Lesson 12: Describing the Center of a Distribution Using the Median

Student Outcomes

- Given a data set, students calculate the median of the data.
- Students estimate the percent of values above and below the median value.

Lesson Overview

The focus of this lesson is the median as a summary statistic to describe a data set. Students report the number of observations for both odd and even numbered sets of data. Informally, they consider the variability among three different data sets to assess a claim about typical behavior. In preparation for a later lesson on finding quartiles, students calculate the median of the values below the median and the median of the values above the median and estimate the approximate count/percent of values above and below the median. This lesson provides the background for the development of a box plot; however, this lesson is not about creating a box plot.

In this lesson students construct arguments and critique the reasoning of others. They respond to the reasoning of others in some of the tasks, distinguish correct reasoning from flawed reasoning, and explain why it is flawed. They also model with mathematics, apply mathematics to problems from everyday life, and interpret results in the context of the situation.

It should be noted that students should have access to calculators throughout this module.

Classwork

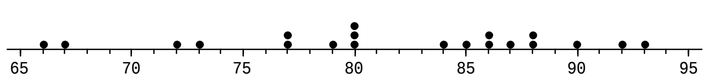
How do we summarize a data distribution? What provides us with a good description of the data? The following exercises help us to understand how a numerical summary answers these questions.

Example 1 (2 minutes): The Median—A Typical Number

The activity begins with a set of data displayed in a dot plot. Introduce the data presented in the example.

Example 1: The Median – A Typical Number

Suppose a chain restaurant (Restaurant A) advertises that a typical number of french fries in a large bag is 82. The graph shows the number of french fries in selected samples of large bags from Restaurant A.



Using the data shown in the plot, students are asked to think about when it might be useful to separate a set of data into two parts that have the same number of elements. In other words, when would it be useful to know the point that separates the top half from the bottom half? The notion of median is developed by a set of questions and then defined.

Let students work independently on the exercises and confirm answers with a neighbor.

Exercises 1–3 (5 minutes)

Exercises 1–3

1. **You just bought a large bag of fries from the restaurant. Do you think you have 82 french fries? Why or why not?**

   The number seems to vary greatly from bag to bag. No bag even had 82 fries, so mine probably will not. The restaurant sells french fries in bags that have from 66 to 93 per bag, so the claim that they typically have 82 fries doesn't seem right.

2. **How many bags were in the sample?**

   20

3. **Which of the following statements would seem to be true given the data? Explain your reasoning.**
   a. **Half of the bags had more than 82 fries in them.**
   b. **Half of the bags had fewer than 82 fries in them.**
   c. **More than half of the bags had more than 82 fries in them.**
   d. **More than half of the bags had fewer than 82 fries in them.**
   e. **If you got a random bag of fries, you could get as many as 93 fries.**

   (a) and (b) are true because there are 10 bags above 82 fries and 10 bags below 82 fries. (e) is true because that happened once and so probably could happen again.

Example 2 (3 minutes): The Median

Read through the text with students.

Example 2: The Median

Sometimes it is useful to know what point separates a data distribution into two equal parts, where one part represents the larger "half" of the data values and the other part represents the smaller "half" of the data values. This point is called the median. When the data are arranged in order from smallest to largest, the same number of values will be above the median as are below the median.

As a class, work through the exercises one at a time. As students work through the problems, ask the following questions:

- How do you find the median if there are an even number of data points?
- About what fraction of the data values should be above the median? What fraction should be below the median?

Exercises 4–5 (5 Minutes)

Exercises 4–5

4. **Suppose you were trying to convince your family that you needed a new pair of tennis shoes. After checking with your friends, you argued that half of them had more than four pairs of tennis shoes, and you only had two pairs. Give another example of when you might want to know that a data value is a half-way point? Explain your thinking.**

   Possible responses: When the data are about how much people earn, it would be interesting to know the amount that is less than what half of the people earn; if you are looking at the number of points earned in a competition, it would be good to know what number separates the top half of the competitors from the bottom.

5. **Use the information from the dot plot in Example 1. The median number of fries was 82.**

   a. **What percent of the bags have more fries than the median? Less than the median?**

      50 percent or $\frac{1}{2}$ of the bags have more fries than the median, and 50% or $\frac{1}{2}$ have fewer fries than the median.

   b. **Suppose the bag with 93 fries was miscounted and there were only 85 fries. Would the median change? Why or why not?**

      The median would not change because there would still be 10 bags with fewer than 82 fries and 10 bags with more than 82 fries.

   c. **Suppose the bag with 93 fries really only had 80 fries. Would the median change? Why or why not?**

      The median would change because now there would be 11 bags that would have fewer than 82 fries and only 9 that have more than 82 instead of the same number in both directions.

