Exponential and Log Functions – Part One

Math Background

Previously, you

- Simplified linear, quadratic, radical, polynomial and rational functions
- Performed arithmetic operations with linear, quadratic, radical, polynomial and rational functions
- Identified the domain, range and x-intercepts of real-life functions
- Graphed linear, quadratic and polynomial functions
- Transformed parent functions of linear, quadratic, radical, polynomial and rational functions

In this unit you will

- Apply and graph arithmetic and geometric sequences
- Graph exponential functions with and without technology
- Find the inverse of an exponential function
- Graph logarithmic functions
- Transform exponential and logarithmic functions

You can use the skills in this unit to

- Derive the sum of an arithmetic and geometric series
- Use the structure of an expression to identify ways to rewrite it.
- Interpret the domain and its restrictions of a real-life function.
- Describe how an exponential and logarithmic function graph is related to its parent function.
- Model and solve real-world problems with exponential and logarithmic functions using graphs

Vocabulary

- **Arithmetic sequence –** A sequence in which the difference between successive terms is constant.
- **Arithmetic series –** The indicated sum of all terms in an arithmetic sequence.
- **Base –** The number b in the logarithm expression $\log_b C$
- **Common logarithm –** Logarithm to the base 10
- **Decay factor –** It is the percentage by which the original amount will decline.
- **Explicit Formula** – An equation to represent the n^{th} term of the sequence.
- **Exponential Function –** A function whose exponent involves at least one variable.
- **Geometric sequence** – A sequence in which each term (after the first one) bears a fixed ratio to its previous term.
- **Growth factor** – It is the percentage by which the original amount will increase.
- **Horizontal Asymptote** – A horizontal line that the graph of a function approaches as x tends to plus or minus infinity. It describes the function's end behavior.
- **Logarithm –** The exponent when expressing a number as the exponent of another number, usually to the base 10.
- **Logarithmic function –** A function in the form of $\log_b x$ in which b is a constant and x is a positive variable.

- **Natural logarithm –** The logarithm of a given number to the base *e*, where *e* is 2.71828….
- **Recursive Formula –** A formula which uses the preceding term to define the next term of the sequence.
- **Vertical asymptote –** A vertical line that the curve approaches more and more closely but never touches as the curve goes off to positive or negative infinity. The vertical lines correspond to the zeroes of the denominator of the rational function.

Essential Questions

- How are arithmetic and geometric sequences expressed algebraically to model a real world situation?
- How do we find the sum of a geometric series?
- How is the rate of growth or decay in an exponential function determined?
- Why is it useful to model real-world problems with equations and graphs?

Overall Big Ideas

Sums of geometric sequences are important because they can model the value of retirement funds, the amount of money owed on a house or credit card and other types of loans.

The rate of growth or decay in an exponential function can be determined through the application of properties of exponents.

Equations and graphs can help to predict a future outcome of a real world problem or provide insight in to the problems past.

Skill

To apply and graph arithmetic sequences, deriving the sum of arithmetic series.

To apply and graph geometic sequences, deriving the sum of geometic series

To graph exponential functions modeling growth and decay.

To find and graph the inverse of an exponential function.

To evaluate and graph logarithmic functions.

To graph and transform exponential and logarithmic functions.

Related Standards

F.BF.A.2

Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.* (Modeling Standard)

A.SSE.B.4

Derive the formulas for the sums of a finite and infinite geometric series (when the common ratio is not 1), and use the formulas to solve problems. For example, calculate mortgage payments. *(Modeling Standard)

F.LE.A.2

Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). *(Modeling Standard)

F.IF.C.8b

Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as $y=(1.02)^t$, $y=(0.97)^t$, $y=(1.01)^{12t}$, $y=(1.2)^{t/10}$, and classify them as representing exponential growth or decay.

A.CED.A.2-2

Create equations in two or more variables to represent relationships between quantities and graph equations on coordinate axes with labels and scales. Use all types of equations. *(Modeling Standard)

A.CED.A.3-2

Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. Use all types of equations. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods. *(Modeling Standard)

F.IF.B.5-2

Relate the domain of any function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble *n* engines in a factory, then the positive integers would be an appropriate domain for the function *(Modeling Standard)

F.IF.C.7e-2

Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. *(Modeling Standard)

F.BF.B.4c

Read values of an inverse function from a graph or a table, given that the function has an inverse.

F.BF.B.5

Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

Notes, Examples, and Exam Questions

Background Information on Sequences and Series before Unit 7.1.

Sequence: a function whose domain is a set of consecutive integers, and whose range is the terms of the sequence

Finite Sequence: a sequence that has n terms, where *n* is a whole number

Infinite Sequence: a sequence that continues without stopping

Writing the Terms of a Sequence

Ex: Let a sequence be defined by $a_n = 3n + 1$. Write the first five terms of the sequence.

1st Term: $a_1 = 3(1) + 1 = 4$

2nd Term: $a_2 = 3(2) + 1 = 7$

3rd Term: $a_3 = 3(3) + 1 = 10$

4th Term: $a_4 = 3(4) + 1 = 13$

5th Term: $a_5 = 3(5) + 1 = 16$

Ex: Let a sequence be defined by $a_n = (-2)^{n+1}$. Write the first five terms of the sequence.

1st Term: $a_1 = (-2)^{1+1} = (-2)^2 = 4$

2nd Term: $a_2 = (-2)^{2+1} = (-2)^3 = -8$

3rd Term: $a_3 = (-2)^{3+1} = (-2)^4 = 16$

4th Term: $a_4 = (-2)^{4+1} = (-2)^5 = -32$

5th Term: $a_5 = (-2)^{5+1} = (-2)^6 = 64$

Writing a Rule for a Sequence

Ex: Write the next term in the sequence. Then write a rule for the *n*th term. $-\frac{2}{5}, \frac{2}{25}, -\frac{2}{125}, \frac{2}{625}, \ldots$

Each term is found by multiplying the previous term by $-\frac{1}{5}$. So the next term is $\frac{2}{625} \cdot \left(-\frac{1}{5}\right) = \boxed{-\frac{2}{3125}}$.

To write a rule, rewrite the sequence as $\frac{2}{(-5)^1}, \frac{2}{(-5)^2}, \frac{2}{(-5)^3}, \frac{2}{(-5)^4}, \ldots$. So the nth term is

$\boxed{a_n = \frac{2}{(-5)^n}}$.

<u>Series</u>: the sum of the terms of a sequence

<u>Summation Notation (Sigma Notation):</u> $\sum_{i=1}^{n} i$, where *i* is the index of summation, 1 is the lower limit (first *i*-value) of summation, *n* is the upper limit (last *i*-value) of summation. This *finite* series is the sum of $1+2+3+4+\ldots+n$.

⚠Caution: Do not confuse the index of summation, *i*, with the imaginary number *i*.

<u>Infinite Series</u>: a series that continues without end. The upper limit of summation is ∞.

Ex: Find the sum. $\sum_{i=1}^{5} 2i$

$$\sum_{i=1}^{5} 2i = 2(1)+2(2)+2(3)+2(4)+2(5) = \boxed{30}$$

Ex: Write the series in summation notation. $\frac{2}{3}+\frac{3}{4}+\frac{4}{5}+\frac{5}{6}\ldots$

The denominator is one more than the numerator. So each term can be written as $a_i = \frac{i}{i+1}$, where $i = 2, 3, 4, \ldots$.

This is an infinite series, so the upper limit of summation is ∞. The summation notation for the series is $\boxed{\sum_{i=2}^{\infty} \frac{i}{i+1}}$.

<u>Explore:</u> A famous mathematician, Gauss, got in trouble in grade school. His teacher told him to go to the back of the room and add the whole numbers 1-100. She thought that would keep him quiet for a while, but he came back quickly with an answer. Here is what he did…

$$\begin{array}{l} 1+\ \ 2+\ \ 3+\ \ 4+\ldots+98+\ \ 99+100 \\ \underline{100+\ \ 99+\ \ 98+\ \ 97+\ldots+\ \ 3+\ \ 2+\ \ \ 1} \\ 101+101+101+101+\ldots+101+101+101 \end{array}$$

This gave him $n = 100$ "101"'s.

But that was twice the original sum since he listed the terms twice. So he calculated the sum as $\frac{100(101)}{2} = \boxed{5050}$ (See the second formula on the next page.)

