

Claim 1: Concepts and Procedures

Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.

Content Domain: Statistics and Probability

Target J [s]: Investigate patterns of association in bivariate data. (DOK Levels 1, 2)

Tasks for this target will often be paired with 8.F Target F and ask students to determine the rate of change and initial value of a line suggested by examining bivariate data. Interpretations related to clustering, outliers, positive or negative association, linear and nonlinear association will primarily be presented in context by pairing this target with those from Claims 2 and 4.

Standards: 8.SP.A, 8.SP.1, 8.SP.2, 8.SP.3, 8.SP.4

8.SP.A Investigate patterns of association in bivariate data

- **8.SP.1** Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.
- **8.SP.2** Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.
- **8.SP.3** Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.
- **8.SP.4** Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?

Related Below-Grade and Above-Grade Standards for Purposes of Planning for Vertical Scaling:

> 7.SP.A, 7.SP.1, 7.SP.2,

F-IF-B, F-IF.4, F-LE.5, S-ID.B, S-ID.5, S-ID.6

Related Grade 7 standards

7.SP.A Use random sampling to draw inferences about a population

7.SP.1 Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.

7.SP.2 Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions.



Related High School Standards

F-IF.B Interpret functions that arise in application in terms of the context.

F-IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include:* intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.

F-LE.5 Interpret the parameters in a linear or exponential function in terms of a context.

S-ID.B Summarize, represent, and interpret data on two categorical and quantitative variables

S-ID.5 Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.

S-ID.6 Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.

DOK Levels: 1, 2

Achievement Level Descriptors:

RANGE Achievement Level Descriptor (Range ALD)

Target J: Investigate patterns of association in bivariate data.

Level 1 Students should be able to investigate a scatter plot for clustering between two quantities and construct a scatter plot from given data. They should be able to construct a two-way frequency table of given categorical data.

Level 2 Students should be able to investigate a scatter plot for positive, negative, and linear association and informally fit a line to data for a given a scatter plot that suggests a linear association. They should be able to calculate frequencies from categorical data in a two-way frequency table.

Level 3 Students should be able to investigate a scatter plot for patterns such as outliers and nonlinear association. They should be able to write an equation for the trend line or line of best fit for a given scatter plot with a linear association. They should also be able to interpret and use relative frequencies from a two way table to describe possible association between two variables.

Level 4 Students should be able to use scatter plots, trend lines, and associations between variables in two-way frequency tables to make predictions in real-world situations.

Evidence Required:

- 1. The student interprets patterns of association between two quantities in a scatter plot (clustering in reference to the line of best fit, positive or negative association, linear association, nonlinear association, and the effect of outliers) and interprets the slope and *y*-intercept in terms of the context.
- 2. The student identifies the slope (rate of change) and intercept (initial value) of a line suggested by examining bivariate measurement data in a scatter plot.
- 3. The student constructs and interprets a two-way table summarizing data on two categorical variables collected from the same subjects.



Allowable Response	Matching Table; Equation/Numeric; Fill-in Table
Types:	rideciming rable, Equation, warnerie, i in in rable
Allowable Stimulus Materials:	Scatter plot, two-way relative frequency table, raw data
Construct-Relevant Vocabulary:	cluster, data, frequency, initial value, line of best fit, trend line, linear extrapolation, linear association, negative association, outlier, positive association, rate of change, relative frequency, scale, scatter plot, slope, two-way relative frequency table, variable, x-axis, y-axis, x-intercept, y-intercept
Allowable Tools:	Calculator
Target-Specific Attributes	
Non-Targeted Constructs:	
Accessibility Concerns:	Visual graphics and 3-D images may be difficult or not accessible for students who are blind or visually impaired. Reviewing tactile graphics may be time-consuming but not prohibitive. The simplest graphics with labels should be used to minimize this issue. Students with dyscalculia may have difficulty with the calculations. Students with visual perceptual disabilities may struggle with answer choices that contain complex number sentences. Students who are visually impaired or blind may need enlarged or brailled text. Students with reading disabilities may struggle with the reading load of word problems. All vocabulary should be at or below grade level to minimize this issue. Students with reading disabilities may need the text read aloud, or have access to trackers to follow along. Student with visual processing impairments may benefit from using a tracker or masker when reading. Consider replacing these response types with multiple choice items for Braille versions. The accommodations listed here are suggestions and could be altered depending on what accommodations will be allowable.
Development Notes:	Assessing the fit of a model $(8.SP.2)$ will be assessed in Claim 4. Interpreting the slope and y -intercept in context $(8.SP.3)$ will be assessed in Claims 2 and 4.