Exercises 6–7 (15 minutes): A Skewed Distribution

MP.3

In this activity, students have to order the data before they find the median. There are 19 values, so the median is the 10^{th} value with 9 counts above and 9 counts below. Another way to determine the median after ordering the data is to cross out the maximum and minimum values continuously until students reach one number in the middle if there are an odd number of data values, or two numbers for an even number of values. Students would then find the mean of the two values. The questions are designed to help students confront some common misconceptions: not ordering the data before counting to the middle; confusing median and mode (most frequent value); confusing median and midrange (half way between the maximum and the minimum). They also compute the mean and compare the median to the mean, noting that several bags with a low number of french fries pulled the mean down and so the median might be more reflective of the typical number of fries.

Consider the following questions as students are completing this exercise:

- Why is it necessary to order the data before you find the median?
- Is the median connected to the range (maximum-minimum) of the data? Why or why not?
- What is the difference in the effect of very extreme values on the mean and on the median?

Exercises 6–7: A Skewed Distribution

6. **The owner of the chain decided to check the number of french fries at another restaurant in the chain. Here is the data for Restaurant B: $82, 83, 83, 79, 85, 82, 78, 76, 76, 75, 78, 74, 70, 60, 82, 82, 83, 83, 83$.**

   a. **How many bags of fries were counted?**

      19

   b. **Sallee claims the median is 75 as she sees that 75 is the middle number in the data set listed above. She thinks half of the bags had fewer than 75 fries. Do you think she would change her mind if the data were plotted in a dot plot? Why or why not?**

      You cannot find the median unless the data are ordered by size. Plotting the number of fries in each bag on a number line in a dot plot would order the data so you would probably get a different halfway point because the data above is not ordered from smallest to largest.

   c. **Jake said the median was 83. What would you say to Jake?**

      83 is the most common number of fries in the bags (5 bags had 83 fries), but it is not in the "middle" of the data, marking where the number of bags with fries more than and less than are the same.

   d. **Betse argued that the median was halfway between 60 and 85 or 72.5. Do you think she is right? Why or why not?**

      She is wrong because the median is not connected to the distance between points on the number line but are connected to finding a point that separates the data into two parts with the same number of values in each part.

   e. **Chris thought the median was 82. Do you agree? Why or why not?**

      Chris is correct because if you order the numbers, the middle number will be the 10^{th} number, with 9 bags that have more than 82 fries and 9 bags with fewer than 82 fries.

7. **Calculate the mean and compare it to the median. What do you observe about the two values? If the mean and median are both measures of center, why do you think one of them is lower than the other?**

   The mean is 78.6 and the median is 82. The bag with the 60 fries lowered the value of the mean.

Exercises 8–10 (15 minutes): Finding Medians from Frequency Tables

MP.4 In this example, students find the median using a frequency table halfway between the 13^{th} and 14^{th} counts. They also find the medians of the top and bottom halves, the 7^{th} value from the top and from the bottom, as a precursor to finding an interquartile range in a later lesson. They will encounter repeated values in finding the quartiles. You may want students to write out the individual counts in a long ordered list. For example, the first 13 counts would be as follows:

Median of the lower half

75 75 76 77 77 78 **78** 78 79 79 79 79 79 …

Then have students find the medians of each half by counting from the top and bottom of the list, noting that a value for bags with the same count can be in both halves. It might help to think about the individual bags – one of the bags with 78 fries is in the first half, one of the bags with 78 fries is in the second half, and one of the bags divides the two halves and marks the median of the data set. At this point, the important idea is that students get a sense of how to find a median: order the values and find a midpoint for the ordered values.

Exercises 8–10: Finding Medians from Frequency Tables

8. A third restaurant (Restaurant C) tallied a sample of bags of french fries and found the results below.

Number of fries	Frequency
75	||
76	|
77	||
78	|||
79	|||||
80	||||
81	|
82	|
83	
84	|||
85	|||
86	|

a. How many bags of fries did they count?

26

b. What is the median number of fries for the sample of bags from this restaurant? Describe how you found your answer.

79.5; I took half of 26, which was 13 and then counted 13 tallies from 86 down to get to 80. I also counted 13 up from 75 to get to 79. The point halfway between 79 and 80 is the median.

9. Robere decided to divide the data into four parts. He found the median of the whole set.

a. List the 13 values of the bottom half. Find the median of these 13 values.

$75\ 75\ 76\ 77\ 77\ 78\ 78\ 78\ 79\ 79\ 79\ 79\ 79$

Median of this half is 78.

b. List the 13 values of the top half. Find the median of these 13 values.