Formulas of Special Series

$$\sum_{i=1}^{n} 1 = n \qquad \sum_{i=1}^{n} i = \frac{n(n+1)}{2} \qquad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

Ex: Find the sum of the series $\sum_{i=1}^{10} i^2$.

Use the special formula. $\sum_{i=1}^{10} i^2 = \frac{10(10+1)(2\cdot 10+1)}{6} = \boxed{385}$

You Try: Write the series $4+8+12+...+100$ in summation notation. Then find the sum of the series.

QOD: What is the difference between a sequence and a series?

SAMPLE EXAM QUESTIONS

1. What is the expanded form of the series $\sum_{k=1}^{6}(-4-3k)$?

A. $-7+10-13+16-19+22$

B. $-7-10-13-16-19-22$

C. $-3+6-9+12-15+18$

D. $-3-6-9-12-15-18$

Ans: B

2. What is the series $1-2+3-4+5-6+\ldots$ when written in summation notation?

A. $\sum_{k=1}^{\infty}(-1)^{k-1}k$

B. $\sum_{k=1}^{\infty}(-1)^{k}k$

C. $\sum_{k=1}^{\infty}(-1)^{-1}k$

D. $\sum_{k=1}^{\infty}(-1)^{k+1}(-k)$

Ans: A

Sample SAT Question(s): Taken from College Board online practice problems.

6, 10, 18, 34, 66

The first number in the list above is 6. Which of the following gives a rule for finding each successive number in the list?

(A) Add 4 to the preceding number.

(B) Take $\frac{1}{2}$ of the preceding number and then add 7 to that result.

(C) Double the preceding number and then subtract 2 from that result.

(D) Subtract 2 from the preceding number and then double that result.

(E) Triple the preceding number and then subtract 8 from that result.

Ans: C

Unit 7.1, 7.2: Discrete to Continuous – Arithmetic and Geometric Sequences

⚠ Note: Algebra I covered arithmetic and geometric sequences, both recursive and explicit forms. This section should be review – the only new topic added is the arithmetic and geometric series and deriving their sums.

Arithmetic Sequence: a sequence in which the difference between consecutive terms is constant

Common Difference: the difference, *d*, between consecutive terms in an arithmetic sequence

Explicit Rule: a rule for a sequence that gives a_n as a function of the term's position number, *n*, in the sequence

Recursive Rule: a rule for a sequence that gives the beginning term(s) of the sequence and then an equation that tells how a_n is related to one or more preceding terms

Ex 1: Determine if the sequence is arithmetic. If it is, state the common difference. $-8, -5, -2, 1, 4, \ldots$

Find the difference of consecutive terms:

$-5-(-8)=3$ $\quad -2-(-5)=3$ $\quad 1-(-2)=3$ $\quad 4-1=3$

The difference is constant, so it **is** an arithmetic sequence with $\boxed{d=3}$.

Rule for the *n*th Term of an Arithmetic Sequence

Given an arithmetic sequence with common difference *d*, $\boxed{a_n = a_1 + (n-1)d}$. This is the explicit rule.

Ex 2: Write a rule for the *n*th term of the sequence $12, 7, 2, -3, \ldots$. Then find a_{23}.

Use the rule, with $a_1 = 12$ and $d = -5$. $a_n = 12 + (n-1)(-5) \Rightarrow \boxed{a_n = 17 - 5n}$

$a_{23} = 17 - 5(23) = \boxed{-98}$

Ex 3: Write a rule for an arithmetic sequence with $a_8 = 50$ and a common difference of 2.

Use the rule, with $a_n = 50, n = 8,$ and $d = 2$.

$$50 = a_1 + (8-1)(2)$$
$$36 = a_1$$

$$a_n = 36 + (n-1)(2) \Rightarrow \boxed{a_n = 34 + 2n}$$

<u>Recursive Rule for Arithmetic Sequences:</u> $a_n = a_{n-1} + d$ (Teachers – have students come up with this.)

Ex 4: Write the explicit and recursive rules for the arithmetic sequence where $a_1 = 15$ and $d = 5$.

Explicit Rule: $a_n = a_1 + (n-1)d = 15 + (n-1)(5) \Rightarrow \boxed{a_n = 10 + 5n}$

Recursive Rule: $\boxed{a_1 = 15,\ a_n = a_{n-1} + 5}$

Ex 5: Write a rule for the nth term of an arithmetic sequence if two terms are $a_5 = 10$ and $a_{30} = 110$. Then find the value of *n* for which $a_n = -2$.

Use the rule to write a system of equations:

$$10 = a_1 + (5-1)d \qquad 110 = a_1 + (30-1)d$$
$$10 = a_1 + 4d \qquad 110 = a_1 + 29d$$

Solve the system of equation:

$$\begin{array}{l} 110 = a_1 + 29d \\ \underline{-10 = -a_1 - 4d} \\ 100 = 25d \quad \Rightarrow \quad d = 4 \end{array} \qquad \begin{array}{l} 10 = a_1 + 4d \\ 10 = a_1 + 4(4) \\ a_1 = -6 \end{array}$$

Write the rule: $a_n = -6 + (n-1)(4) \Rightarrow \boxed{a_n = -10 + 4n}$

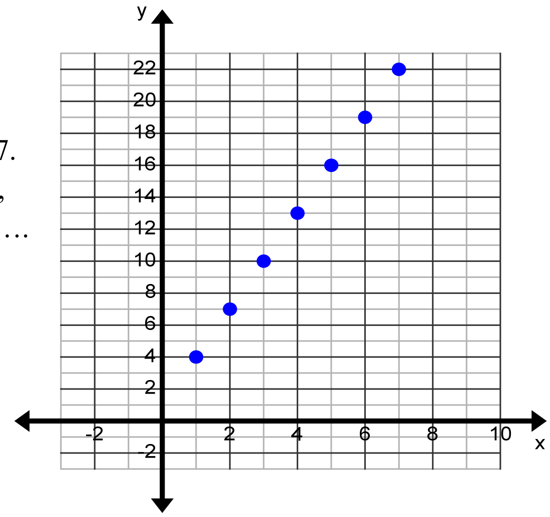
Let $a_n = -2$. $-2 = -10 + 4n \Rightarrow \boxed{n = 2}$

Connection between arithmetic sequences and linear functions:

Linear functions graph as lines and have a very special property: equal changes in the input give rise to equal changes in the output. Arithmetic sequences have this same property: equal changes in the input (e.g. moving from term to term) give rise to equal changes in the output (determined by the common difference). Thus, arithmetic sequences always graph as **points long a line**. Instead of a continuous linear function, an arithmetic sequence is a discrete function which is shown on the graph as discrete points along the path of a line.

Ex 6: Graph the arithmetic sequence 4, 7, 10, 13, …

When the input is 1 (for the first term in the sequence), the output is 4. When the input is 2 (for the second term in the sequence), the output is 7. When the input is 3 (for the third term in the sequence), the output is 10, and so on. That gives us the ordered pairs (1, 4), (2, 7), (3, 10), (4, 13),….



Arithmetic Series: the sum of the terms of an arithmetic sequence

Formula for the Sum of a Finite Arithmetic Series

The sum of the first *n* terms of an arithmetic series is $S_n = \frac{n}{2}(2a + (n-1)d)$.

This can be rewritten as: $S_n = n\left(\frac{a_1 + a_n}{2}\right)$.

Ex 7: Given the arithmetic series $20 + 18 + 16 + 14 + \ldots$. Find the sum of the first 25 terms. Then, find n such that $S_n = -760$.

The sum of the first 25 terms is $S_{25} = 25\left(\frac{a_1 + a_{25}}{2}\right)$. We must find $a_{25} = a_1 + (25-1)d$.

Using $a_1 = 20$ and $d = -2$, we have $a_n = 20 + (n-1)(-2) = 22 - 2n$; $a_{25} = 22 - 2(25) = -28$

$$S_{25} = 25\left(\frac{20 + (-28)}{2}\right) = \boxed{-100}$$

Now use the formula to solve for *n* when $S_n = -760$. $-760 = n\left(\frac{20 + a_n}{2}\right)$

From above, we have $a_n = 22 - 2n$. $-760 = n\left(\frac{20 + (22 - 2n)}{2}\right)$

Solve for *n*: $-760 = n(21 - n) \Rightarrow -760 = 21n - n^2$

$n^2 - 21n - 760 = 0 \Rightarrow (n - 40)(n + 19) = 0$ $n = 40$ or $n = -19$

n cannot be negative, so $S_n = -760$ when $\boxed{n = 40}$.