Response Type: Matching Table

DOK Level 1

8.SP.1

Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.

8.SP.2

Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.

Evidence Required:

1. The student interprets patterns of association between two quantities in a scatter plot (clustering in reference to the line of best fit, positive or negative association, linear association, nonlinear

Prompt Features: The student is prompted to determine whether statements about the data in a scatter plot are true.

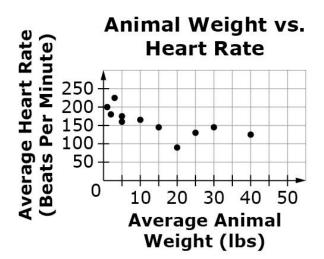
Stimulus Guidelines:

- Context should be familiar to students 13–15 years old.
- Scatter plot will have an informative title relevant to the situation.
- Axes will have informative titles relevant to the situation and appropriate interval scales.
- The data set may include clustering.
- Item difficulty can be adjusted via these example methods:
 - The association may be positive, negative, linear, or nonlinear.
 - There may be clustering, gaps, and outliers in the data.

TM1a

Stimulus: The student is presented with a situation that involves a relationship between two quantities or continuous variables and a scatter plot of bivariate measurement data with sufficient points to demonstrate a linear or nonlinear relationship.

Example Stem: This scatter plot shows the relationship between the average weight and average heart rate for 11 different animals.



Select True or False for each statement based on the scatter plot.

Statement	True	False
There is a positive association between		
average weight and average heart rate for		
animals.		
Animals with higher body weights tend to		
have lower heart rates than animals with		
lower body weights.		
There is a linear association between		
average weight and average heart rate for		
animals.		

Rubric: (1 point) Student determines each statement as being either true or false (e.g., F, T, T) Each statement that interprets the



association, and the effect of outliers) and interprets the slope and *y*-intercept in terms of the context.

scatter plot and may involve clustering in reference to the line of best fit, positive or negative associations, linear associations, nonlinear associations, or the effect of outliers.

Response Type: Matching Table

Tools: Calculator



Response Type: Matching Table

DOK Level 1

8.SP.1

Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.

8.SP.2

Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.

8.SP.3

Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept.

Evidence Required:

1. The student interprets patterns of

Prompt Features: The student is prompted to determine whether statements about the data in a scatter plot are true.

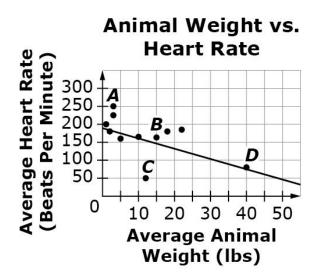
Stimulus Guidelines:

- Context should be familiar to students 13–15 years old.
- Scatter plot will have an informative title relevant to the situation.
- Axes will have informative titles relevant to the situation and appropriate interval scales.
- The data set may include clustering.
- Item difficulty can be adjusted via these example methods:
 - The association may be positive, negative, linear, or nonlinear.
 - The data set may reflect an explicit or implicit linear or explicit or implicit nonlinear relationship.
 - o There may be clustering, gaps, and outliers in the data.

TM1b

Stimulus: The student is presented with a situation that involves a relationship between two quantities or continuous variables and a scatter plot of bivariate measurement data with sufficient points to demonstrate a linear or nonlinear relationship.

Example Stem: This scatter plot shows the relationship between the average weight and average heart rate for 11 different animals. The line of best fit is shown on the graph.



Select True or False for each statement based on the graph.

Statement	True	False
The line of best fit provides a good estimate of any animal's average heart rate based on its weight.		
The y -intercept is at approximately (0, 185).		
Point <i>D</i> is one outlier because it is far away from the other data points.		

Rubric: (1 point) Student determines each statement as being



association between two quantities in a scatter plot (clustering in reference to the line of best fit, positive or negative association, linear association, nonlinear association, and the effect of outliers) and interprets the slope and y-intercept in terms of the context.

either true or false (e.g., F, T, F) Each statement interprets the scatter plot and may involve clustering in reference to the line of best fit, positive or negative associations, linear associations, nonlinear associations, the effect of outliers, the identification or the interpretation of the slope or *y*-intercept in terms of the context.

Response Type: Matching Table

Tools: Calculator



Response Type: Equation/Numeric

DOK Level 2

8.SP.3

Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.

Evidence Required:

2. The student identifies the slope (rate of change) and intercept (initial value) of a line suggested by examining bivariate measurement data in a scatter plot.

Tools: Calculator

Prompt Features: The student is prompted to find the slope and *y*-intercept of the line of best fit on a scatter plot.

