sin(45^\circ + 30^\circ)$$

$$\sin 75^\circ = \sin 45^\circ \cos 30^\circ + \cos 45^\circ \sin 30^\circ$$

$$\sin 75^\circ = \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2}$$

$$\sin 75^\circ = \frac{\sqrt{6} + \sqrt{2}}{4}$$

11. (8.12) Find the exact value of $\tan \frac{7\pi}{12}$.

$$\tan \frac{7\pi}{12} = \tan \left(\frac{\pi}{3} + \frac{\pi}{4} \right) \quad \text{rewrite } \frac{7\pi}{12}$$

$$\tan \frac{7\pi}{12} = \frac{\tan \frac{\pi}{3} + \tan \frac{\pi}{4}}{1 - \tan \frac{\pi}{3} \tan \frac{\pi}{4}} \quad \text{apply the tangent sum identity}$$

$$\tan \frac{7\pi}{12} = \frac{\sqrt{3} + 1}{1 - \sqrt{3} \cdot 1}$$

$$\tan \frac{7\pi}{12} = \frac{\sqrt{3} + 1}{1 - \sqrt{3}} \cdot \frac{1 + \sqrt{3}}{1 + \sqrt{3}} \quad \text{rationalize the denominator}$$

$$\tan \frac{7\pi}{12} = \frac{\sqrt{3} + 3 + 1 + \sqrt{3}}{1 - 3} \quad \text{multiply}$$

$$\tan \frac{7\pi}{12} = \frac{4 + 2\sqrt{3}}{-2} \quad \text{combine terms}$$

$$\tan \frac{7\pi}{12} = \frac{2(2 + \sqrt{3})}{-2} \quad \text{factor out 2 in the numerator}$$

$$\tan \frac{7\pi}{12} = -2 - \sqrt{3} \quad \text{lowest terms}$$

12. (8.13) Find the exact value of $\cos 15^\circ$ using half-angle identity for cosine.

Solution

$$\cos 15^\circ = \cos \frac{1}{2}(30^\circ) = \sqrt{\frac{1 + \cos 30^\circ}{2}}$$

Choose the positive square root.

$$= \sqrt{\frac{1 + \frac{\sqrt{3}}{2}}{2}} = \sqrt{\frac{(1 + \frac{\sqrt{3}}{2}) \cdot 2}{2 \cdot 2}} = \frac{\sqrt{2 + \sqrt{3}}}{2}$$

Simplify the radicals.

15. Solve on the interval $[0, 2\pi)$.

$$\sin x \tan x = \sin x$$

$$\sin x \tan x = \sin x$$

$$\sin x \tan x - \sin x = 0 \quad \text{subtract } \sin x \text{ from both sides of equation}$$

$$\sin x (\tan x - 1) = 0 \quad \text{factor out } \sin x$$

$$\sin x = 0 \quad \text{or} \quad \tan x - 1 = 0 \quad \text{zero product property}$$

$$\tan x = 1$$

$$x = 0 \quad \text{or} \quad x = \pi \quad x = \frac{\pi}{4} \quad \text{or} \quad x = \frac{5\pi}{4}$$

16. Solve on the interval $[0, 2\pi)$.

$$\cos^4 x + \cos^2 x - 2 = 0$$

$$\cos^4 x + \cos^2 x - 2 = 0 \quad \text{original equation}$$

$$(\cos^2 x + 2)(\cos^2 x - 1) = 0 \quad \text{factor}$$

$$\cos^2 x + 2 = 0 \quad \text{or} \quad \cos^2 x - 1 = 0 \quad \text{zero product property}$$

$$\cos^2 x = -2 \quad \cos^2 x = 1 \quad \text{solve for } \cos^2 x$$

$$\cos x = \pm\sqrt{-2} \quad \cos x = \pm\sqrt{1} = \pm 1 \quad \text{take square root of each side}$$

$$\text{no real solution} \quad 0, \pi$$

17. Solve on the interval $[0, 2\pi)$.

$$2\cos^2 x - \sin x - 1 = 0$$

$$2\cos^2 x - \sin x - 1 = 0$$

original equation

$$2(1 - \sin^2 x) - \sin x - 1 = 0$$

substitute $\cos^2 x = 1 - \sin^2 x$ (Pythagorean Identity)

$$2 - 2\sin^2 x - \sin x - 1 = 0$$

multiply

$$-2\sin^2 x - \sin x + 1 = 0$$

combine like terms

$$-1(2\sin^2 x + \sin x - 1) = 0$$

factor out -1

$$-1(2\sin x - 1)(\sin x + 1) = 0$$

factor

$$2\sin x - 1 = 0 \text{ or } \sin x + 1 = 0$$

zero product property

$$\sin x = \frac{1}{2} \quad \sin x = -1$$

solve for $\sin x$

$$x = \frac{\pi}{6} \text{ or } \frac{5\pi}{6} \quad x = \frac{3\pi}{2}$$