#	Question Type	Unit	Common Core State Standard(s)	DOK Level	Key
1	MC	6	F.IF.C.7d	1	C
2	MC	6	F.IF.C.7d	1	D
3	FR	6	F.IF.C.7d, F.IF.B.5	2	-
4	MC	6	A.APR.D.7	2	A
5	FR	6	A.APR.D.7, A.SSE.A.1b-2, A.CED.A.1-2	3	-
6	FR	6	A.APR.D.6, A.SSE.A.1b-2	3	-
7	MC	6	A.APR.D.7	1	B
8	MC	6	A.APR.D.6	1	A
9	FR	6	A.APR.D.7, A.SSE.A.1b-2, A.CED.A.1-2	3	-
10	MC	6	A.APR.D.7	1	B
11	MC	6	A.APR.D.7, F.IF.A.2	2	D
12	MC	6	A.REI.A.2	1	A
13	FR	6	A.REI.A.2, A.CED.A.1-2	3	-
14	FR	6	A.REI.A.2, A.CED.A.1-2	3	-
15	MC	6	F.IF.B.5	2	C
16	MC	6	F.IF.B.5	2	D
17	MC	7	A.SSE.B.4	1	A
18	MC	7	A.SSE.B.4	1	D
19	FR	7	A.SSE.B.4	2	-
20	MC	7	A.SSE.B.4	1	B
21	FR	7	A.SSE.B.4	3	-
22	MC	7	F.BF.B.5	1	D
23	MC	7	F.BF.B.5	2	C
24	MC	7	F.BF.B.5	1	A
25	FR	7	F.IF.C.7e-2, F.BF.B.3-2	2	-
26	MC	7	F.IF.C.7e-2	1	A
27	MC	7	F.BF.B.5	1	B
28	FR	7	A.CED.A.2-2	3	-
29	MC	7	F.BF.B.5	2	C
30	MC	7	F.LE.A.4, F.BF.B.5	1	D
31	MC	7	A.CED.A.4	2	A
32	MC	7	F.IF.C.7e-2	1	A
33	MC	7	F.IF.C.7e-2	1	C
34	MC	7	F.IF.C.7e-2, F.BF.B.3-2	1	D
35	MC	7	F.IF.C.7e-2, F.BF.B.3-2	1	B
36	MC	7	F.LE.A.4	1	A
37	FR	7	F.IF.C.7e-2	2	-
38	FR	7	F.LE.A.4	3	-
39	FR	7	F.LE.A.4, A.CED.A.4-2	2	-
40	MC	7	S.ID.B.6a	2	B
41	MC	7	SID.B.6a	2	B
42	FR	7	A.CED.A.2-2	3	-
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45	MC	7	S.ID.B.6a	1	C
46	FR	7	S.ID.B.6a, A.CED.A.2-2	3	-
47	MC	8	F.TF.3	1	D
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49	MC	8	F.TF.3	1	C
50	MC	8	F.TF.3	2	B
51	MC	8	F.TF.A.1	1	A
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53	FR	8	F.TF.A.1	1	-
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55	FR	8	F.TF.A.1	4	-
56	MC	8	F.TF.A.2	1	A
57	MC	8	F.TF.A.2	1	A
58	MC	8	F.TF.A.2	1	C
59	MC	8	F.TF.A.2	2	A
60	MC	8	F.TF.A.2	2	C
61	MC	8	F.TF.A.2	2	B
62	FR	8	F.TF.A.2	3	-
63	FR	8	F.TF.A.2	3	-
64	MC	8	F.TF.C.8	2	D
65	FR	8	F.TF.A.2	4	-
66	MC	8	F.TF.B.7	2	D
67	MC	8	G.SRT.D.10	2	D
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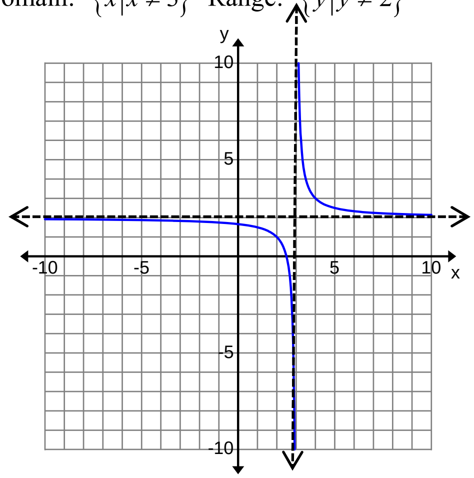
72	MC	8	G.SRT.D.10	1	A
73	MC	8	G.SRT.D.10	2	D
74	MC	8	G.SRT.D.9	2	A
75	FR	8	G.SRT.D.9	4	-
76	MC	8	G.SRT.D.9	1	B
77	MC	8	F.TF.B.5	1	A
78	MC	8	F.TF.B.5	1	B
79	MC	8	F.TF.B.5	1	A
80	MC	8	F.TF.B.5	1	D
81	MC	8	F.TF.B.5	2	A
82	MC	8	F.TF.B.5	2	A
83	MC	8	F.TF.B.5	1	A
84	MC	8	F.TF.B.5	2	D
85	FR	8	F.TF.B.5	2	-
86	FR	8	F.TF.B.5	2	-
87	FR	8	F.TF.B.5, F.IF.C.7e-2, F.BF.B.3	2	-
88	FR	8	F.TF.B.5, F.IF.C.7e-2, F.BF.B.3	3	-
89	MC	8	F.IF.C.7e-2	2	A
90	MC	8	F.IF.C.7e-2	2	A
91	FR	8	F.IF.C.7e-2	2	-
92	FR	8	F.IF.C.7e-2	2	-
93	FR	8	F.IF.C.7e-2	2	-
94	FR	8	F.IF.C.7e-2	2	-
95	FR	8	F.IF.C.7e-2	2	-
96	FR	8	F.IF.C.7e-2	2	-
97	FR	8	F.IF.C.7e-2	2	-
98	FR	8	F.IF.C.7e-2	3	-
99	FR	8	F.IF.C.7e-2	3	-
100	FR	8	F.TF.C.8	4	-
101	FR	8	F.TF.C.8	3	-
102	FR	8	A.CED.A.2-2, F.TF.B.5	3	-
103	FR	8	A.CED.A.2-2, F.TF.B.5	2	-
104	FR	8	A.CED.A.2-2, F.TF.B.5	2	-
105	FR	8	A.CED.A.2-2, F.TF.B.5	3	-
106	FR	8	A.CED.A.2-2, F.TF.B.5	3	-

107	FR	8	F.BF.A.1b-2, A.CED.A.2-2	3	
108	FR	8	A.CED.A.2-2, F.TF.B.5	3	-
109	FR	8	A.CED.A.2-2, F.TF.B.5	3	-
110	FR	8	F.TF.C.8	4	-

3. Domain: $\{x|x \neq 3\}$ Range: $\{y|y \neq 2\}$



5. Use the following expressions to answer the questions.

$$A = \frac{x+5}{x^2+4x+3} \qquad B = \frac{2}{x^2+x} \qquad C = \frac{x-3}{2x+2} \qquad D = \frac{7}{2x}$$

(A)

$$A+B = \frac{x+5}{x^2+4x+3} + \frac{2}{x^2+x} = \frac{x+5}{(x+3)(x+1)} + \frac{2}{x(x+1)} =$$

$$\frac{(x+5)}{(x+3)(x+1)} \bullet \frac{x}{x} + \frac{2}{x(x+1)} \bullet \frac{(x+3)}{(x+3)} = \frac{x^2+5x+2x+6}{x(x+1)(x+3)} =$$

$$= \frac{x^2+7x+6}{x(x+1)(x+3)} = \frac{x+6}{x(x+3)}$$

(B)

$$A-B = \frac{x+5}{x^2+4x+3} - \frac{2}{x^2+x} = \frac{x+5}{(x+3)(x+1)} - \frac{2}{x(x+1)} =$$

$$\frac{(x+5)}{(x+3)(x+1)} \bullet \frac{x}{x} - \frac{2}{x(x+1)} \bullet \frac{(x+3)}{(x+3)} = \frac{x^2+5x-2x-6}{x(x+1)(x+3)} =$$

$$= \frac{x^2+3x-6}{x(x+1)(x+3)}$$

(C) $$B \cdot C = \frac{2}{x^2+x} \cdot \frac{x-3}{2x+2} = \frac{2}{x(x+1)} \cdot \frac{x-3}{2(x+1)} = \frac{x-3}{x(x+1)^2}$$

(D) $$B : C = \frac{2}{x^2+x} : \frac{x-3}{2x+2} = \frac{2}{x(x+1)} \cdot \frac{2(x+1)}{x-3} = \frac{4}{x(x-3)}$$

(E) First, I need to check the denominators: they tell me that x cannot equal zero or –1 (since these values would cause division by zero). I'll re-check at the end, to make sure any solutions I find are "valid".

$$C = \frac{x-3}{2x+2} \quad D = \frac{7}{2x}$$

$$C = D \Leftrightarrow \frac{x-3}{2x+2} = \frac{7}{2X} \Leftrightarrow \frac{x-3}{x+1} = \frac{7}{x}$$

Cross product $x^2 - 3x = 7x + 7$ so $x^2 - 10x - 7 = 0$

In the end, using the quadratic formula the solutions are:

$x = 5 \pm 4\sqrt{2}$ “valid” solutions

Now $B + C = D$

When you were adding and subtracting rational expressions, you had to find a common denominator. Now that you have equations (with an "equals" sign in the middle), you are allowed to multiply through by the LCD (because you have two sides to multiply on) and get rid of the denominators entirely. In other words, you still need to find the common denominator, but you don't necessarily need to use it in the same way.

$$\frac{2}{x(x+1)} + \frac{x-3}{2(x+1)} = \frac{7}{2x}$$

Now let’s multiply through by LCD=$2x(x+1)$

So we get $$\frac{2}{x(x+1)} \cdot 2x(x+1) + \frac{x-3}{2(x+1)} \cdot 2x(x+1) = \frac{7}{2x} \cdot 2x(x+1)$$

Which is equivalent to: $4 + (x-3)x = 7(x+1)$ This is a quadratic equation with the following real solutions:

$x_{1,2} = 5 \pm 2\sqrt{7}$

6. The rate of heat loss from a metal object is proportional to the ratio of its surface area to its volume.

(a) What is the ratio of a steel sphere's surface area to volume?