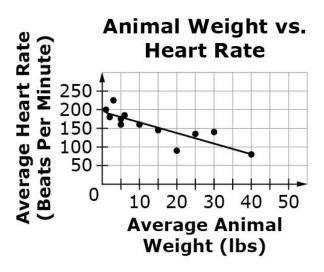
Stimulus Guidelines:

- Context should be familiar to students 13–15 years old.
- Scatter plot will have an informative title relevant to the situation.
- Axes will have informative titles relevant to the situation and appropriate interval scales.
- The data set may include clustering.
- Item difficulty can be adjusted via these example methods:
- The association may be positive, negative, linear, or nonlinear.
- The data set may reflect an explicit or implicit linear relationship or explicit or implicit nonlinear relationship.
- There may be clustering, gaps, and outliers in the data.

TM2

Stimulus: The student is presented with a situation that involves a relationship between two intervals or continuous variables and a scatter plot of bivariate measurement data with sufficient points to demonstrate a linear relationship. The graph will provide the line of best fit.

Example Stem: This scatter plot shows the relationship between animal weight and animal heart rate.



The y-intercept of the estimated line of best fit is at (0, b). Enter the approximate value of b in the first response box.

Enter the approximate slope of the estimated line of best fit in the second response box.

Rubric: (2 points) Student enters the correct value for the *y*-intercept and the slope (e.g., 185 to 195; -2 to -3). (1 point) Student enters a value within the range for either the *y*-intercept or the slope.

Response Type: Equation/Numeric (2 response boxes)



Response Type: Fill-in Table

DOK Level 1

8.SP.4

Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables.

Evidence Required:

3. The student constructs and interprets a two-way table summarizing data on two categorical variables collected from the same subjects.

Tools: Calculator

Prompt Features: The student is prompted to complete a two-way table to summarize the data on two categorical variables for the same subjects.

Stimulus Guidelines:

- Context should be familiar to students 13–15 years old.
- Item difficulty can be adjusted via these example methods:
 - Student must determine sums of rows and columns.
 - Student must determine sums of rows and/or columns; and/or determines one non-sum cell.
 - Student must determine sums of rows and/or columns and/or determines two to three non-sum cells.

TM3a

Stimulus: The student is presented with a situation that involves a relationship between two categorical variables.

Example Stem: All 8th-grade students at a school answered Yes or No to the two survey questions shown.

Do you have a cell phone?Do you have an MP3 player?Yes No

The same students responded to both questions. Complete the two-way frequency table to show the correct totals for the given data. You must complete **all** five cells of the table for a full credit response.

	MP3 Player	No MP3 Player	Total
Cell Phone	57	122	
No Cell Phone	30	65	
Total			

Rubric: (1 point) Student correctly fills in the table for all five cells with the correct totals (see below).

	MP3 Player	No MP3 Player	Total
Cell Phone	57	122	179
No Cell Phone	30	65	95
Total	87	187	274

Response Type: Fill-in Table



Task Model 3

Response Type: Fill-in Table

DOK Level 1

8.SP.4

Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables.

Evidence Required:

3. The student constructs and interprets a two-way table summarizing data on two categorical variables collected from the same subjects.

Tools: Calculator

TM3b

Stimulus: The student is presented with a situation that involves a relationship between two categorical variables.

Example Stem: All 8th-grade students at a school answered Yes or No to the two survey questions shown.

Do you have a cell phone? Yes NoDo you have an MP3 player? Yes No

The same students responded to both questions. The results are shown in the table.

	Cell Phone		MP3	Player
Totals	Yes	No	Yes	No
iotais	179	95	87	187

Of the 187 students with **no** MP3 Player, 65 have **no** cell phone.

Complete the two-way frequency table to represent the correct number of students in each cell. You must complete **all** nine cells of the table for a full credit response.

	MP3 Player	No MP3 Player	Total
Cell Phone			
No Cell Phone			
Total			

Rubric:

(2 points) Student correctly fills in the table for all nine cells with the correct totals (see below).

(1 point) Student correctly fills in the table for 6–8 cells with the correct totals (to allow for a minor calculation error).

	MP3 Player	No MP3 Player	Total
Cell Phone	57	122	179
No Cell Phone	30	65	95
Total	87	187	274

Response Type: Fill-in Table



Task Model 3

Response Type: Fill-in Table

DOK Level 1

8.SP.4

Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables.

Evidence Required:

3. The student constructs and interprets a two-way table summarizing data on two categorical variables collected from the same subjects.