Why does the formula work? Let's see why the arithmetic series formula works, because we get to use an interesting "trick" which is worth knowing.

First, let's call the whole sum "S":

$$S = a + (a + d) + (a + 2d) + ... + (a + (n - 3)d) + (a + (n - 2)d) + (a + (n - 1)d)$$

Next, rewrite S in reverse order:

$$S = (a + (n - 1)d) + (a + (n - 2)d) + (a + (n - 3)d)...(a + 2d) + (a + d) + a$$

Now, add the two sums together, term by term:

$$\begin{aligned} S &= a + (a + d) + (a + 2d) + ... + (a + (n - 3)d) + (a + (n - 2)d) + (a + (n - 1)d \\ S &= (a + (n - 1)d) + (a + (n - 2)d) + (a + (n - 3)d) + ... + (a + 2d) + (a + d) + a \\ 2S &= (2a + (n - 1)d) + (2a + (n - 1)d) + (2a + (n - 1)d) + ... + (2a + (n - 1)d) + (2a + (n - 1)d) + (2a + (n - 1)d) \end{aligned}$$

Note that **each term is the same!** And there are "n" of them so, $2S = n \times (2a + (n - 1)d)$

Now, divide both sides by two and we get: $S = \frac{n}{2}(2a + (n - 1)d)$. This is our formula!!

Application Problems:

Ex 8: The first row of a concert hall has 20 seats. Each row after the first has two more seats than the row before it. There are 30 rows of seats. What is the total number of seats in the concert hall?

Use $a_1 = 20$ and $d = 2$. $a_n = 20 + (n - 1)(2) = 18 + 2n$ $a_{30} = 18 + 2(30) = 78$

Find S_{30}. $S_{30} = 30\left(\frac{20 + 78}{2}\right) = \boxed{1470 \text{ seats}}$

Ex 9: A construction company is laying sections of pipe in a pile at a construction site. There are 12 sections of pipe in the bottom row of the pile. Each row has one less pipe than the row below it. There are 8 rows of pipe. What is the total number of pipe sections in the pile?

Use $a_1 = 12$ and $d = -1$.	$a_n = 12 + (n-1)(-1) = 13 - n$	$a_8 = 13 - (8) = 5$

Find S_8.	$S_8 = 8\left(\frac{12+5}{2}\right) =$ 68 pipe sections

SAMPLE EXAM QUESTIONS

3. Which equation is the sum of the first 10 terms of the series $1+3+5+7+9+\ldots$?

A. $S_{10} = \frac{5}{2}(1+9)$

B. $S_{10} = \frac{10}{2}(1+17)$

C. $S_{10} = \frac{10}{2}(1+19)$

D. $S_{10} = \frac{10}{2}(1+21)$

Ans: C

4. Which is a formula of an arithmetic sequence when $a_1 = 3$ and $a_{11} = 23$?

A. $a_n = 3 + 23(n-1)$

B. $a_n = 3 + 2(n-1)$

C. $a_n = 23 + 10(n-1)$

D. $a_n = 23 + 2(n-1)$

Ans: B

5. What is the value of $\sum_{n=3}^{8}(15-4n)$?

A. -42	**C.** -17

B. 88	**D.** 363

Ans: A

<u>Geometric Sequence and Series:</u>

<u>Geometric Sequence:</u> a sequence in which the ratio between consecutive terms is constant

<u>Common Ratio</u>: the ratio, *r*, between consecutive terms in an geometric sequence

Ex 10: Write the first 5 terms of a geometric sequence whose first term is 3 and whose common ratio is 2.

3
$3 \cdot 2 = 6$
$6 \cdot 2 = 12$
$12 \cdot 2 = 24$
$24 \cdot 2 = 48$

The first 5 terms are: $\boxed{3,\ 6,\ 12,\ 24,\ 48}$

Note: If you divide (find the ratio) of consecutive terms, you will obtain a quotient (ratio) of 2.

<u>Rule for the *n*th Term of an Geometric Sequence</u>

Given an geometric sequence with common ratio *r*, $\boxed{a_n = a_1 \cdot r^{n-1}}$. This is the explicit rule.

<u>Recursive Rule for Geometric Sequences:</u> $a_n = ra_{n-1}$ (Teachers – have students come up with this.)

Ex 11: Write a rule for the *n*th term of the sequence. $3, -12, 48, -192, \ldots$

This is a geometric sequence with $a_1 = 3$ and $r = \frac{-12}{3} = \frac{48}{-12} = \frac{-192}{48} = -4$. So, $\boxed{a_n = 3(-4)^{n-1}}$.

Ex 12: Write the explicit and recursive rules for the geometric sequence. $4, 2, 1, \frac{1}{2}, \ldots$

Explicit Rule: $a_n = a_1 \cdot r^{n-1} \Rightarrow \boxed{a_n = 4\left(\frac{1}{2}\right)^{n-1}}$ Recursive Rule: $\boxed{a_1 = 4, a_n = \frac{1}{2} \cdot a_{n-1}}$

Ex 13: One term of a geometric sequence is $a_3 = -2$. The common ratio is 3. Write a rule for the *n*th term. Then find a_8.

Use the general rule and solve for a_1.

$$a_n = a_1 \cdot r^{n-1} \Rightarrow a_3 = a_1 \cdot 3^{3-1}$$
$$-2 = a_1 \cdot 9 \Rightarrow a_1 = -\frac{2}{9}$$

So the rule for the nth term is $\boxed{a_n = \left(-\frac{2}{9}\right)(3)^{n-1}}$ and $a_8 = \left(-\frac{2}{9}\right)(3)^{8-1} = \boxed{-486}$.

Ex 14: Two terms of a geometric series are $a_2 = 18$ and $a_5 = \frac{2}{3}$. Write a rule for the *n*th term.

Substitute each term into the general rule to create a system. $18 = a_1 \cdot r^{2-1}$ and $\frac{2}{3} = a_1 \cdot r^{5-1}$

Solve the system by substitution.

$$18 = a_1 \cdot r$$
$$a_1 = \frac{18}{r}$$

$$\frac{2}{3} = a_1 \cdot r^4 \Rightarrow \frac{2}{3} = \frac{18}{r} \cdot r^4$$
$$\frac{2}{3} = 18r^3 \Rightarrow r^3 = \frac{1}{27} \Rightarrow r = \frac{1}{3}$$

$$a_1 = \frac{18}{\frac{1}{3}} = 54$$

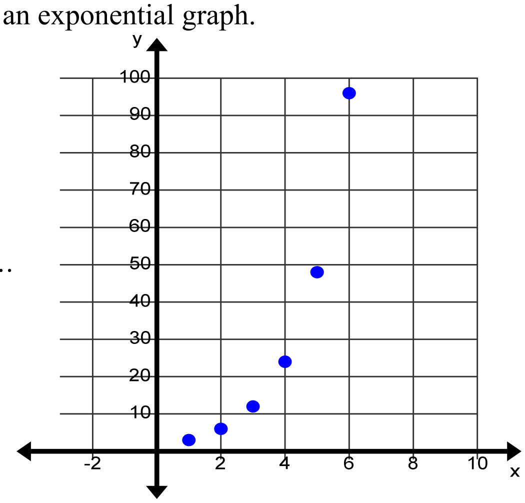
So $\boxed{a_n = 54\left(\frac{1}{3}\right)^{n-1}}$.

Connection between geometric sequences and exponential functions:

There is a class of functions, called **exponential** functions, which have a very special property: equal changes in the input cause the output to be successively multiplied by a constant. Geometric sequences have this same special property: equal changes in the input (e.g., moving from term to term) cause the output to be successively multiplied by a constant (determined by the common ratio). Thus, geometric sequences always graph as **points along the graph of an exponential function**. Instead of a continuous exponential function, a geometric sequence is a discrete function which is shown on the graph as discrete points along the path of an exponential graph.

Ex 15: Graph the geometric sequence 3, 6, 12, 24, …

When the input is 1 (for the first term in the sequence), the output is 3. When the input is 2 (for the second term in the sequence), the output is 6. When the input is 3 (for the third term in the sequence), the output is 12, and so on. That gives us the ordered pairs (1, 3), (2, 6), (3, 12), (4, 24),….



Geometric Series: the sum of the terms of a geometric sequence

Formula for the Sum of a Finite Geometric Series

The sum of the first *n* terms of a **finite** geometric series is $S_n = a_1\left(\frac{1-r^n}{1-r}\right)$.