$80\ 80\ 80\ 80\ 81\ 82\ 84\ 84\ 84\ 85\ 85\ 85\ 86$

Median of this is 84.

10. Which of the three restaurants seems most likely to really have 82 fries in a typical bag? Explain your thinking.

Answers will vary: Restaurant B seems to have most bags closest to 82 because the middle half of the number of fries in bags covers a span of 7 fries (from 76 to 83) with a median of 82; Restaurant A goes from 87.5 to 75 fries for a span of 12.5 with a median of 80, and Restaurant C goes from 78 to 84 for a span of 6 but the median is 79. Some students might say Restaurant B because the median is 82.

Closing (3 minutes)

Lesson Summary

In this lesson, you learned about a summary measure for a set of data called the *median*. To find a median you first have to order the data. The median is the midpoint of a set of ordered data; it separates the data into two parts with the same number of values below as above that point. For an even number of data values, you find the average of the two middle numbers; for an odd number of data values, you use the middle value. It is important to note that the median might not be a data value and that the median has nothing to do with a measure of distance. Medians are sometimes called a measure of the center of a frequency distribution but do not have to be the middle of the spread or range (maximum-minimum) of the data.

Exit Ticket (5 minutes)

Name ____ Date____________________

Lesson 12: Describing the Center of a Distribution Using the Median

Exit Ticket

1. What is the median age for the following data set representing the age of students requesting tickets for a summer band concert?

   $13 \quad 14 \quad 15 \quad 15 \quad 16 \quad 16 \quad 17 \quad 18 \quad 18$

2. What is the median number of diseased trees from a data set of diseased trees on 10 city blocks?

   $11 \quad 3 \quad 3 \quad 4 \quad 6 \quad 12 \quad 9 \quad 3 \quad 8 \quad 8 \quad 8 \quad 1$

3. Describe how you would find the median for a set of data that has 35 values. How would this be different if there were 36 values?

Exit Ticket Sample Solutions

1. **What is the median age for the following data set representing the age of students requesting tickets for a summer band concert?**

   $13 \quad 14 \quad 15 \quad 15 \quad 16 \quad 16 \quad 17 \quad 18 \quad 18$

   The median is the 5^{th} value, or 16 years old, as there are 4 values less than 16 and 4 values greater than or equal to 16.

2. **What is the median number of diseased trees from a data set of diseased trees on 10 city blocks?**

   $11 \ 3 \ 3 \ 4 \ 6 \ 12 \ 9 \ 3 \ 8 \ 8 \ 8 \ 1$

   To find the median, the values first need to be ordered: $1 \ 3 \ 3 \ 3 \ 4 \ 6 \ 8 \ 8 \ 8 \ 9 \ 11 \ 12$

   As there is an even number of data values, the median would be the mean of the 6^{th} and 7^{th} values: $\frac{6+8}{2}$, or 7 diseased trees.

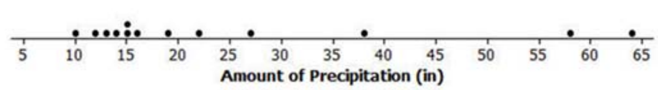
3. **Describe how you would find the median for a set of data that has 35 values. How would this be different if there were 36 values?**

   Answers will vary; first you would order the data from smallest to largest. Because there are 35 values, you would look for the 18^{th} value from the top or bottom. This would be the median with 17 values above and 17 values below. If the set had 36 values, you would go halfway between the 18^{th} and 19^{th} values.

Problem Set Sample Solutions

1. **The amount of precipitation in the western states in the U.S. is given in the table as well as the graph.**

State	Amount of Precipitation (in)
WA	38.4
OR	27.4
CA	22.2
MT	15.3
ID	18.9
WY	12.9
NV	9.5
UT	12.2
CO	15.9
AZ	13.6
NM	14.6
AK	58.3
HI	63.7



Data Source: http://www.currentresults.com/Weather/US/average-annual-state-precipitation.php

   a. **How do the amounts vary across the states?**

   Answers will vary: The spread is pretty large: 54.2 inches. Nevada has the lowest at 9.5 inches per year. Hawaii, Alaska, and Washington have more rain than most of the states; Hawaii has the most with 63.7 inches followed by Alaska at 58.3 inches.

b. **Find the median. What does the median tell you about the amount of precipitation?**

The median is 15.9 inches; half of the states have more than 15.9 inches of precipitation per year and half have less.

c. **Use the median and the range to describe the average monthly precipitation in western states in the U.S.**

The amount of precipitation varies from 63.7 to 9.5 inches per year. Half of the states have from 9.5 to 15.9 inches per year, but only two have more than 40 inches.

d. **Do you think the mean or median would be a better description of the typical amount of precipitation? Explain your thinking.**

The mean at 24.8 inches reflects the extreme values, while the median seems more typical at 15.9 inches.