(b) Compare the rate of heat loss for two steel spheres of radius 2 meters and 3 meters, respectively.

a) Here are the formulas for sphere's surface area and volume, where r is radius:

$$S.A._{sphere} = 4\pi r^2 \qquad V_{sphere} = \frac{4}{3}\pi r^3$$

So the ratio of sphere's surface area to volume is: $\dfrac{4\pi r^2}{\frac{4}{3}\pi r^3} = \dfrac{3}{r}$

b) The smaller sphere (the one with the radius of 2) will lose heat at a rate of 3/2=1.5 and the bigger one at a rate of 1. Since time is inverse proportion to heat rate we could also say that if it takes the big sphere an hour to cool down, is going to take the smaller one 2/3 of that time which is 40 minutes.

9. Last week, Wendy jogged for a total of 10 miles and biked for a total of 10 miles. She biked at a rate that was twice as fast as her jogging rate.

(A) Suppose Wendy jogs at a rate of r miles per hour. Write an expression that represents the amount of time she jogged last week and an expression that represents the amount to time she biked last week. (*hint: $dis\tan ce = rate \bullet time$*)

$$t_{jogged} = \frac{d}{r} = \frac{10}{r}$$

$$t_{biked} = \frac{d}{2r} = \frac{10}{2r} = \frac{5}{r}$$

(B) Write and simplify an expression for the total amount of time Wendy jogged and biked last week.

$$t_{total} = \frac{10}{r} + \frac{5}{r} = \frac{15}{r}$$

(C) Wendy jogged at a rate of 5 miles per hour. What was the total amount of time Wendy jogged and biked last week?

$$t_{total} = \frac{15}{r} = \frac{15}{5} = 3(hours)$$

13. A sight-seeing boat travels at an average speed of 20 miles per hour in the calm water of a large lake. The same boat is also used for sight-seeing in a nearby river. In the river, the boat travels 2.9 miles downstream (with the current) in the same amount of time it takes to travel 1.8 miles upstream (against the current). Find the current of the river.

$$t_{downstream} = t_{upstream}$$
$$\frac{2.9}{20+r} = \frac{1.8}{20-r}$$
$$2.9(20-r) = 1.8(20+r)$$
$$1.8r + 2.9r = 2.9 \cdot 20 - 1.8 \cdot 20$$
$$4.7r = 58 - 36$$
$$4.7r = 22$$
$$r = 4.68 mi / h$$

14. A baseball player's batting average is found by dividing the number of hits the player has by the number of at-bats the player has. Suppose a baseball player has 45 hits and 130 at-bats. Write and solve an equation to model the number of consecutive hits the player needs in order to raise his batting average to 0.400. Explain now you found your answer.

Let x be the number of consecutive hits the player needs. In that case the number of hits will become $(45+x)$ and the number of at-bats will be $(130+x)$, so the equation representing his batting average will be:

$$\frac{45+x}{130+x} = 0.400$$
$$x + 45 = 0.4x + 52$$
$$0.6x = 7$$
$$x = 11.67$$

cross product, followed by solving the linear equation gives us:

$$x + 45 = 0.4x + 52$$
$$0.6x = 7$$
$$x = 11.67$$

and since x is a whole number we need to round it up to 12. So the number of consecutive hits the player needs in order to raise his batting average to 0.400 is 12 hits.

19. During a flu outbreak, a hospital recorded 12 cases the first week, 54 cases the second week, and 243 cases the third week.

a) Write a geometric sequence to model the flu outbreak.

The common ratio, $r = \frac{a_n}{a_{n-1}} = \frac{54}{12} = \frac{243}{54} = \frac{9}{2}$

So $a_n = 12\left(\frac{9}{2}\right)^{n-1} = \frac{8}{3}\left(\frac{9}{2}\right)^n$

b) How many cases will occur in the sixth week if the hospital cannot stop the outbreak?

In the sixth week $n = 6$, so $a_6 = \frac{8}{3}\left(\frac{9}{2}\right)^6 = \frac{3^{11}}{2^3}$ = 22,143.375 which is about 22,143 cases of the flu.

21. In a classic math problem a king wants to reward a knight who has rescued him from an attack. The king gives the knight a chessboard and plans to place money on each square. He gives the knight two options. Option 1 is to place a thousand dollars on the first square, two thousand on the second square, three thousand on the third square, and so on. Option 2 is to place one penny on the first square, two pennies on the second, four on the third, and so on.

Think about which offer sounds better and then answer these questions.

a) List the first five terms in the sequences formed by the given options. Identify each sequence as arithmetic, geometric, or neither.
Option 1: \$1000, \$2000, \$3000, \$4000, \$5000
So the sequence is arithmetic with the common difference $d = \$1000$

Option 2: 1, 2, 4, 8, 16
So the sequence is geometric with a common ratio $r = 2$

b) For each option, write a rule that tells how much money is placed on the nth square of the chess board and a rule that tells the total amount of money placed on squares one through n.

Option 1:
$$a_n = a_1 + (n-1)d = 1000 + (n-1)1000 = 1000n$$
$$S_n = \frac{n(a_1 + a_n)}{2}$$

Option 2:
$$a_n = a_1(r)^{n-1} = 1(2)^{n-1}$$
$$S_n = a_1\left(\frac{1-r^n}{1-r}\right)$$

c) Find the amount of money placed the on the 20th square of the chessboard and the total amount of money placed on squares 1 through 20 for each option.

Option 1:
$$a_{20} = 1000(20) = \$20,000$$
$$S_{20} = \frac{n(a_1 + a_{20})}{2} = \frac{20(1000 + 20000)}{2} = \$210,000$$

Option 2:
$$a_{20} = a_1(r)^{20-1} = 1(2)^{19} = \$5,242.88$$
$$S_{20} = a_1\left(\frac{1-r^{20}}{1-r}\right) = 1\left(\frac{1-2^{20}}{1-2}\right) = 2^{20} - 1 = \$10,485.75$$

d) There are 64 squares on a chessboard. Find the total amount of money placed on the chessboard for each option.

Option 1: $S_n = \frac{n(a_1 + a_n)}{2} = \frac{64(1000 + 1000 \cdot 64)}{2} = \$2,080,000$

Option 2: $S_n = a_1\left(\frac{1-r^n}{1-r}\right) = 1\left(\frac{1-2^{64}}{1-2}\right) = 2^{64} - 1 = 1.8446744 \text{ E } 19\text{-}1$

NOTE: This answer doesn't reflect the unit change from cents to dollars.

e) Which gives the better reward, Option 1 or Option 2? Explain why.
Using all the data we can conclude that a quantity increasing exponentially eventually exceeds a quantity increasing linearly after enough iteration has been done. If we would have come up with a conclusion based on the results regarding the 20th square, we would have been wrong.

25. Consider the function $f(x) = \log x$.

a) Identify the transformation applied to $f(x)$ to create $g(x) = \log x + 1$.

Vertical shift 1 unit up applied to $f(x)$ to create $g(x)$

b) Identify the transformation applied to $f(x)$ to create $h(x) = \log(10x)$.

The graph of $h(x)$ is a horizontal compression of the original function, by a factor of 10.

c) Compare the graphs of $g(x)$ and $h(x)$. What do you notice?

By comparing the graphs of $g(x)$ and $h(x)$ we see that the graphs are identical.

d) Use the properties of logarithms to explain your answer to part (c).
$h(x) = \log(10x) = \log(10) + \log(x) = 1 + \log(x) = g(x)$

28. Psychologists try to predict the activation of memory when a person is tested on a list of words they learned. The following model is used to make this prediction: $A = \ln(n) - 0.5\ln(T) - 0.5\ln(L)$ where A is the number of words learned, n is the number of exercises, T is the amount of time between learning and testing and L is the length of the list that was tested.

a) Write the formula as the ln of a single expression.

Using the power, product and quotient property we get:

$$A = \ln\frac{n}{T^{0.5}L^{0.5}} = \ln\frac{n}{T^{1/2}L^{1/2}} = \ln\frac{n}{\sqrt{TL}}$$

b) Discuss the influence on A (going up or down) when increasing *n, T*, and *L*, according to the formula. Do these results make sense?

Since the natural logarithm function is strictly increasing, as the input values are increasing, the output ones are increasing. In this example we need to take in consideration all three independent variables n, T and L. Because n is in the numerator of the argument, as n is increasing A is increasing. T and L are in the denominator of the argument of the natural logarithm function, so as T and L are increasing, $\frac{n}{\sqrt{TL}}$ is decreasing, so A is decreasing.

c) If you want A to be bigger than 0, what conditions must be placed on L, T, and n?

Since ln1=0 and the natural logarithm function is strictly increasing, the argument of the ln function must be greater than 1 for A to be bigger than 0, so the condition is:

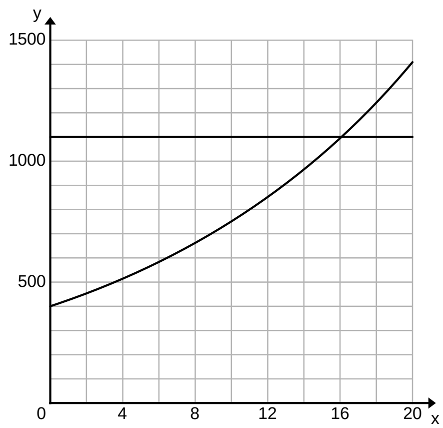
$$\frac{n}{\sqrt{TL}} > 1$$

37. Sarai bought $400 of Las Vegas Cellular stock in January of 2005. The value of the stock is expected to increase by 6.5% per year.

a) Write a model to describe Sarai's investment.