Ex 16: Use the geometric series $1+4+16+64+\ldots$. Find the sum of the first 10 terms.

Use $a_1 = 1$ and $r = 4$.

$$S_{10} = 1\left(\frac{1-4^{10}}{1-4}\right) = \boxed{349{,}525}$$

Deriving the Finite Geometric Series Formula:

For an *n*-term geometric sequence, we can write the series as $S_n = a + ar + ar^2 + ar^3 + ar^4 + ... + ar^{n-1}$

Now, multiply both sides by the common ratio. $rS_n = ar + ar^2 + ar^3 + ar^4 + ar^5 ... + ar^n$

Subtract the two equations. $S_n - rS_n = \left(a + ar + ar^2 + ar^3 + ar^4 + ... + ar^{n-1}\right) - \left(ar + ar^2 + ar^3 + ar^4 + ar^5 ... + ar^n\right)$

Notice that except for the very first term in the expression for S_n and the very last term in the expression for rS_n, there are matching terms in the two sets of parentheses. That means all of those terms cancel each other out, and we are left with: $S_n - rS_n = a - ar^n$

Factoring and Solving for S_n,

$$S_n - rS_n = a - ar^n$$
$$S_n(1-r) = a(1-r^n)$$
$$S_n = a\frac{1-r^n}{1-r}$$

Explore: Find S_n for $n = 1, 2, 3, 4, 5$ for the **infinite** geometric series. $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + ...$.

$S_1 = 0.5$ $S_2 = 0.75$ $S_3 \approx 0.88$ $S_4 \approx 0.94$ $S_5 \approx 0.97$

These appear to be approaching 1. This is the case. An infinite geometric series does have a finite sum, if $|r| < 1$ and we say that it converges.

Formula for the Sum of an Infinite Geometric Series

The sum of an infinite geometric series with first term a_1 and common ratio r, when $|r| < 1$: $\boxed{S = \frac{a_1}{1-r}}$

Note: If $|r| \geq 1$, then the series has no sum. We say that the series diverges.

Ex 17: Write the infinite geometric series in summation notation. Then find the sum (if possible).

$$12 - 4 + \frac{4}{3} - \frac{4}{9} + ...$$

$a_1 = 12,\ r = -\frac{1}{3}$ $\qquad \sum_{n=1}^{\infty} 12\left(-\frac{1}{3}\right)^{n-1}$

$|r| = \left|-\frac{1}{3}\right| = \frac{1}{3} < 1$, so the sum is $S = \frac{12}{1 - \left(-\frac{1}{3}\right)} = \frac{12}{\frac{4}{3}} = \boxed{9}$

Ex 18: An infinite geometric series with first term $a_1 = -\frac{4}{3}$ has a sum of -2. What is the common ratio of the series?

$$S = \frac{a_1}{1-r} \qquad -2 = \frac{-\frac{4}{3}}{1-r} \Rightarrow -2 + 2r = -\frac{4}{3} \Rightarrow \boxed{r = \frac{1}{3}}$$

Writing a Repeating Decimal as a Fraction

Ex 19: Use an infinite geometric series to write 0.272727… as a fraction.

$0.272727... = 27(0.01) + 27(0.01)^2 + 27(0.01)^3 + ...$, so $a_1 = 0.27,\ r = 0.01$

$$S = \frac{27(0.01)}{1-0.01} = \frac{0.27}{0.99} = \frac{27}{99} = \boxed{\frac{3}{11}}$$

Application Problem

Ex 20: A pendulum swings 10 feet going left to right. On its swing back, it swings 90% as far as the first swing. Each successive swing is 90% of the previous swing. Find the total distance traveled by the pendulum when it finally stops.

The total distance traveled = $10 + 10(0.9) + 10(0.9)^2 + 10(0.9)^3 + 10(0.9)^3 ...$

Use $a_1 = 10, r = 0.9$. $S = \frac{10}{1-0.9} = \boxed{100 \text{ ft}}$

QOD: Explain why you can't find the sum of an infinite geometric series with $|r| \geq 1$.

SAMPLE EXAM QUESTIONS

1. Which is a formula of the geometric sequence 3, 1, $\frac{1}{3}$, $\frac{1}{9}$, …?

A. $g_n = \left(\frac{1}{3}\right)^{n-2}$

B. $g_n = 3^{n-2}$

C. $g_n = \left(\frac{1}{3}\right)^{1-n}$

D. $g_n = 3^{1-n}$

Ans: A

2. A simulation starts with 192 coins. The first step of the simulation removes half (96) of the coins. Each subsequent step removes half of the remaining coins. How many coins are left after the fifth step of the simulation?

A. 24 **C.** 12

B. 6 **D.** 3

Ans: C

3. Given the sequence 1, 2, 4, 8, ….

Find the sum of the infinite series.

A. 15 **C.** 18

B. 30 **D.** ∞

Ans: D

4. During a flu outbreak, a hospital recorded 12 cases the first week, 54 cases the second week, and 243 cases the third week.

a) Write a geometric sequence to model the flu outbreak.

The common ratio, $r = \frac{a_n}{a_{n-1}} = \frac{54}{12} = \frac{243}{54} = \frac{9}{2}$

So $a_n = 12\left(\frac{9}{2}\right)^{n-1} = \frac{8}{3}\left(\frac{9}{2}\right)^n$

b) How many cases will occur in the sixth week if the hospital cannot stop the outbreak?

In the sixth week $n = 6$, so $a_6 = \frac{8}{3}\left(\frac{9}{2}\right)^6 = \frac{3^{11}}{2^3} = 22{,}143.375$ which is about 22,143 cases of the flu.

5. Given the geometric sequence with common ratio r, write a rule for the nth term of the sequence 4, -28, 196, -1372…

A. $a_n = 7(-4)^{n-1}$

B. $a_n = 4(-7)^{n-1}$

C. $a_n = 7(4)^{n-1}$

D. $a_n = 4(7)^{n-1}$

Ans: B

6. What is the sum of the infinite geometric series $0.4+0.04+0.004+\ldots$?

A. 0.012

B. 0.444

C. $\frac{2}{5}$

D. $\frac{4}{9}$

Ans: D

7. In a classic math problem a king wants to reward a knight who has rescued him from an attack. The king gives the knight a chessboard and plans to place money on each square. He gives the knight two options. Potion 1 is to place a thousand dollars on the first square, two thousand on the second square, three thousand on the third square, and so on. Option 2 is to place one penny on the first square, two pennies on the second, four on the third, and so on.

Think about which offer sounds better and then answer these questions.

a) List the first five terms in the sequences formed by the given options. Identify each sequence as arithmetic, geometric, or neither.

Option 1: \$1000, \$2000, \$3000, \$4000, \$5000

So the sequence is arithmetic with the common difference

Option 2: 1, 2, 4, 8, 16

So the sequence is geometric with a common ratio

b) For each option, write a rule that tells how much money is placed on the nth square of the chessboard and a rule that tells the total amount of money placed on squares one through n.

Option 1:

$$a_n = a_1 + (n-1)d = 1000 + (n-1)1000 = 1000n$$

$$S_n = \frac{n(a_1 + a_n)}{2}$$

Option 2:

$$a_n = a_1 (r)^{n-1} = 1(2)^{n-1}$$

$$S_n = a_1\left(\frac{1-r^n}{1-r}\right)$$

c) Find the amount of money placed on the 20th square of the chessboard and the total amount placed on squares 1 through 20 for each option.

Option 1

$$a_{20} = 1000(20) = \$20,000$$

$$S_{20} = \frac{n(a_1 + a_{20})}{2} = \frac{20(1000+20000)}{2} = \$210,000$$

Option 2

$$a_{20} = a_1(r)^{20-1} = 1(2)^{19} = \$5,242.88$$

$$S_{20} = a_1\left(\frac{1-r^{20}}{1-r}\right) = 1\left(\frac{1-2^{20}}{1-2}\right) = 2^{20} - 1 = \$10,485.75$$

d) There are 64 squares on a chessboard. Find the total amount of money placed on the chessboard for each option.

Option 1 $$S_n = \frac{n(a_1 + a_n)}{2} = \frac{64(1000 + 1000 \cdot 64)}{2} = \$2,080,000$$

Option 2 $$S_n = a_1\left(\frac{1-r^n}{1-r}\right) = 1\left(\frac{1-2^{64}}{1-2}\right) = 2^{64} - 1 = 1.8446744 \text{ E } 19\text{-}1$$

NOTE: This answer doesn't reflect the unit change from cents to dollars.

e) Which gives the better reward, Option 1 or Option 2? Explain why.