2. **Identify the following as true or false. If a statement is false, give an example showing why.**

a. **The median is always equal to one of the values in the data set.**

False. If the numbers are 1 and 5, the median is 3 and it is not in the set.

b. **The median is the midpoint between the smallest and largest values in the data set.**

False. Look at the number of french fries per bag for Restaurant A above where the median is 82, which is not halfway between 66 and 93 (79.5).

c. **At most, half of the values in a data set have values less than the median.**

True.

d. **In a data set with 25 different values, if you change the two smallest values of a data set to smaller values, the median will not be changed.**

True.

e. **If you add 10 to every element of a data set, the median will not change.**

False. The median will increase by 10 as well. If the set is $1, 2, 3, 4, 5$, the median is 3; for the set $11, 12, 13, 14, 15$, the median will be 13.

3. **Make up a data set such that the following is true:**

a. **The set has 11 different values and the median is 5.**

Answers will vary depending on the whether the numbers are whole numbers or fractions. If the numbers are whole numbers, the set would be $0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10$.

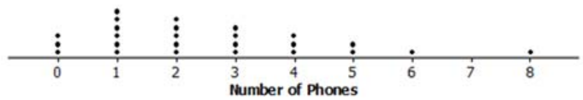
b. **The set has 10 values and the median is 25.**

Answers will vary. One answer is to have all 25's.

c. **The set has 7 values and the median is the same as the smallest value.**

Answers will vary. One answer is to have $1, 1, 1, 1, 2, 3, 4$.

4. The dot plot shows the number of landline phones that a sample of people have in their homes.



a. How many people were in the sample?

25

b. Why do you think three people have no landline phones in their homes?

Possible answers: Some people might only have cell phones, or some people may not be able to afford a phone.

c. Find the median number of phones for the people in the sample.

The median number of phones per home is 2*.*

d. Use the median and the range (maximum-minimum) to describe the distribution of the number of phones.

Possible answer: The median number of phones was 2 *per home, and over half of the people have fewer than* 3 *phones in their homes. Three had none, and one house had* 8 *phones.*

5. The salaries of the Los Angeles Lakers for the 2012–2013 basketball season are given below.

Player	Salary ($)
Kobe Bryant	$\$27,849,149$
Dwight Howard	$\$19,536,360$
Pau Gasol	$\$19,000,000$
Steve Nash	$\$8,700,000$
Metta World Peace	$\$7,258,960$
Steve Blake	$\$4,000,000$
Jordan Hill	$\$3,563,600$
Chris Duhon	$\$3,500,000$
Jodie Meeks	$\$1,500,000$
Earl Clark	$\$1,240,000$
Devin Ebanks	$\$1,054,389$
Darius Morris	$\$962,195$
Antawn Jamison	$\$854,389$
Robert Sacre	$\$473,604$
Darius Johnson-Odom	$\$203,371$

Data Source: www.basketball-reference.com/contracts/LAL.html

a. Just looking at the data, what do you notice about the salaries?

Possible answer: A few of the salaries for the big stars like Kobe are really big, while others are very small in comparison.

b. Find the median salary, and explain what it tells you about the salaries.

$\$3,500,000$ *for Chris Duhon. Half of the players make more than* $\$3,500,000$ *and half make less than that.*

c. Find the median of the lower half of the salaries and the median of the upper half of the salaries.

$\$962,195$ *for the bottom half of the salaries;* $\$8,700,000$ *for the top half of the salaries.*

d. Find the width of each of the following intervals. What do you notice about the size of the interval widths, and what does that tell you about the salaries?

i. minimum salary to median of the lower half: $\$758,824$

ii. median of the lower half to the median of the whole set: $\$2,537,805$

iii. median of the whole set to the median of the upper half: $\$5,200,000$

iv. median of the upper half to the highest salary: $\$19,149,149$

The largest width is from the median of the upper half to the highest salary. The smaller salaries are closer together than the larger ones.

6. Use the salary table from above to answer the following.

a. If you were to find the mean salary, how do you think it would compare to the median? Explain your reasoning.

Possible answer: The mean will be a lot larger than the median because when you add in the really big salaries, the size of the mean will increase a lot.

b. Which measure do you think would give a better picture of a typical salary for the Lakers, the mean or the median? Explain your thinking.

Possible answer: The median seems better as it is more typical of most of the salaries.