Tools: Calculator

Prompt Features: The student is prompted to complete a two-way frequency table to summarize the data on two categorical variables for the same subjects.

Stimulus Guidelines:

- Context should be familiar to students 13–15 years old.
- Percentage values may go to the tenths or hundredths place. If so, give directions on rounding.

TM3c

Stimulus: The student is presented with a situation that involves a relationship between two categorical variables.

Example Stem: All 8th-grade students at a school answered Yes or No to the two survey questions shown.

Do you have a cell phone? Yes NoDo you have an MP3 player? Yes No

The same students responded to both questions. Complete the two-way relative frequency table to show the correct percentages in all cells. You must complete **all** five cells of the table for a full credit response.

	MP3 Player	No MP3 Player	Total
Cell Phone	20%	57%	
No Cell Phone	12%	11%	
Total			

Rubric: (1 point) Student correctly fills in the table for all five cells with the correct totals (see below). The percentage sign is not required for each cell in order for a student to get full credit. Correct answers may be given as decimal equivalents of the percentages (0.2 for 20%, etc.).

	MP3 Player	No MP3 Player	Total
Cell Phone	20%	57%	77%
No Cell Phone	12%	11%	23%
Total	32%	68%	100%

Response Type: Fill-in Table



Task Model 3

Response Type: Fill-in Table

DOK Level 1

8.SP.4

Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables.

Evidence Required:

3. The student constructs and interprets a two-way table summarizing data on two categorical variables collected from the same subjects.

Tools: Calculator

TM3d

Stimulus: The student is presented with a situation that involves a relationship between two categorical variables.

Example Stem: All 8th-grade students at a school answered Yes or No to the two survey questions shown.

Do you have a cell phone? Yes NoDo you have an MP3 player? Yes No

The same students responded to both questions. The results are shown in the table.

	Cell Phone		MP3	Player
Totals	Yes	No	Yes	No
iotais	77%	23%	68%	32%

Of all the students surveyed, 20% have both a cell phone and a MP3 player.

Complete the two-way relative frequency table to show the correct percentages in all cells. You must complete **all** nine cells of the table for a full credit response.

	MP3 Player	No MP3 Player	Total
Cell Phone			
No Cell Phone			
Total			

Rubric:

(2 points) Student correctly fills in the table for all nine cells with the correct totals (see below).

(1 point) Student correctly fills in the table for 7-8 cells with the correct totals (to allow for a minor calculation error).

The percentage sign is not required for each cell in order for a student to get full credit. Correct answers may be given as decimal equivalents of the percentages (0.2 for 20%, etc.).

	MP3 Player	No MP3 Player	Total
Cell Phone	20%	57%	77%
No Cell Phone	12%	11%	23%
Total	32%	68%	100%

Response Type: Fill-in Table

Task Model 3

Response Type: Fill-in Table

DOK Level 1

8.SP.4

Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables.

Evidence Required:

3. The student constructs and interprets a two-way table summarizing data on two categorical variables collected from the same subjects.

Tools: Calculator

Prompt Features: The student is prompted to complete a two-way frequency table to summarize the data on two categorical variables for the same subjects.

Stimulus Guidelines:

- Context should be familiar to students 13–15 years old.
- Percentage values may go to the tenths or hundredths place.
 If so, give directions on rounding.
- Item difficulty can be adjusted via these example methods, but are not limited to these methods:
 - Totals are not given to the student.

TM3e

Stimulus: The student is presented with a situation that involves a relationship between two categorical variables.

Example Stem: All 8th-grade students at a school answered Yes or No to the two survey questions shown.

Do you have a cell phone? Yes NoDo you have an MP3 player? Yes No

The same students responded to both questions. The results are shown in the table.

	MP3 Player	No MP3 Player	Total
Cell Phone	40	114	154
No Cell Phone	24	22	46
Total	64	136	200

Complete two-way table to represent the correct percentage of students in each cell. You must complete **all** nine cells of the table for a full credit response.

	MP3 Player	No MP3 Player	Total
Cell Phone			
No Cell Phone			
Total			

Rubric: (2 points) Student correctly fills in the table for all nine cells with the correct totals (see below).

(1 point) Student correctly fills in the table for 6-8 cells with the correct totals (to allow for a minor calculation error).

The percentage sign is not required for each cell in order for a student to get full credit. Correct answers may be given as decimal equivalents of the percentages (0.2 for 20%, etc.).

	MP3 Player	No MP3 Player	Total
Cell Phone	20%	57%	77%
No Cell Phone	12%	11%	23%
Total	32%	68%	100%

Response Type: Fill-in Table