Using all the data we can conclude that a quantity increasing exponentially eventually exceeds a quantity increasing linearly after enough iteration has been done. If we would have come up with a conclusion based on the results regarding the 20th square, we would have been wrong.

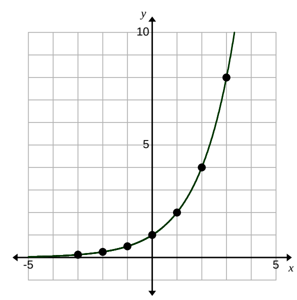
Section 7.3 and 7.9: To graph and transform exponential functions modeling growth and decay.

Exponential Function: $y = a \cdot b^x$, $b > 0, b \neq 1$

Graph of an Exponential Function

x	-3	-2	-1	0	1	2	3
y	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8

Ex 21: Create a table of values to graph the exponential function $y = 2^x$.



End Behavior: As $x \to +\infty$, $f(x) \to +\infty$.

As $x \to -\infty$, $f(x) \to 0$, therefore $y = 0$ is an **asymptote**.

EXPLORE: Graph the following exponential functions and describe the change from the graph of $y = 2^x$.

1. $y = \frac{1}{3} \cdot 2^x$ 2. $y = 3 \cdot 2^x$ 3. $y = -\frac{1}{4} \cdot 2^x$

4. $y = -4 \cdot 2^x$ 5. $y = 2^x + 3$ 6. $y = 2^x - 4$ 7. $y = 2^{x-1}$

General characteristics of $y = ab^{x-h} + k$ **:**

Graph of $y = ab^x$ is shifted horizontally by *h* units. Graph of $y = ab^x$ is shifted vertically by *k* units.

If $a > 0$ and $b > 1$, it is an **exponential growth function.**

Domain of a Growth Function: All Reals Range of a Growth Function: $y > k$

If $a > 0$ and $0 < b < 1$, it is an **exponential decay function.**

Domain of a Decay Function: All Reals Range of a Decay Function: $y > k$

Ex 22: State whether $f(x)$ is an exponential growth or decay function.

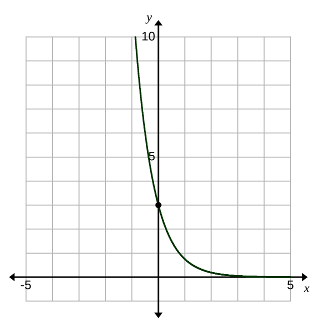
1. $f(x) = 4\left(\frac{3}{4}\right)^x$ DECAY (because $a > 0$ and $b < 1$)
2. $f(x) = 3\left(\frac{5}{2}\right)^x$ GROWTH (because $a > 0$ and $b > 1$)
3. $f(x) = 6(4)^{-x}$ DECAY (Can be rewritten as $f(x) = 6\left(\frac{1}{4}\right)^x$.)
4. $f(x) = -8\left(\frac{2}{3}\right)^x$ Exponential Function, NEITHER GROWTH OR DECAY
5. $f(x) = -2 \cdot 3^x$ Exponential Function, NEITHER GROWTH OR DECAY

Ex 23: Graph the function $y = 3\left(\frac{1}{4}\right)^x$ and state its domain and range.

y – intercept: $(0,3)$ Asymptote: $y = 0$

Domain: All real numbers

Range: $y > 0$

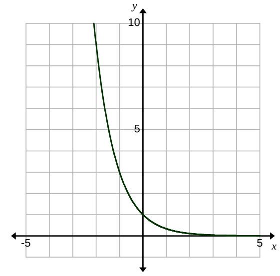


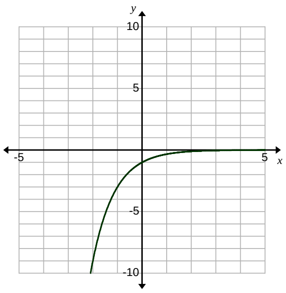
Ex 24: Describe how the graph of $y = -2(3)^{x+1} - 4$ differs from the function $y = 3^x$.

The graph of $y = -2(3)^{x+1} - 4$ is vertically stretched by a factor of two, is reflected over the x-axis, translated horizontally to the left by one unit and translated vertically down by four units.

Ex 25: Use transformations and sketch the graph of $y=-\left(\frac{1}{3}\right)^{x}+4$

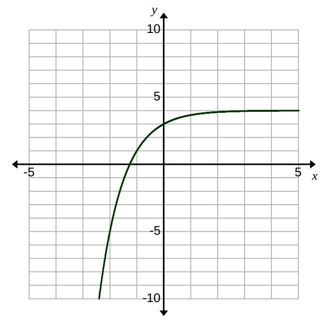
Step One: Sketch the graph of $y=\left(\frac{1}{3}\right)^{x}$





Step Two: Sketch the graph of $y=-\left(\frac{1}{3}\right)^{x}$ by reflecting over the *x*-axis.

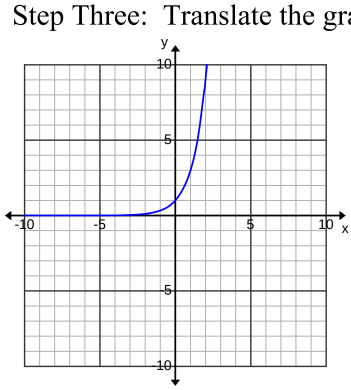
Step Three: Translate the graph vertically 4 units up.

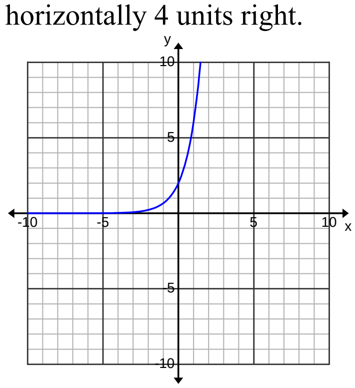


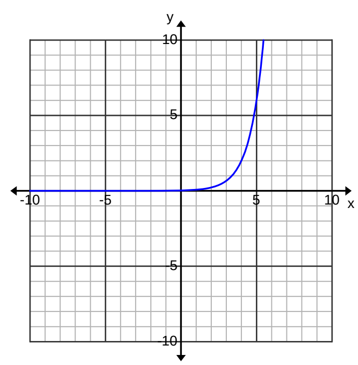
Ex 26: Use transformations and sketch the graph of $y=2\cdot 3^{x-4}$.

Step One: Sketch the graph of $y=3^{x}$ Step Two: Stretch the graph vertically by a factor of 2.

Step Three: Translate the graph horizontally 4 units right.







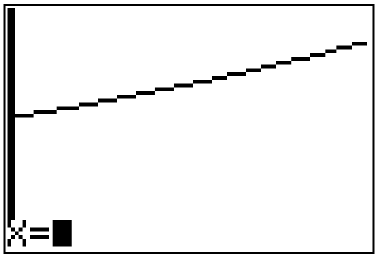
<u>Exponential Growth Models:</u> In a real-life situation, if a quantity increases by *r* percent each time period *t*, the situation can be modeled by the equation $y=a\left(1+r\right)^{t}$, where *a* is the initial amount. The quantity $\left(1+r\right)$ is called the **growth factor**.

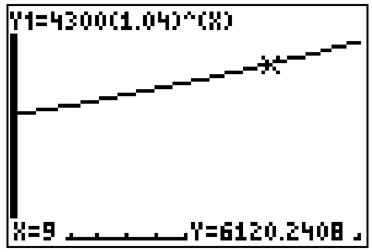
Ex 27: In 1990 the cost of tuition at a state university was \$4300. During the next 8 years, the tuition rose 4% each year. Write a model that gives the tuition *y* (in dollars) *t* years after 1990. Then estimate the cost of tuition in 1999.

Model: $y=4300\left(1+0.04\right)^{t} \Rightarrow \boxed{y=4300\cdot 1.04^{t}}$

1999: $t=9 \Rightarrow y=4300\cdot 1.04^{9} \approx \boxed{\$6120.24}$

Graph: On the graphing calculator, graph $y = 4300 \cdot 1.04^x$ and find the value of the function when $x = 9$.



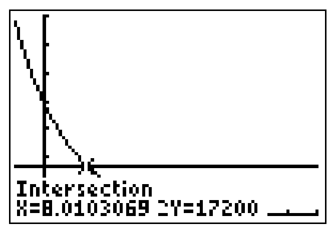


We find the same answer $6120.24.