$A(x) = P(x)(1+r)^t = 400(1.065)^t$, t is the number of years

b) Use the graph to show when Sarai's investment will reach $1100.



Sarai's investment will reach $1100 in just over 16 years.

38. The loudness of sound is measured on a logarithmic scale according to the formula $L = 10\log\left(\frac{I}{I_0}\right)$, where L is the loudness of sound in decibels (db), I is the intensity of sound, and I_0 is the intensity of the softest audible sound.

a) Find the loudness in decibels of each sound listed in the table.

Sound	Intensity
Jet taking off	$10^{15} I_0$
Jackhammer	$10^{12} I_0$
Hairdryer	$10^{7} I_0$
Whisper	$10^{3} I_0$
Leaves rustling	$10^{2} I_0$
Softest audible sound	I_0

According to the logarithmic scale formula $L = 10\log\left(\frac{I}{I_0}\right)$ by replacing the intensity I by their formula in terms of I_0 we obtain:

Sound	Intensity
Jet taking off	150
Jackhammer	120
Hairdryer	70
Whisper	30
Leaves rustling	20
Softest audible sound	0

b) The sound at a rock concert is found to have a loudness of 110 decibels. Where should this sound be placed in the table to keep the sound intensities in order from least to greatest?
It should be placed between the jackhammer and the hairdryer.
Here is the explanation: $110 = 10\log\left(\frac{I}{I_0}\right)$, so $\frac{I}{I_0} = 10^{11}$

c) A decibel is $\frac{1}{10}$ of a *bel*. Is a jet plane louder than a sound that measures 20 *bels*? Explain.
The loudness of a jet plane is 150 db=15 bels, so the het is not louder that a sound that measures 20 bels.

39. Aaron invested \$4000 in an account that paid an interest rate r compounded continuously. After 10 years he has \$5809.81. The compounded interest formula is $A = Pe^{rt}$, where P is the principle (the initial investment), A is the total amount of money (principle plus interest), r is the annual interest rate, and t is the time in years.

a) Divide both sides of the formula by P and then use logarithms to rewrite the formula without an exponent. Show your work.

$A = Pe^{rt}$

$\frac{A}{P} = e^{rt}$

$\ln\frac{A}{P} = \ln e^{rt}$

$\ln\frac{A}{P} = rt$

b) Using your answer for part (a) as a starting point, solve the compound interest formula for the interest rate r.

$r = \frac{1}{t}\ln\frac{A}{P}$

c) Use your equation from part (a) to determine the interest rate.

$r = \frac{1}{t}\ln\frac{A}{P} = \frac{1}{10}\ln\frac{5809.81}{4000} = \frac{1}{10}\cdot 0.37325350697 \approx 0.037$

Which gives us an interest rate of 3.7%

42. Public Service Utilities uses the equation $y = a + b\log x$ to determine the cost of electricity where x represents the time in hours and y represents the cost. The first hour of use costs $6.66 and three hours cost $18.11.

a) Determine the value of a and b in the model.

We are given $(1, 6.66)$ and $(3, 18.11)$, then set up a system to solve for a and b.

$6.66 = a + b\log(1)$ and $18.11 = a + b\log(3)$

6.66=a

$18.11 = 6.66 + b\log(3)$

$\frac{11.45}{\log(3)} = b$

$b = 24$

b) What is the x-intercept of the graph of the model? What is the real world meaning of the x-intercept?

To find the x-intercept, set $y=0$.

$y = 6.66 + 24\log(x)$

$0 = 6.66 + 24\log(x)$

$\frac{-6.66}{24} = \log(x)$

$10^{-6.66/24} = x$

$x = .5278$ hours

This means that the first half hour of service is free.

c) Use the model to find the cost for 65 hours of electricity use.

$y = 6.66 + 24\log(x)$

$y = 6.66 + 24\log(65)$

$y = \$50.17$

d) If a customer can afford \$40 per month for electricity, how long can he or she have the electricity turned on?

$y = 6.66 + 24\log(x)$

$\$40 = 6.66 + 24\log(x)$

$\frac{33.34}{24} = \log(x)$

$10^{33.34/24} = x$

$x = 24.5$ hours

43. On an x-y coordinate plane the earth is located at (2, -1) and an asteroid is traveling on the path of $g(x) = e^{3x} + 3$.

a) Write an equation representing the distance from the earth to the asteroid.

The position of earth is at (2 , -1) and the position of the asteroid is at $(x, e^{3x} + 3)$.

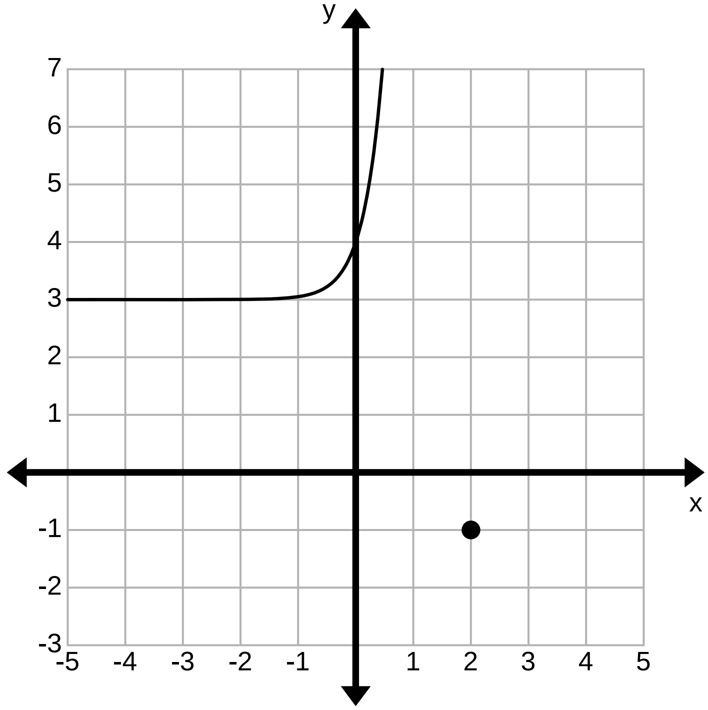
$d = \sqrt{(x-2)^2 + (e^{3x} + 3 + 1)^2}$

b) If the asteroid is currently located at $(4, e^{12} + 3)$, what is the distance from the earth to the asteroid?

$d = \sqrt{(4-2)^2 + (e^{12} + 3 + 1)^2}$

$d = 162,758.79$ units

c) Sketch a graph of $g(x)$.



d) Find the point when the asteroid is closest to the earth.

The distance from earth to the asteroid can be modeled by $d = \sqrt{(x-2)^2 + (e^{3x} + 3 + 1)^2}$.
The minimum value for this model occurs at (-.535,4.906). This means that when x=-.535, you will have the smallest distance from earth to the asteroid.
We must now determine the position of the asteroid when x=-.535.

$g(x) = e^{3x} + 3$

$g(-.535) = e^{3(-.535)} + 3$

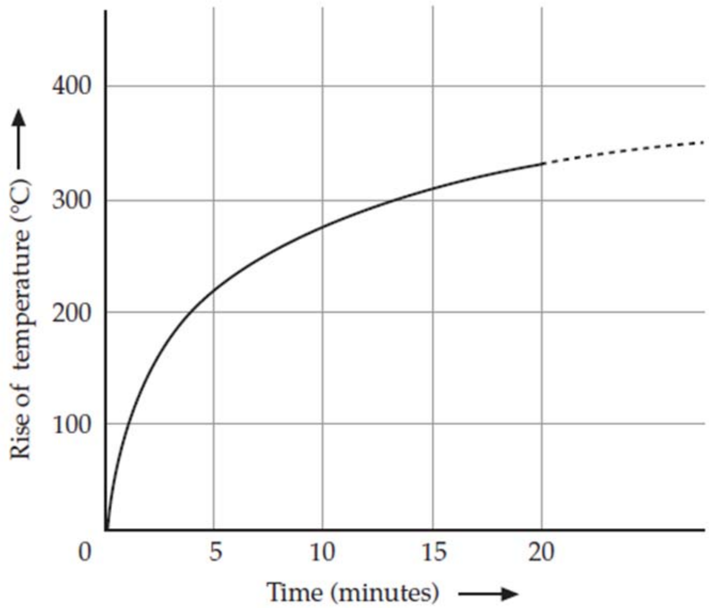
$g(-.535) = 3.2$

The position of the asteroid closest to earth is $(-.535, 3.2)$

46. The graph below shows the change in temperature of a burning house over time.

a) Describe the graph.

The graph looks like the n-th root function, where n must be even. Some might confuse it with the logarithmic function, but there is no vertical asymptote and our function passes through origin.



b) This graph was found in an old math book and next to it was written:

Rise of temperature = $t^{0.25}$
Show that this function does not describe the graph correctly.

$t^{0.25} = t^{\frac{1}{4}} = \sqrt[4]{t}$

$\left(\sqrt[4]{1} = 1; \sqrt[4]{16} = 2; \sqrt[4]{27} = 3\right)$ $t^{0.25} = t^{\frac{1}{4}} = \sqrt[4]{t}$

$(1,1);(16,2);(81,3)$

As we know: $\sqrt[4]{1} = 1; \sqrt[4]{16} = 2; \sqrt[4]{81} = 3$ but there are no points of coordinates $(1,1);(16,2);(81,3)$ on the graph.

c) Assume that the power function $r = At^{0.25}$ is a good description of the graph. Find a reasonable value for A. Graph the new function.

