Lesson 22: Getting the Job Done—Speed, Work, and Measurement Units

Student Outcomes

- Students decontextualize a given speed situation, representing symbolically the quantities involved with the formula distance $=$ rate $\cdot$ time.

Materials

- Stopwatches
- 50-foot measured course
- Calculators

Lesson Notes

Vocabulary: distance, rate, time, $d = r \cdot t$, $r = \dfrac{d}{t}$

Classwork

> If an object is moving at a constant rate of speed for a certain amount of time, it is possible to find how far the object went by multiplying the rate and the time. In mathematical language, we say Distance $=$ Rate • Time.

Exploratory Challenge

Students will make measurements of distance and time during this lesson and will calculate speed. When using a stopwatch, the teacher can decide whether to round to the nearest second or tenth of a second. If desired, multiple trials can be measured and results averaged.

Opening Exercise (2 minutes)

- How many seconds are in 1 minute?
    - 60 *seconds*
- Can you verbalize this relationship?
    - *For every* 60 *seconds, there is* 1 *minute*
- Here are two different ways (display for students):

    $\dfrac{60 \text{ seconds}}{\text{minute}}$ and $60\,\dfrac{\text{seconds}}{\text{minute}}$.

- Are these the same values?
  - *Allow for discussion.*
- The first representation states that for every 60 seconds, there is 1 minute. Is that what the second representation states? I read this as "60 seconds per minute." Knowing what we learned previously in Lessons 1 and 2, "per" and "for every" are verbal representations of a ratio, so they mean the same thing.

Example 1 (15 minutes)

Measure out a 50-foot course in the hallway (or shorter in the classroom). Have student volunteers use a stopwatch to measure the time it takes to have others walk the distance. Also, ask a fast runner to run the course as fast as he or she can.

- *I wonder how fast you were moving*. In this exercise, we know the distance (in feet) and time (in seconds), and we must find the speed, which is the rate of distance traveled per unit of time. This will be expressed in feet per second for this activity.
- Many people like to use the $d = r \cdot t$ formula, substitute in the values for rate and time, and then multiply. Would you agree that $r = \frac{d}{t}$?

Remind students that $12 = 3 \cdot 4$, $3 = \frac{12}{4}$, and $4 = \frac{12}{3}$ are all related and can be an anchor in relating $d = r \cdot t$ and $r = \frac{d}{t}$. Substitute the values to test if needed.

MP.2 Ask students to substitute the runner's distance and time into the equation and solve for the rate of speed. Also, substitute the runner's time and distance into the equation to find his or her rate of speed.

Example 1

Walker: Substitute the walker's distance and time into the equation and solve for the rate of speed.

Distance = Rate • Time

$d = r \cdot t$

Hint: Consider the units that you want to end up with. If you want to end up with the rate (feet/second), then divide the distance (feet) by time (seconds).

Runner: Substitute the runner's time and distance into the equation to find the rate of speed.

Distance = Rate • Time

$d = r \cdot t$

Here is a sample of student work using 8 seconds as an example:

$d = r \cdot t$ *and* $r = \frac{d}{t}$; *Distance: 50 feet; Time: 8 seconds*

$r = \frac{50\ feet}{8\ sec} = 6.25\ feet/sec$

MP.5 & MP.6

It might be important to discuss the desired precision of each measurement and the limitations to precision inherent in the tools used (e.g., 50-foot race course measured to the nearest inch and time measured to the nearest hundredth of a second on the stopwatch). Measurements are, by their nature, never exact. Also, when arriving at an answer, it should be expressed with a degree of precision appropriate for the context of the problem.

Example 2 (15 minutes)

Example 2

Part 1: Chris Johnson ran the 40-yard dash in 4.24 seconds. What is the rate of speed? Round any answer to the nearest hundredth.

Distance = Rate • Time

$$d = r \bullet t$$

$$d = r \bullet t \text{ and } r = \frac{d}{t}; \ \ r = \frac{40 \text{ yards}}{4.24 \text{ sec}} \approx 9.43 \text{ yard/sec}$$

MP.1

Part 2: In Lesson 21, we converted units of measure using unit rates. If the runner were able to run at a constant rate, how many yards would he run in an hour? This problem can be solved by breaking it down into two steps. Work with a partner, and make a record of your calculations.

    a.   **How many yards would he run in one minute?**

$$9.43 \frac{yards}{\cancel{second}} \bullet 60 \frac{\cancel{seconds}}{minute} = 565.80 \text{ yards in one minute}$$

    b.   **How many yards would he run in one hour?**

$$565.80 \frac{yards}{\cancel{minute}} \bullet 60 \frac{\cancel{minutes}}{hour} = 33,948 \text{ yards in one hour}$$

        We completed that problem in two separate steps, but it is possible to complete this same problem in one step. We can multiply the yards per second by the seconds per minute, then by the minutes per hour.

$$9.43 \frac{yards}{\cancel{second}} \bullet 60 \frac{\cancel{seconds}}{\cancel{minute}} \bullet 60 \frac{\cancel{minutes}}{hour} = 33,948 \text{ yards in one hour}$$

        Cross out any units that are in both the numerator and denominator in the expression because these cancel each other out.