Exponential Decay Models: In a real-life situation, if a quantity decreases by r percent each time period t, the situation can be modeled by the equation $y = a(1-r)^t$, where a is the initial amount. The quantity $(1-r)$ is called the **decay factor**.

Ex 28: There are 40,000 homes in a certain city. Each year 10% of the homes are expected to disconnect from septic systems and connect to the sewer system. Write an exponential decay model for the number of homes that still use septic systems. Use the graph of the model to estimate when about 17,200 homes will still not be connected to the sewer system.

Model: $y = 40000(1-0.1)^t \Rightarrow \boxed{y = 40000(0.9)^t}$



Graph: On the graphing calculator, graph $y = 40000(0.9)^x$ and $y = 17200$. Find the point of intersection.

There will be 17,200 homes not connected to the sewer system after about 8 years.

QOD: What is the difference between percent increase and growth factor?

QOD: Describe the end behaviors of an exponential decay function.

Exploration: Evaluate the following for larger and larger values of n. $\left(1+\frac{1}{n}\right)^n$

Note: Students should get values closer and closer to 2.71828… , which is the value of e.

Now use the e key on the calculator, and it will automatically give an approximation of e.

Euler Number: the irrational **natural** base $e \approx 2.718281828459$

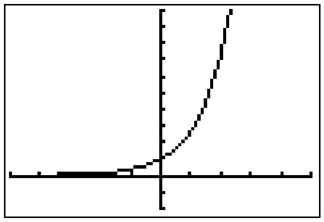
Note: e is named after its discoverer, Leonhard Euler.

Graphing Natural Base Functions

Ex 29: Graph the functions $y = e^x$ and $y = e^{-x}$. State the domain and range.

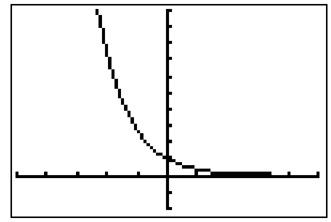
$y = e^x$

Exponential Growth



$y = e^{-x}$

Exponential Decay

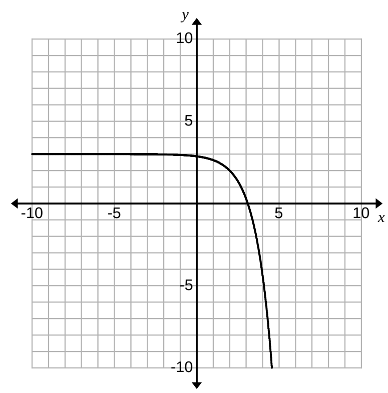


Domain: All real numbers. Range: $y > 0$

Domain: All real numbers. Range: $y > 0$

Ex 30: Sketch the graph of the function. State its domain and range.

$$y = -e^{x-2} + 3$$



Graph the function $y = e^x$. Because the coefficient of *e* is -1, we will reflect over the *x*-axis. Then, the graph will be shifted left 2 and up 3.

Domain: All real numbers

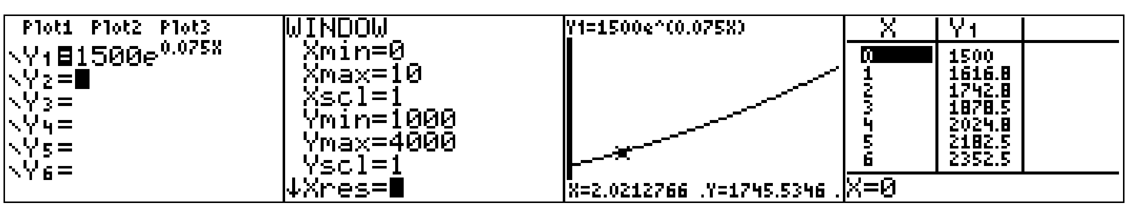
Range: $y < 3$

Continuously Compounded Interest

Recall: $A = P\left(1 + \frac{r}{n}\right)^{nt}$ is the formula for interest compounded *n* times per year. If interest is compounded continuously, then $n \to \infty$. Therefore, the formula for continuously compounded interest is $A = Pe^{rt}$.

Ex 31: Chelli deposited \$1500 into an account that pays 7.5% annual interest compounded continuously. What is her balance after 2 years?

Use $A = Pe^{rt}$ with $P = 1500$ and $r = 0.075$. Graph: $y = 1500e^{(0.075x)}$ and find the value when $x = 2$.

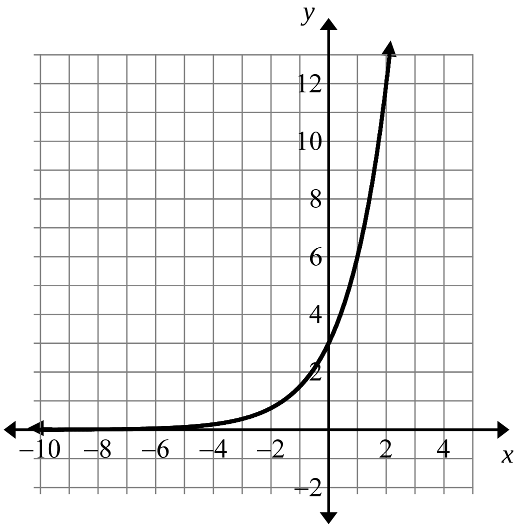


The balance after 2 years is \$1,742.80.

QOD: e is an irrational number. Define irrational and give 3 other examples of irrational numbers.

SAMPLE EXAM QUESTIONS

1. What function describes the graph below?

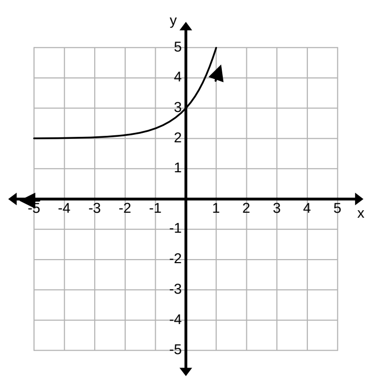


A. $y = x^2 + 3$ **C.** $y = 3x^3 + 3$

B. $y = 3 \cdot 2^x$ **D.** $y = 6^x$

Ans: B

2. What function is represented by the following graph?



A. $f(x) = 3^x$ **C.** $f(x) = 3^x - 2$

B. $f(x) = 2 + 3^x$ **D.** $f(x) = (\frac{3}{2})^x$

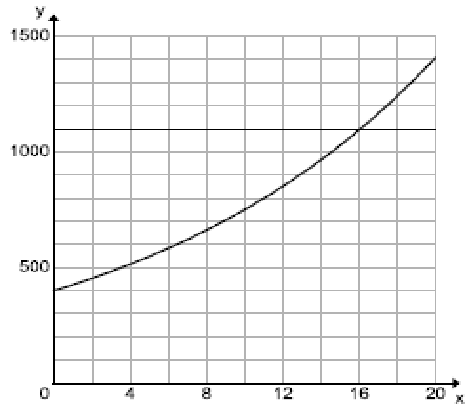
Ans: B

3. Sarai bought $400 of Las Vegas Cellular stock in January 2005. The value of the stock is expected to increase by 6.5% per year.

a) Write a model to describe Sarai's investment.

$A(x) = P(x)(a+r)^t = 400(1.065)^t$, t is the number of years

b) Use the graph to show when Sarai's investment will reach $1100?



Sarai's investment will reach $1100 in just over 16 years.

4. John graphs the equation $y = 5^x$. Lana graphs the equation $y = 5^x + 2$. How does Lana's graph compare to John's graph?

A. Lana's graph shifts 2 units downward

B. Lana's graph shifts 2 units upward

C. Lana's graph shifts 2 units to the left

D. Lana's graph shifts 2 units to the left

Ans: B

5. The value of a new car was $27,000 in 2005. The car loses 15% of its value each year. Which formula represents the car's value, $v(t)$, *t* years after 2005?