A reasonable value for A is 150. $r = 150t^{0.25}$ is a good approximation of the graph mentioned above.

d) Compare the graph in part (c) to the original one.
Do you think that a different power of t might result in a better model? Would a larger or smaller power produce a better fit? Explain.

Answers may vary: The graph of $r = 150t^{0.25}$ is too steep in the beginning and too flat for the bigger values of t. Therefore, a power that is a little bit bigger might produce a better fit.

e) Use the original graph to find data. Carry out a power regression on the data to find a function that would produce a better fit.

The regression model will depend on the data used. A sample function is $r = 137.5t^{0.3}$

53. Convert $\frac{3\pi}{8}$ radians to degrees. $\frac{3\pi^{rad}}{8} \cdot \frac{180°}{\pi^{rad}} = \frac{135°}{2} = 67.5°$

54. Convert $\frac{16\pi}{3}$ radians to degrees. $\frac{16\pi^{rad}}{3} \cdot \frac{180°}{\pi^{rad}} = 960°$

55. Suppose each paddle on the wall of a clothes dryer makes 80 revolutions per minute.

Part A: What angle does one paddle subtend in 10 seconds? Give your answer in radians.

$$\frac{80\ revolutions}{1min} * \frac{1min}{60\ sec} = \frac{{}^{80}/_{6}\ revolutions}{10\ sec}$$

So in 10 seconds you have 80/6 revolutions, which in radians is

$$\frac{80revolutions}{6} * \frac{2\pi^{rad}}{1revolution} = \frac{80\pi^{rad}}{3}$$

Part B: Write an algebraic expression to determine the measure in radians of the subtended angle after x seconds. Show how the units simplify in your expression.

$$\frac{80 \text{ revolutions}}{1 \text{ minute}} \times \frac{1 \text{ minute}}{60 \text{ seconds}} = \frac{{}^{4}/_{3} \text{ revolutions}}{1 \text{ second}}$$

$$\frac{{}^{4}/_{3} \text{ revolutions}}{1 \text{ second}} \times \frac{2\pi \text{ radians}}{1 \text{ revolution}} \times (x \text{ seconds}) = \frac{8}{3}\pi x \text{ radians}$$

Part C: You are interested in determining the total distance a point on the drum travels in a 20-minute drying cycle. Can you use your expression from **Part B**? What other information, if any, is needed? Explain.

In 20 minutes, which is 1200 seconds, the measure of the subtended angle is

$\frac{8\pi * 1200^{rad}}{3} = 3200\pi^{rad}$, which happens in 1600 revolutions
The distance a point on the drum travels in a full revolution is the circumference of the circle $C = 2\pi r$, so the formula for 1600 revolutions would be d=1600*$2\pi r$. To be able to come up with the distance traveled we need to know the radius of the circle.

62. The diameter of a bicycle tire is 20 in. A point on the outer edge of the tire is marked with a white dot. The tire is positioned so that the white dot is on the ground, then the bike is rolled so that the dot rotates clockwise through an angle of 16.75π radians.

Part A: To the nearest tenth of an inch, how high off the ground is the dot when the wheel stops? Show your work.

Let's find how many degrees the dot rotates clockwise:

$$16.75\pi^{rad} * \frac{180°}{\pi^{rad}} = 3015°$$

If the angle is greater than 360 degrees, you subtract 360 degrees from it until the angle is less than 360 degrees or you could use this shortcut for very large positive angles:

Divide the angle by 360.

Take the integer part of the result and multiply 360 by that.

Subtract the result from the angle.

Example:

Your angle is 3015.

Divide by 360 to get 8.375

Multiply 360 by 8 to get 2880

Subtract 2880 from 3015 to get 135.
135 is the angle you need to work with to get your reference angle from. So this dot moves from the ground, clockwise 135 degrees, so it is going to end in the second quadrant. The reference angle will be the acute angle between the terminal side and the x axis which will be 45 degrees.

So the height of the dot will be $10 + 10\sin 45° = 10 + 10 * \frac{\sqrt{2}}{2} = 17.07(in)$

Part B: What distance was the bicycle pushed? Round your answer to the nearest foot.

For every full rotation the dot moved a distance equal to the circumference of the circle $2\pi r, so\ 3015° * \frac{2\pi * 10}{360°} = 526.22(in)$ = 44 feet

Part C: Would changing the size of the tire (value of *r*) change either of the answers found in **Parts A** or **B**? Explain your reasoning.

In both answers the radius is present as a factor, so if you would triple the original value for the radius, both answers would triple their values.

63. A ribbon is tied around a bicycle tire at the standard position $0°$. The diameter of the wheel is 26 inches. The bike is then pushed forward 20 feet from the starting point. In what quadrant is the ribbon? Explain how you obtained your answer.

If the bike is pushed 20 ft from the starting point, then the ribbon moved 20 ft from the starting point.

20 ft = 240 in

The circumference of a circle is the length of the curve that encloses that circle, which is $C = 2\pi$r. Another way to think of the curve that encloses a circle is through the 360 degree arc of that curve. Thus, the circumference of a circle is the length of the 360 degree arc of that circle. So there is a “unit” fraction that we could use:

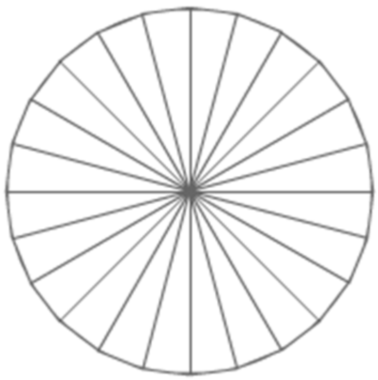
$$240in * \frac{360°}{2\pi * 13in} = 1057.77°$$

Through its circular movement the ribbon will define a curve whose measure in degrees is 1057.77

1057.77-360-360=337.77 which tells us that the ribbon is in the 4th quadrant.

65. Two friends counted 24 evenly spaced seats on a Ferris wheel. As they boarded one of the seats, they noticed the edge of the wheel was 1 meter off the ground. They learned from the operator that the diameter of the wheel was 28 meters. After they got seated and started moving, in a counter-clockwise direction, they counted 13 chairs pass the operator, and then the Ferris wheel was stopped on the fourteenth chair to load another passenger.

Part A: Design a representation of the Ferris wheel and locate where the friends were when the wheel stopped to load the next passenger.



Part B: How many radians had they rotated through in the time before they stopped?

$$\frac{\mathbf{360^\circ}}{\mathbf{24}} * \mathbf{14 = 210^\circ}$$

$210^\circ = {}^{7\pi}/_{6}\, rad$ **,** so from the lowest point of the wheel they move 210 degrees or ${}^{7\pi}/_{6}\, rad$ counter clockwise.

Part C: To the nearest tenth of a meter, how far above the ground were they? Show your work.

The wheel is 1 meter above the ground so we need to add that to our sum. The friends are at point M whose reference angle is 60 degrees

$$\mathbf{1 + 14 + 14\, sin(60^\circ) = 1 + 14 + 14 * \frac{\sqrt{3}}{2} = 27.12(m)}$$

68. Solve ΔABC, given that $A = 47^\circ, B = 52^\circ$, and $b = 78$.

$$\frac{\sin 47^\circ}{a} = \frac{\sin 52^\circ}{78} = \frac{\sin C}{c}$$
$$a = 72.39, c = 97.76, C = 81^\circ$$

69. Given ΔABC with $a = 10, b = 13$, and $A = 19^\circ$, find c. Round your answer to two decimal places.

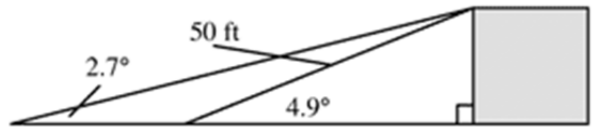
$$\frac{\sin 19^\circ}{10} = \frac{\sin B}{13} = \frac{\sin C}{c}$$

$\angle B = 25^\circ$ or 155°, there are 2 possible triangles, therefore $c = 21.35$ or 3.23.

70. Solve ΔABC with $A = 110°$, $a = 5$, and $b = 7.3$.

Apply the Law of Sines to this problem, there is no solution.

71. A 50 foot ramp makes an angle of $4.9°$ with the horizontal. To meet new accessibility guidelines, a new ramp must be built so it makes an angle of $2.7°$ with the horizontal. What will be the length of the new ramp?



There are two ways to approach this problem.

1) Apply the Law of Sines to the obtuse triangle.

$$\frac{\sin 2.7°}{50} = \frac{\sin 175.1}{\text{ramp}}$$, ramp = 90.66 feet

2) Use the right triangle with the old ramp, 50 feet long, to find the height of the step. Use this height and the larger right triangle with the new ramp as the hypotenuse to find its length.

(Small right triangle) $\sin 4.9° = \frac{\text{step height}}{50 \text{ ft}}$, step height is 4.27 feet.

(Large right triangle) $\sin 2.7° = \frac{4.27\text{feet}}{\text{new ramp length}}$

new ramp =90.66 feet

75. ***Part A:*** Because $\triangle RST$ is a right triangle, the Pythagorean Theorem can be used to determine the value of t.