Part 3: How many miles did the runner travel in that hour? Round your response to the nearest tenth.

$$33,948 \frac{\cancel{yards}}{hour} \bullet \frac{1 \text{ mile}}{1,760 \cancel{yards}} \approx 19.3 \text{ miles per hour}$$

Cross out any units that are in both the numerator and denominator in the expression because they cancel out.

Exercises: Road Trip (5 minutes)

Exercise 1

I drove my car on cruise control at 65 miles per hour for 3 hours without stopping. How far did I go?

$d = r \cdot t$

$d = \underline{\quad\quad} \dfrac{\text{miles}}{\text{hour}} \cdot \underline{\quad\quad} \text{ hours}$

$d = 65 \dfrac{\text{miles}}{\cancel{\text{hour}}} \cdot 3 \cancel{\text{ hours}}$

Cross out any units that are in both the numerator and denominator in the expression because they cancel out.

$d = \underline{\quad\quad} \text{ miles}$

$d = 195 \text{ miles}$

Exercise 2

On the road trip, the speed limit changed to 50 miles per hour in a construction zone. Traffic moved along at a constant rate (50 mph), and it took me 15 minutes (0.25 hours) to get through the zone. What was the distance of the construction zone? (Round your response to the nearest hundredth of a mile.)

$d = r \cdot t$

$d = \underline{\quad\quad} \dfrac{\text{miles}}{\text{hour}} \cdot \underline{\quad\quad} \text{ hours}$

$d = 50 \dfrac{\text{miles}}{\cancel{\text{hour}}} \cdot 0.25 \cancel{\text{ hour}}$

$d = 12.50 \text{ miles}$

Closing (3 minutes)

- Describe the relationship between distance, rate, and time. State this relationship as many different ways as you can. How does each of these representations differ? How are they alike?
  - *We can find distance if we know the rate and time using the formula/equation*
    $d = r \cdot t.$
  - *We can find the rate if we know the distance and the time using the formula/equation*
    $r = \dfrac{d}{t}.$

Lesson Summary

Distance, rate, and time are related by the formula $d = r \cdot t$.

Knowing any two of the values allows the calculation of the third.

Exit Ticket (5 minutes)

Name _____   Date_____

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Exit Ticket

Franny took a road trip to her grandmother's house. She drove at a constant speed of 60 miles per hour for 2 hours. She took a break and then finished the rest of her trip driving at a constant speed of 50 miles per hour for 2 hours. What was the total distance of Franny's trip?

Exit Ticket Sample Solutions

Franny took a road trip to her grandmother's house. She drove at a constant speed of 60 miles per hour for 2 hours. She took a break and then finished the rest of her trip driving at a constant speed of 50 miles per hour for 2 hours. What was the total distance of Franny's trip?

$$d \; = \; 60\frac{miles}{hour} \bullet 2 \; hours \; = \; 120 \; miles$$

$$d \; = \; 50\frac{miles}{hour} \bullet 2 \; hours \; = \; 100 \; miles$$

$$120 \; miles \; + \; 100 \; miles \; = \; 220 \; miles$$

Problem Set Sample Solutions

1. If Adam's plane traveled at a constant speed of 375 miles per hour for six hours, how far did the plane travel?

   $$d = r \cdot t$$

   $$d = \frac{375 \; miles}{hour} \times 6 \; hours = 2,250 \; miles$$

2. A Salt March Harvest Mouse ran a 360 centimeter straight course race in 9 seconds. How fast did it run?

   $$r = \frac{d}{t}$$

   $$r = \frac{360 \; centimeters}{9 \; seconds} = 40 \; cm/sec$$

3. Another Salt Marsh Harvest Mouse took 7 seconds to run a 350 centimeter race. How fast did it run?

   $$r = \frac{d}{t}$$

   $$r = \frac{350 \; centimeter}{7 \; seconds} = 50 \; cm/sec$$

4. A slow boat to China travels at a constant speed of 17.25 miles per hour for 200 hours. How far was the voyage?

   $$d = r \cdot t$$

   $$d = \frac{17.25 \; miles}{hour} \times 200 \; hours = 3,450 \; miles$$

5. The Sopwith Camel was a British, First World War, single-seat, biplane fighter introduced on the Western Front in 1917. Traveling at its top speed of 110 mph in one direction for $2\frac{1}{2}$ hours, how far did the plane travel?

   $$d = r \cdot t$$

   $$d = \frac{110 \; miles}{hour} \times 2.5 \; hours = 275 \; miles$$

6.   A world-class marathon runner can finish 26.2 miles in 2 hours. What is the rate of speed for the runner?

$$r = \frac{d}{t}$$

$$r = \frac{26.2 \ miles}{2 \ hours} = 13.1 \ mph \ or \ 13.1\frac{mi}{h}$$

7.   Banana slugs can move at 17 cm per minute. If a banana slug travels for 5 hours, how far will it travel?

$$d = r \cdot t$$

$$d = \frac{17 \ cm}{min} \times 300 \ min = 5,100 \ cm$$