A. $v(t) = 27,000(0.15)^t$

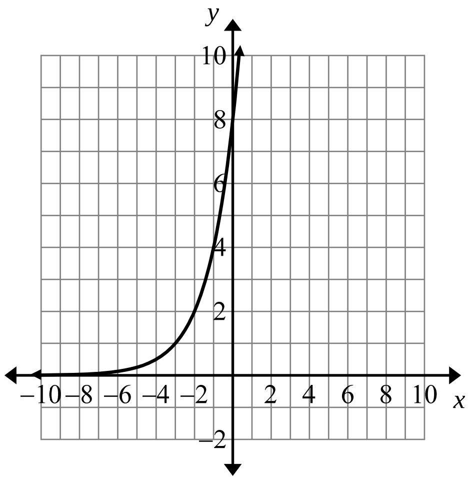
B. $v(t) = 27,000(1.15)^t$

C. $v(t) = 27,000(0.85)^t$

D. $v(t) = 27,000 - 0.15^t$

Ans: C

6. What function describes the graph below?



A. $f(x) = 4x^3 + 8$

B. $f(x) = 2x^2 + 8$

C. $f(x) = 4 \cdot 2^{x+1}$

D. $f(x) = 2^{x+1}$

Ans: C

<u>**Section 7.4: To find and graph the inverse of an exponential function.**</u>

<u>Exponential and logarithmic functions are **inverse** functions.</u>

Recall: By definition, if $f(x)$ and $g(x)$ are inverse functions, then $f(g(x)) = g(f(x)) = x$

Therefore, $\log_b b^x = x$ and $b^{\log_b x} = x$.

Ex: Evaluate the expressions.

1. $e^{\ln 4}$ $\qquad e^{\ln 4} = \boxed{4}$

2. $\log 10^5$ $\qquad \log 10^5 = \boxed{5}$

3. $\log_5 25^x$ $\qquad$ Rewrite the argument to match the base. $\log_5 (5^2)^x = \log_5 5^{2x} = \boxed{2x}$

Finding Inverses

Ex: Find the inverse of the function $y = \log_5 x$.

The inverse of a logarithmic function is an exponential function. $\boxed{y = 5^x}$

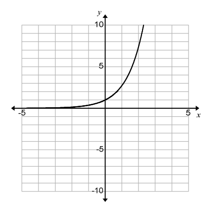
Ex: Find the inverse of the function $y = \ln(x-1)$.

Step One: Switch the *x* and the *y*. $x = \ln(y-1)$

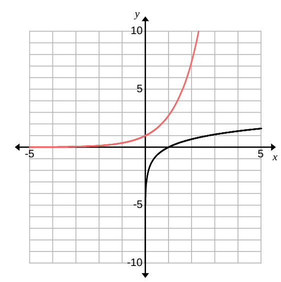
Step Two: Solve for *y*. Write in exponential form. $y - 1 = e^x \Rightarrow \boxed{y = e^x + 1}$

<u>Graphing:</u>

Ex 32: Recall: Graph the exponential function $y = e^x$.



The graph of $y = \ln x$ (the inverse of $y = e^x$) is the reflection of the graph of $y = e^x$ over the line $y = x$.

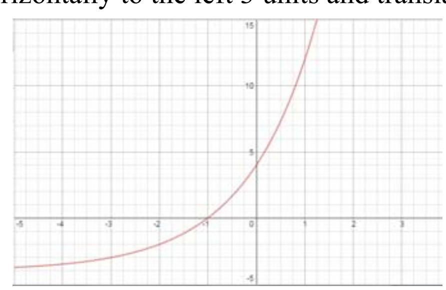


Domain: $x > 0$; Range: All real numbers Asymptote: $x = 0$

Note: The domain and range of a logarithmic function is the domain and range of its inverse (exponential function) switched!

Ex 33: Sketch the graph of the function $y = 2^{x+3} - 4$ and its inverse.

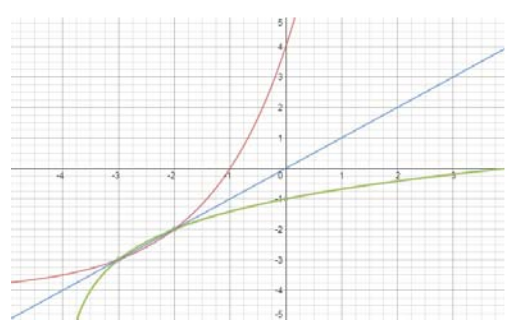
Using transformations, the graph of $y = 2^x$ will be translated horizontally to the left 3 units and translated vertically down 4 units.



x	-3	-2	-1	0	1
y	-3	-2	0	4	12

Domain: All real numbers Range: $y > -4$

The inverse function will be reflected across the line $y = x$ and the values of the domain and range are switched.



The inverse function is: $y = \log_2(x+4) - 3$

Section 7.5: To evaluate graph and transform logarithmic functions.

Definition of a Logarithm: Let b and y be positive numbers, with $b \neq 1$.

$$\log_b y = x \text{ if and only if } b^x = y$$

Note: Evaluating a **logarithm** is the same as finding an **exponent**.

$\log_b y = x$ is read "log base *b* of *y*."

Writing Logarithmic Equations in Exponential Form

Ex 34: Rewrite the following in exponential form.

1. $\log_2 8 = 3$ The base is 2, the exponent is 3, and the power is 8. $\boxed{2^3 = 8}$

2. $\log_3 \frac{1}{3} = -1$ The base is 3, the exponent is −1, and the power is $\frac{1}{3}$. $\boxed{3^{-1} = \frac{1}{3}}$

3. $\log_{10} 0.01 = -2$ The base is 10, the exponent is −2, and the power is 0.01 $\boxed{10^{-2} = 0.01}$

Note: The logarithmic and exponential forms of the equations are **equivalent**.

Evaluating Logarithms: To evaluate a logarithmic expression, remember you are finding the **exponent** the base would need to be raised to in order to obtain the *argument* of the logarithm.

Ex 35: Evaluate the following expressions.

1. $\log_4 64$ Answer the question: What exponent would I raise 4 to in order to obtain 64?

   $\log_4 64 = \boxed{3}$ because $4^3 = 64$

2. $\log_{1/3} 9$ Answer the question: What exponent would I raise 1/3 to in order to obtain 9?

   $\log_{1/3} 9 = \boxed{-2}$ because $\left(\frac{1}{3}\right)^{-2} = 9$

3. $\log_9 3$ Answer the question: What exponent would I raise 9 to in order to obtain 3?

   $\log_9 3 = \boxed{\frac{1}{2}}$ because $9^{\frac{1}{2}} = 3$

Special Values of Logarithms: (Let b be a positive real number other than 1.)

1. $\log_b 1$

   Answer the question: What exponent would I raise b to in order to obtain 1? Because any positive real number raised to the 0 power is equal to 1, we can say that $\boxed{\log_b 1 = 0}$.

2. $\log_b b$

   Answer the question: What exponent would I raise b to in order to obtain b? Because any number raised to the power of 1 is equal to itself, we can say that $\log_b b = \boxed{1}$.

Special Logarithms

Common Logarithm: the logarithm with base 10 $\log_{10} x = \log x$

Natural Logarithm: the logarithm with base e $\log_e x = \ln x$

Ex: Evaluate $\log 100$. $\log 100 = \boxed{2}$ because $10^2 = 100$

Ex: Evaluate $\ln e^3$. $\ln e^3 = \boxed{3}$

Logarithms on the Calculator: The calculator can only evaluate common and natural logarithms.

Ex: Approximate the value of $\log 5$.

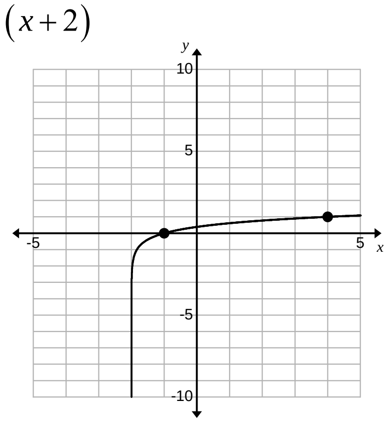
```
log(5)
      .6989700043
```

Ex: Approximate the value of $\ln 0.01$.

```
ln(.01)
     -4.605170186
```

QOD: Explain why the logarithm of a negative number is undefined.

Ex 36: Graph the logarithmic function. State the domain and range. $y = \log_6(x+2)$



Plot "convenient" points: Let $x = -1$. $\log_6(-1+2) = \log_6 1 = 0$

Let $x = 4$. $\log_6(4+2) = \log_6 6 = 1$

Because the graph is shifted to the left 2, there is an asymptote at $x = -2$.