$t^2 = 1^2 + 1^2 = 1 + 1 = 2$

$t = \sqrt{2}$

Part B: Since the angles opposite the congruent sides of an isosceles triangle are congruent, and since the three angle measures of a triangle must sum to 180°, $m\angle R + m\angle S = \frac{180° - 90°}{2} = 45°$

Therefore $\sin R = \frac{\text{opp}}{\text{hyp}} = \frac{1}{t} = \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

Part C: The area of triangle DEF is:

$A = \frac{1}{2} \cdot DF \cdot EF \cdot \sin(45°)$

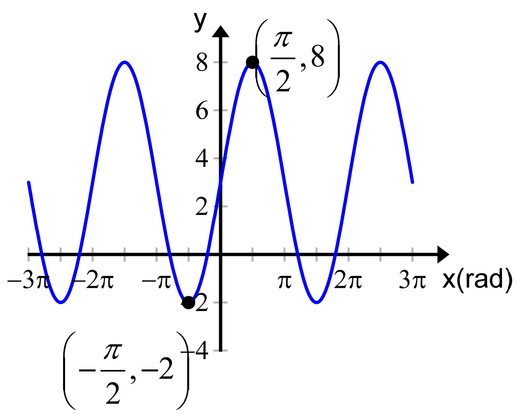
$= \frac{1}{2} \cdot 4 \text{ cm} \cdot 6 \text{ cm} \cdot \frac{\sqrt{2}}{2} = 6\sqrt{2} \text{ cm}^2$

Part D: $6\sqrt{2} \text{ cm}^2 \approx 8.5 \text{ cm}^2$

85. Write an equation of the form $y = a\sin bx$, where $a > 0$ and $b > 0$, with amplitude $\frac{2}{3}$ and period 12

$$y = \frac{2}{3}\pi \sin\frac{\pi}{6}x$$

86. Write a function for the sinusoid.



The period is the length of a full cycle. The given points represent the end points of half cycle so by subtracting their x values we get the length of half cycle $\frac{\pi}{2} - \left(-\frac{\pi}{2}\right) = \pi$. That result is doubled to get the length of the full cycle, which is $P = 2\pi$. Knowing that $P * b = 2\pi$, we get $b = \frac{2\pi}{2\pi} = 1$.

The amplitude $A = \frac{max - min}{2} = \frac{8-(-2)}{2} = 5$

The sinusoidal curve was shifted 3 units up so the simplest form of the equation is: $y = 5\sin\phi + 3$

87. A sound wave models a sinusoidal function.

Part A: If the wave reaches its maximum at $\left(\frac{\pi}{2}, 12\right)$ and its minimum at $\left(\frac{3\pi}{2}, 0\right)$, what are the shift, amplitude, and period of the function?

The amplitude $A = \frac{max - min}{2} = \frac{12-0}{2} = 6$
Between max and min we have half cycle so the difference between the x values represents the length of half cycle.

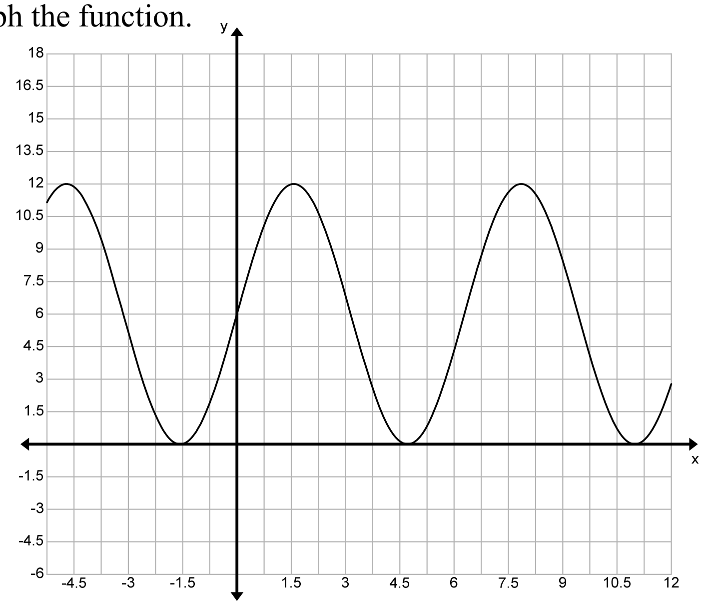
$$\frac{3\pi}{2} - \frac{\pi}{2} = \pi$$

Since the period is the length of a full cycle, the period is $P = 2\pi$, so b=1.
Since the max value is 12 and min is 0 is clear that the sinusoidal curve was shifted vertically 6 units up.

Part B: Write the function that models this sound wave.

$$y = 6\sin x + 6$$

Part C: Graph the function.



88. Is the function $y=-2\sin\left(2x-\frac{\pi}{2}\right)+3$ in the form $y=a\sin b(x-h)+k$? Why or why not? How does the amplitude and period of the function compare to the amplitude and period of $y=\sin x$? How does the graph of the function compare to the graph of $y=2\sin 2x$?

In order to put the equation in standard form we need to factor out the coefficient of x which is 2.

$$y=-2\sin\left(2x-\frac{\pi}{2}\right)+3=-2\sin\left[2\left(x-\frac{\pi}{4}\right)\right]+3$$

The amplitude is $A=|-2|=2$ and the period is $P=\frac{2\pi}{b}=\frac{2\pi}{2}=\pi$

How does the graph of the function compare to the graph of $y=2\sin 2x$?

$$y=-2\sin\left[2\left(x-\frac{\pi}{4}\right)\right]+3$$

The phase shift $\frac{\pi}{4}$ tells us to shift the graph of the function $y=2\sin 2x$ to the right by $\frac{\pi}{4}$, then to reflect it with respect to the x axis because of the negative coefficient -2 and then to shift it vertically 3 units up.

91. Find the amplitude and period of the graph of $y = -3\cos \pi x$.

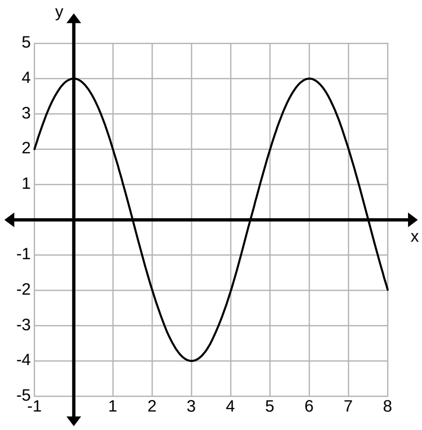
$$A = |-3| = 3$$
$$P = \frac{2\pi}{b} = \frac{2\pi}{\pi} = 2$$

92. Find the amplitude and period of the graph of $y = -2\cos 6x$.

$$A = |-2| = 2$$
$$P = \frac{2\pi}{b} = \frac{2\pi}{6} = \frac{\pi}{3}$$

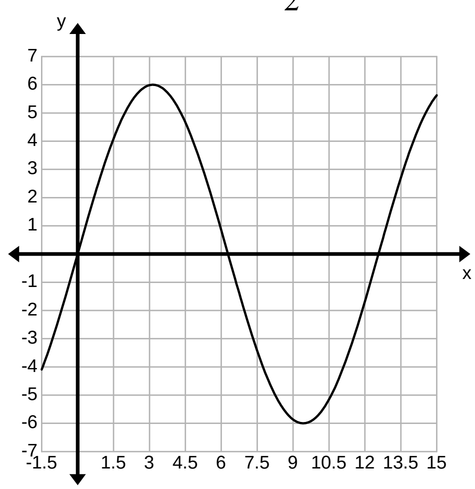
93. Graph one cycle of the graph of the function $f(x) = 4\cos \frac{\pi x}{3}$.

$$A = |4| = 4$$
$$P = \frac{2\pi}{b} = \frac{2\pi}{\pi/3} = 6$$

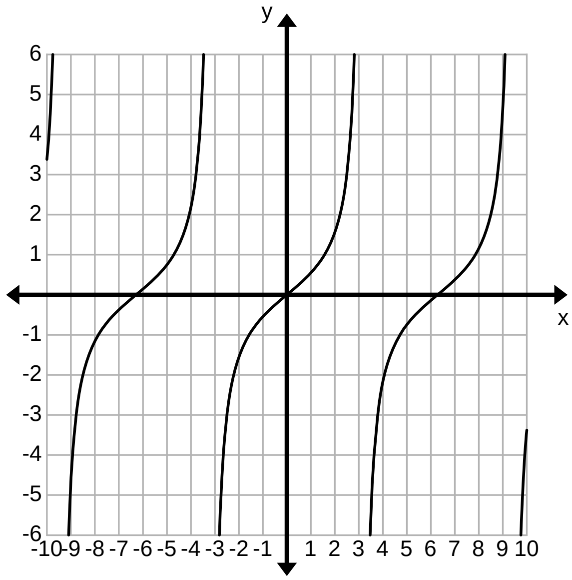


94. Graph one cycle of the graph of the function $f(x) = 6\sin \frac{x}{2}$

$$A = |6| = 6$$
$$P = \frac{2\pi}{b} = \frac{2\pi}{1/2} = 4\pi$$



95. Graph $y = \tan\left(\frac{x}{2}\right)$. Include vertical asymptotes in your sketch.



96. The graph of a sine function has amplitude 5, period $72°$, and a vertical translation 4 units down. Write an equation for the function.

$$P = 72^\circ = 72^\circ \frac{\pi}{180^\circ} = \frac{2\pi}{5}^{rad}$$

$$b = \frac{2\pi}{P} = \frac{2\pi}{2\pi/5} = 5$$

so $y = 5\sin(5x) - 4$

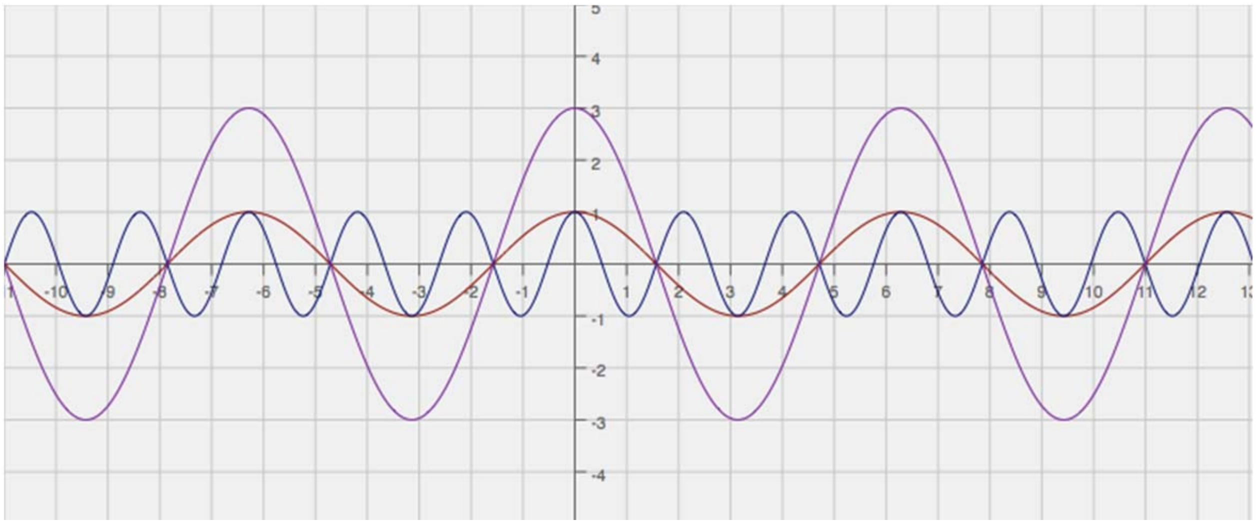
97. The graph of a cosine function has amplitude 4, period $90°$, and a vertical translation 3 units down. Write an equation for the function. Then sketch the graph without using graphing technology.

$$P = 90^\circ = 90^\circ \frac{\pi}{180^\circ} = \frac{\pi}{2}^{rad}$$

$$b = \frac{2\pi}{P} = \frac{2\pi}{\pi/2} = 4$$

so $y = 4\cos(4x) - 3$

98. Sketch the graphs of $y = \cos x, y = \cos 3x$, and $y = 3\cos x$. Tell how the graphs are alike and how they are different.



Both functions $y = \cos x$ *and* $y = \cos(3x)$ have the same amplitudes A=1, while the function $y = 3\cos x$ has an amplitude of 3.

Both functions $y = \cos x$ *and* $y = 3\cos x$ have the same periods $P = 2\pi$, while the function $y = \cos(3x)$ has a period $P = 2\pi/3$

99. Consider the related equations $y = \sin x, y = 2\sin x$, and $y = \sin 2x$. Explain the effect that the coefficient 2 has on the graphs of $y = 2\sin x$ and $y = \sin 2x$ when compared to the graph of $y = \sin x$.

When compared to the parent function, the graph of $y = 2\sin x$ has an amplitude of 2 which means that the parent function has been stretched by a factor of 2.

When it comes to the function *y*=sin (2x) the 2 gives the number of complete cycles on an *x*-interval of length 2π , so since b=2 the period is π.

100. The unit circle centered at the origin has a radius of 1, and the coordinates (x, y) locate any point on the circle.

Part A: Prove that $\cos^2\theta + \sin^2\theta = 1$ for θ representing the central angle of the arc intercepted by the point (x, y) and the *x*-axis.

The unit circle has the equation $x^2 + y^2 = 1$ and we know that $x = \cos\theta$ and y=sinθ .

Therefore, with substitution, $\sin^2 + \cos^2\theta = 1$.

Part B: Does this formula work for all values for θ? Explain.

To complete the proof, the identities found at Trigonometric symmetry, shifts, and periodicity may be used. By the periodicity identities we can say if the formula is true for $-\pi < \theta \le \pi$ then it is true for all real θ. Next we prove the range $\pi/2 < \theta \le \pi$, to do this we let $t = \theta - \pi/2$, t will now be in the range $0 < t \le \pi/2$. Substitution and basic shift identity yields:

$$\sin^2\theta + \cos^2\theta = \sin^2\left(t + \frac{\pi}{2}\right) + \cos^2\left(t + \frac{\pi}{2}\right) = \cos^2 t + \sin^2 t = 1$$

All that remains is to prove it for $-\pi < \theta < 0$; this can be done by squaring the symmetry identities to get

$$\sin^2\theta = \sin^2(-\theta)\, and$$

$$\cos^2\theta = \cos^2(-\theta)$$

Part C: Without using a calculator, evaluate $\cos\theta$ if $\tan\theta = -\frac{3}{4}$.

If $\tan\theta$ is a negative value, it must be in the second or fourth quadrant. Since tangent is the ratio of opposite to adjacent sides, the hypotenuse must be 5.

The definition of Cosine is the ratio of adjacent to hypotenuse. Therefore, $\cos\theta = \pm\frac{4}{5}$.

101. For an angle θ, $\sin\theta = \frac{5}{13}$ and $\frac{\pi}{2} < \theta < \pi$.

Part A: Use the Pythagorean identity to find $\cos\theta$.

$$\sin^2\theta + \cos^2\theta = 1$$

$$\left(\frac{5}{13}\right)^2 + \cos^2\theta = 1$$

$$\cos\theta = \pm\frac{12}{13}$$

but since $\frac{\pi}{2} < \theta < \pi$

the value of cosine will be negative, so it must be -12/13

Part B: If $\sin\theta = -\frac{5}{13}$ and $\frac{3\pi}{2} < \theta < 2\pi$, does $\cos\theta$ change? Explain.

$$\sin^2\theta + \cos^2\theta = 1$$

$$\left(-\frac{5}{13}\right)^2 + \cos^2\theta = 1$$

$$\cos\theta = \pm\frac{12}{13}$$

but since $\frac{3\pi}{2} < \theta < 2\pi$

the value of cosine will be positive, so it must be 12/13

102. Tides can be modeled by periodic functions. Suppose high tide at the city dock occurs at 2:22 AM at a depth of 35 meters and low tide occurs at 9:16 AM at a depth of 9 meters. Write an equation that models the depth of the water as a function of time after midnight. When will the next high and low tides occur?

- The period P is twice the time between the low and the high tide, so P=2*(9:16-2:22)=2*(6h 54min)=13 h 44min=$13\frac{44}{60}h = 13.73h$, so $b = \frac{2\pi}{13.73}$

- The amplitude $A = \frac{1}{2}(35 - 9) = 13$m

- If they asked us to write an equation that models the depth of the water as a function of time after 2:22 AM (when the maximum occurs) the equation would have been:

$$h(t) = 13\cos\frac{2\pi}{13.73}(t - 2.37)$$

$$2h22\min = 2\frac{22}{60} = 2.37h$$

$$y(t) = r\sin bt$$

but it has to be written after midnight so we need to accommodate a shift in time of

$$2h22\min = 2\frac{22}{60} = 2.37h$$

so the equation that models the depth of the water as a function of time after midnight is:

$$h(t) = 13\cos\left[\frac{2\pi}{13.73}(t - 2.37)\right]$$

- When will the next high and low tides occur?

  We know that the high tide occurs at 2:22 AM and the period is P=13h 44min, so the next high tide occurs 13h44min later from the time the high tide occurred, which is @4:06AM the next day. The low tide occurs at 9:16 AM and the period is 13h44min, so 13h44min from 9:16 AM will be @ 11:00AM the next day.

103. A Ferris wheel with a radius of 25 feet is rotating at a rate of 3 revolutions per minute. When $t = 0$, a chair starts at the lowest point on the wheel, which is 5 feet above ground. Write a model for the height *h* (in feet) of the chair as a function of the time *t* (in seconds).

Let's think about the Ferris wheel as a trigonometric circle. The y-coordinate of any point on the trigonometric circle tells you how far the point is above or below the x-axis and the function that gives the y-coordinate is the sine function. Since the wheel rotates at a rate of 3 revolutions per minute, but the problems asks us to write the height *h* (in feet) of the chair as a function of the time *t* (in seconds), we need to do the following transformation:

$$\frac{3revolutions}{1\min} = \frac{3revolutions}{60\sec} = \frac{.05revolutions}{1\sec}$$

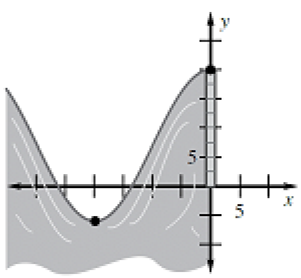
which gives us the number of revolutions/sec, so we can conclude that b=.05

The x-axis is located at the wheel's center, not at ground level, so the wheel's center is 30 ft above ground (the lowest point on the wheel is 5 feet above ground +25 the radius). We need to accommodate this situation by adding the constant k=30, so the equation that describes height *h* (in feet) of the chair as a function of the time *t* (in seconds) is:

$$h(t) = r\sin(bt) + k$$

$$h(t) = 25\sin(.05t) + 30$$

104. Storm surge from a hurricane causes a large sinusoidal wave pattern to develop near the shore. The highest wave reached the top of a wall 20 feet above sea level. The low point immediately behind this wave was 6 feet below sea level, and was 20 feet behind the peak. What is the amplitude of the sinusoid? What is the vertical shift of the sinusoid from a wave at ground level?