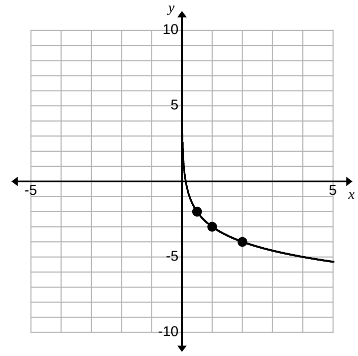
Domain: $x > -2$; Range: All real numbers

Ex 37 : Sketch the graph of the logarithmic function. State the domain and range. $y = \log_{1/2} x - 3$

Plot "convenient" points: Let $x = 1$. $y = \log_{1/2} 1 - 3 = 0 - 3 = -3$

Let $x = \frac{1}{2}$. $y = \log_{1/2} \frac{1}{2} - 3 = 1 - 3 = -2$

Let $x = 2$. $y = \log_{1/2} 2 - 3 = -1 - 3 = -4$



The graph is shifted down 3, and there is an asymptote at $x = 0$.

Domain: $x > 0$; Range: All real numbers

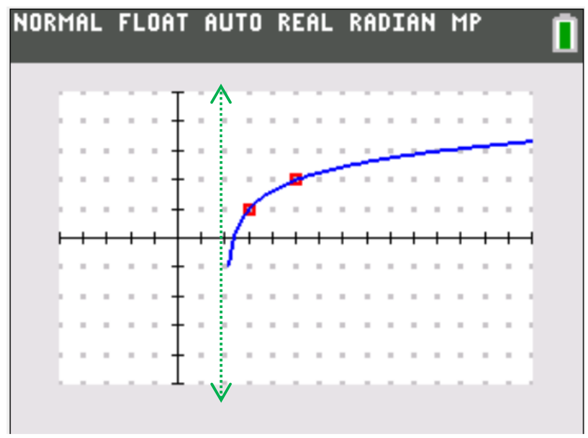
Ex 38 : Sketch the graph of the logarithmic function. State the domain and range. $f(x) = \log_3 (x - 2) + 1$

Plot "convenient" points: Let $x = 3$.
$\log_3 (3 - 2) + 1 = \log_3 1 + 1 = 0 + 1 = 1$

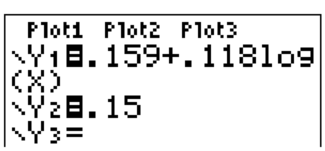
Let $x = 5$. $\log_3 (5 - 2) + 1 = \log_3 3 = 1 + 1 = 2$

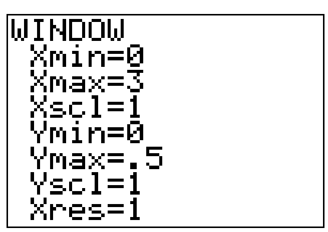
Because the graph is shifted to the right 2, there is an asymptote at $x = 2$.

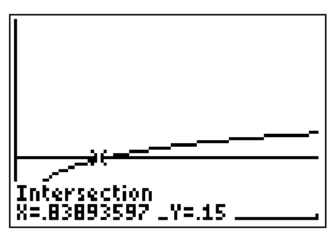
Domain: $x > 2$; Range: All real numbers



Ex 39: The slope s of a beach is related to the average diameter d (in millimeters) of the sand particles on the beach by the equation $s = 0.159 + 0.118 \log d$. Graph the model and estimate the average diameter of the sand pebbles for a beach whose slope is 0.15.







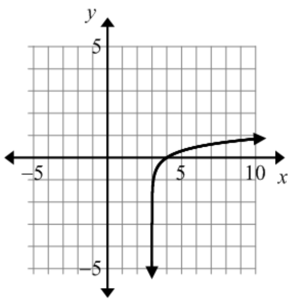
The average diameter is about **0.84 millimeters**.

QOD: For what values of b does the graph of $f(x) = b^x$ intersect its inverse $f^{-1}(x) = \log_b x$?

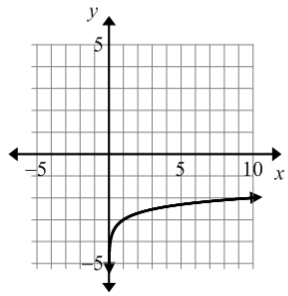
SAMPLE EXAM QUESTIONS

1. What graph represents the function $f(x) = 3 + \log x$?

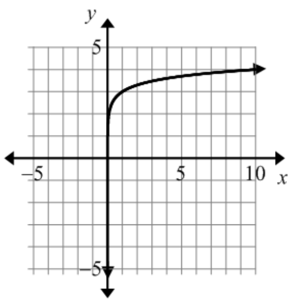
A.



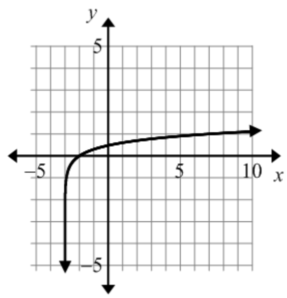
C.



B.

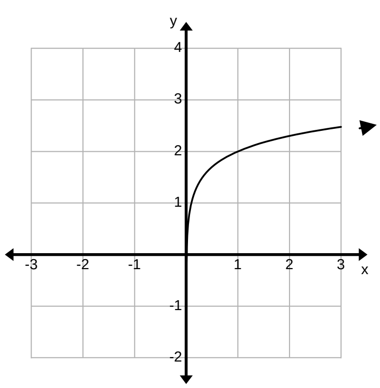


D.



Ans: B

2. Choose the function that describes the graph below:



A. $f(x) = \log x + 2$

B. $f(x) = \log x - 2$

C. $f(x) = \log(x + 2)$

D. $f(x) = \log(x - 2)$

Ans: A

3. Consider the function $f(x) = \log x$.

a) Identify the transformation applied to $f(x)$ to create $g(x) = \log x + 1$.

Vertical shift 1 unit up applied to *f(x)* to create *g(x)*.

b) Identify the transformation applied to $f(x)$ to create $h(x) = \log(10x)$.

The graph of *h(x)* is a horizontal compression of the original function, by a factor of 10.

c) Compare the graphs of $g(x)$ and $h(x)$. What do you notice?

By comparing the graphs of *g(x)* and *h(x)* we see that the graphs are identical.

d) Use the properties of logarithms to explain your answer to part c.

$h(x) = \log(10x) = \log(10) + \log(x) = 1 + \log(x) = g(x)$

4. **The graph of the equation $y = \log(2x+3)$ is translated right 3 units and down 3.5 units to form a new graph. Which equation** best **represents the new graph?**

A. $y = \log(2x+9) + 3.5$

B. $y = \log(2x-3) + 3.5$

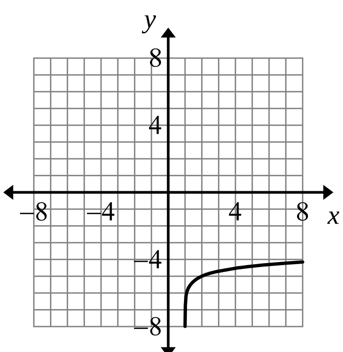
C. $y = \log(2x+9) - 3.5$

D. $y = \log(2x-3) - 3.5$

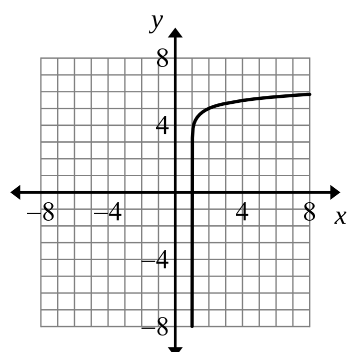
Ans: D

5. Which graph represents the function $f(x) = \log(x-1) - 5$?

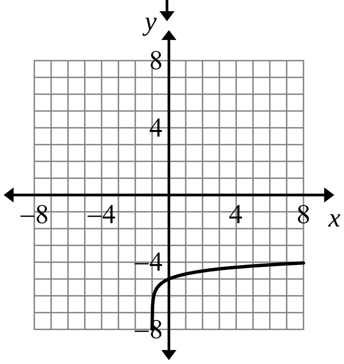
A.



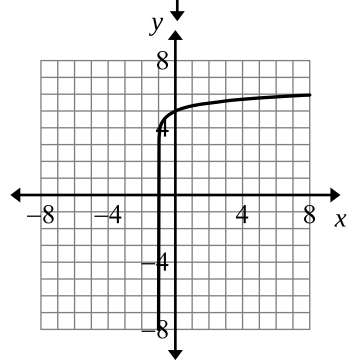
B.



C.



D.



Ans: A