The amplitude is defined as half the distance between the minimum and maximum values of the range, so:

$$A = \frac{20-(-6)}{2} = 13$$

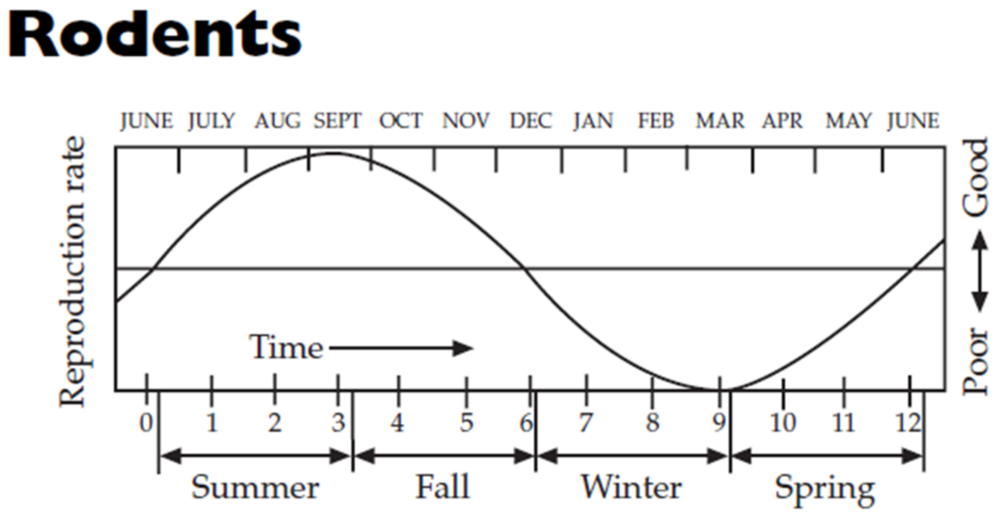
The vertical shift of the sinusoid from a wave at ground level is 6 feet down.

P=2*20=40minutes, so $b = \frac{2\pi}{40}$

So the equation would be

$$h(t) = 13\cos\left(\frac{2\pi t}{40}\right) + 7$$

105. The graph below shows how the reproductive rate of rodents varies depending on the season. On the x-axis, the months are grouped by season and on the y-axis, the reproductive rate is represented on a scale from poor to good.



a) When is the reproduction of the rodents at the lowest? When is it at the highest?

The reproduction of the rodents is at the lowest in March and at its highest in September.

b) Put 0.2 and 2 as the minimum and maximum values on the y-axis. Design a formula that describes the graph. Explain how you determined your formula.

$$A = \frac{\max - \min}{2} = \frac{2 - 0.2}{2} = \frac{1.8}{2} = 0.9$$

P=12 months, so $b = \dfrac{2\pi}{12}$

To obtain the min value of 0.2 we need to do a vertical shift on the parent function cos(t) by 1.1(0.9+0.2=1.1 because we need the cosine 0.2 above the x-axis)

Since the max value is obtain in March (not in January) there is a horizontal shift 3 units to the right, which in the equation is translated by (t-3)

$$y = 0.9\cos\left(\frac{2\pi}{12}(t-3)\right) + 1.1$$

c) Suppose the reproductive rate were put on a scale from 0, for extremely poor, to 10 for extremely good. In this case, use 0 and 10 as the minimum and maximum values on the y-axis. Design a formula that describes the corresponding graph. What changes did you have to make to your formula form part (b)?

The formula that describes the corresponding graph is:

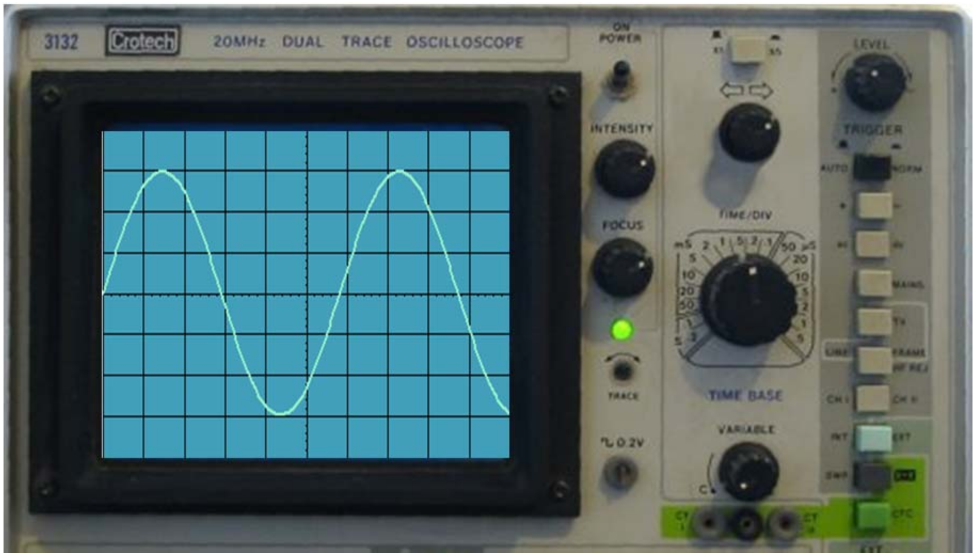
$y = 5\cos\left(\frac{2\pi}{12}(t-3)\right)+5$ The necessary changes are in the amplitude and the vertical shift.

The amplitude is different since the minimum and maximum are changed:

$$A = \frac{\max-\min}{2} = \frac{10-0}{2} = 5$$

Halfway between the min and max values we have the midline which tells us that there is a vertical shift 5 units up.

106. An oscilloscope is a machine that measures the magnitude of fluctuating voltages by displaying a graph of the voltage over time. The figure below shows the shape of the fluctuating voltage. The horizontal axis displays time, t, and the vertical axis shows voltage. The person who is working with scope can see from the buttons that in this case, one step on the horizontal-axis scale is 0.5 sec and one step on the vertical-axis scale is 0.2 V.



Design the formula for this fluctuating voltage. Explain how you determined your formula.

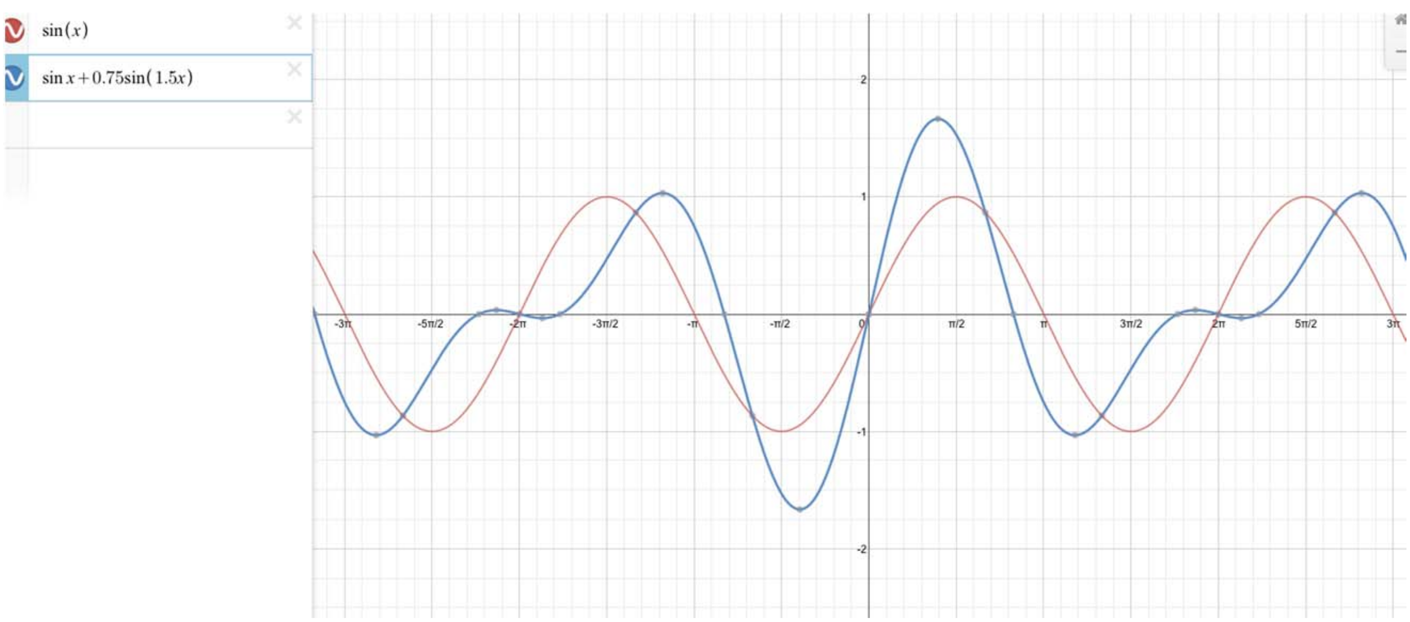
$$V(t) = 0.6\sin\left(\frac{2\pi}{3}(t-0.4)\right)$$

From the picture we can see that the sinusoidal is symmetric with respect to the x axis so the amplitude is 3units on the y-axis and since the vertical scale is 0.2 we get the amplitude A=3*0.2=0.6

The period P is defined as the amount of time it takes to complete a full revolution so the distance between the maximum points is about 6 units on the x-axis and since we know that the horizontal scale is 0.5, the period P=6*0.5=3, so $b=\frac{2\pi}{3}$. The graph is shifted horizontally 4/5 of a horizontal unit =4/5*0.5=0.4 to the right, so we need (t-0.4) factored in the equation.

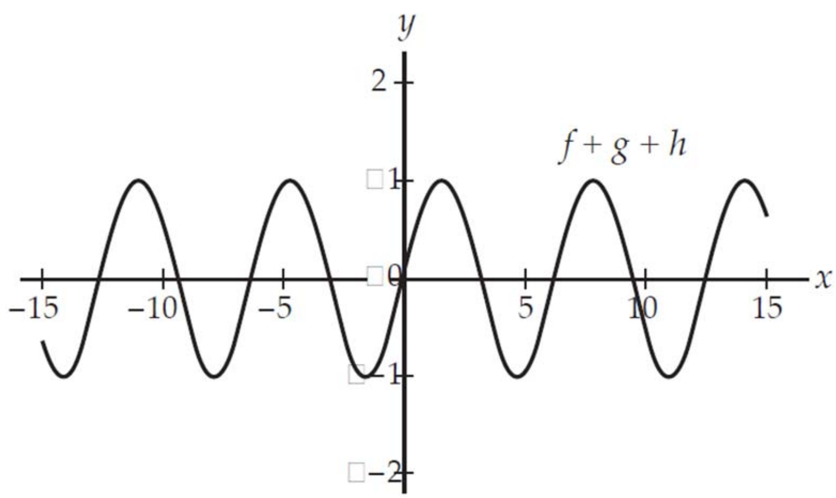
107. One modern application of the addition and subtraction of functions appears in the field of audio engineering. To help understand the idea behind this use, consider the following simplified example. Suppose that the function $f(x) = \sin(x)$ represents the sound of music to which you are listening. Unfortunately, there is background noise. Let $g(x) = 0.75\sin(1.5x)$ represent that noise.

a) Sketch a graph of f and the sum $f + g$, on a single set of axes. Your graph represents both what you want to hear and what you actually do hear. They clearly are not the same.



b) Now apply some mathematics to engineer away the noise. You can add a microphone to your headphones. In turn, the microphone picks up the background noise, $g(x)$, then plays back through our headphones an altered version, $h(x)$. Thus what you now hear in the headphones is the sum of three functions: f (what you want to hear), g (the noise you don't want), and h (the correction for the noise). Suppose the compensating function is $h(x) = 0.75\sin(1.5x - \pi)$. Graph the new sum, $f + g + h$, to see what you hear now. Explain the idea behind the adjustment.

The function h is exactly the opposite of g, so $g + h$ is the constant function $y = 0$. Hence, $f + g + h$, is what you want to hear.



c) Another engineer suggests adding $h(x) = -0.75\sin(1.5x)$ instead of the h defined in part (b). Discuss the merits of that solution.

This is also a good solution. Once possible drawback, though is that it requires playing the solution at the same time the background noise is heard. This means that the detection by the microphone and the construction of the solution curve must occur in the amount of time that the original noise travels from the microphone to the headphone. That may be possible, but it sure is fast!

108. The price of oranges fluctuates, depending on the season. The average price during a complete year is $2.25 per kilogram (about 2.2 pounds). The lowest price will be paid in mid-February ($1.60 per kilo); the highest price is paid in mid-August. Assume the price fluctuations are sinusoidal in nature.

a) Design a formula using the cosine to describe the price of oranges by months during a year. Let $t = 0$ represent January 1, $t = 1$, February, and so forth. Explain how you determined your answer.

- A full cycle of the cosine function starts with a maximum, followed by an intercept, minimum, intercept, maximum; they are known as the five key points. In our case we have a minimum in February, so since we start with a minimum we need to reflect the cosine with respect to the x axis, so the coefficient of the cosine will be negative.
- The midline is the horizontal line who divides the sinusoidal function in two equal parts. The average price 2.25 tells us the equation of the midline y=2.25. The distance between the midline and the minimum is the amplitude A=2.25-1.60=0.65
- The distance between the midline and the x-axis is the vertical shift that is done with respect to the x-axis(because the x-axis is the midline of the parent function), so we calculate this distance by adding the minimum to the amplitude VS=1.60+0.65=2.25. It

would have been easier to observe that the midline is 2.25 above the x-axis and that number will represent the vertical shift.

- The period is twice the time between the minimum and the maximum. The minimum happened in mid February and the maximum in mid August so the period P=6months, so $b=\frac{2\pi}{6}$
- There is a horizontal shift of the parent function 1 unit to the right since the minimum happens in February and not in January so the argument of the cosine will be $(t-1)$
- So the formula that describes the price of oranges by months is:

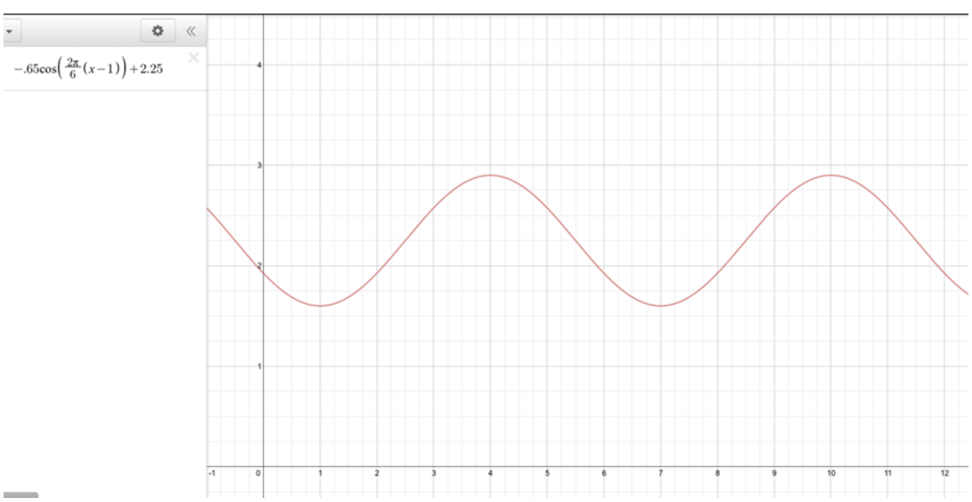
$$C(t)=-0.65\cos\left(\frac{2\pi}{6}(t-1)\right)+2.25$$

b) How would your formula change if you used the sine function instead of cosine? Explain how you arrived at your answer.

If the sine function is shifted left by $\frac{\pi}{2}$ we get the cosine function. Remember that a horizontal shift to the left changes the argument to $\left(\theta+\frac{\pi}{2}\right)$ so the equation is: $\cos\theta=\sin\left(\theta+\frac{\pi}{2}\right)$. Let's apply this to our function:

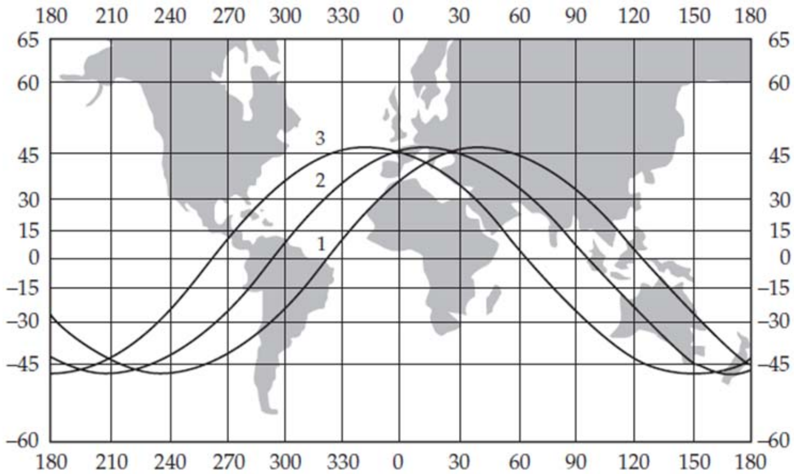
$$C(t)=-0.65\cos\left(\frac{2\pi}{6}(t-1)\right)+2.25=-0.65\sin\left(\frac{2\pi}{6}(t-1)+\frac{\pi}{2}\right)+2.25$$

c) Sketch the graph of your model from part (a). Then read from your graph the times of the year when the price of the oranges will be below $2.45.



From the graph we see that the price of the oranges will be below $2.45 in: January, February, the first three weeks in March, the last three weeks in June, July, August, the first three weeks in September and the last three weeks in December.

109. A satellite is circling west-to-east around the earth. Below are the projections of three orbits of the satellite, labeled as curves 1, 2, and 3. The projections are given in terms of longitude and latitude readings (both are in degrees). Curve 1, can be expresses as a sinusoidal function of the form $y = A\sin(B(x-C))+D$.



a) Determine values for *A, B, C,* and *D* to produce a sinusoidal model that you think best describes Curve 1.

$A = 47; B = 0.5; C = -40; D = 0$ so the equation describing curve 1 will be:

$y = 47\sin\left(0.5\left(x-\left(-40\right)\right)\right)+0$

$y = 47\sin\left(0.5\left(x+40\right)\right)$

$y = 47\sin\left(0.5\left(x+68\right)\right)$

$y = 47\sin\left(0.5\left(x+100\right)\right)$

b) What constants in your sinusoidal model for (a) will you need to modify in order to describe Curves 2 and 3? What are your models describing these curves?

$y = 47\sin\left(0.5\left(x+68\right)\right)$

$y = 47\sin\left(0.5\left(x+100\right)\right)$

110. Suppose $\cos\theta + \sin\theta = 1$.

Part A: Solve the equation for θ in the interval $[0, 2\pi)$.

$\cos\theta + \sin\theta = 1$

$(\cos\theta + \sin\theta)^2 = 1^2$

$\cos^2\theta + 2\cos\theta\sin\theta + \sin^2\theta = 1$

$1 + 2\cos\theta\sin\theta = 1$

$2\cos\theta\sin\theta = 0$

$\cos\theta\sin\theta = 0$

$\cos\theta = 0$ or $\sin\theta=0$

$\theta=0, \frac{\pi}{2}.\pi, \frac{3\pi}{2}$

Since both $\sin\theta$ and $\cos\theta$ both are positive, θ must be in the first quadrant. Therefore, only 0 and $\frac{\pi}{2}$ are the solutions. The others are extraneous solutions.

Part B: Why might you have to check solutions in **Part A** for extraneous roots?

When squaring both sides, the equation can introduce extraneous roots.

Part C: Suppose θ is between 5.25π and 6π and $\cos\theta + \sin\theta = 1$. Find $\cos\theta$ and explain your reasoning.

Sample answers: $5.25\pi - 4\pi = 1.25\pi$ and $6\pi - 4\pi = 2\pi$, so θ is between 1.25π and 2π. 2π is the only value for which $\cos\theta + \sin\theta$ can be 1. So, $\cos\theta = 1$.