SELECTED RESPONSE

#	Question Type	Learning Target	Nevada Academic Content Standard(s)	DOK	Key
1	MC	1.2	F.BF.B.4a-2	1	C
2	MC	1.2	F.BF.B.4a-2	1	A
3	MC	1.4	F.BF.B.4b	1	C
4	MC	3.4	N.CN.A.2	1	C
5	MC	3.4	N.CN.A.2	1	A
6	MC	3.6	N.CN.C.7	1	C
7	CR	3.7	F.IF.C.8a	2	—
8	CR	3.7	C.CN.C.7	2	—
9	MC	3.7	N.CN.C.7	1	C
10	MC	3.7	F.IF.C.8a	2	A
11	MC	3.7	F.IF.C.8a	2	B
12	MC	3.7	N.CN.C.7	1	D
13	MC	3.7	A.REI.B.4b	1	B
14	MC	3.7	A.REI.D.12-2	2	D
15	MC	3.7	A.SSE.A.2-2	2	A
16	MC	3.8	N.CN.C.7	1	D
17	MC	3.8	F.IF.B.5-2	1	B
18	CR	3.8	F.IF.C.7a, F.BF.3-2	2	—
19	MC	3.8	A.APR.B.3	2	C
20	MC	3.9	A.REI.B.4b	2	B
21	CR	3.9	F.IF.C.7a	3	—
22	MC	3.13	F.FI.C.7A	1	B
23	CR	3.14	F.IF.C.8a	2	—
24	MC	3.14	F.IF.C.9-2	2	C
25	MC	3.14	F.IF.C.7a	2	B
26	CR	3.14	FIF.C.7a	3	—
27	MC	4.2	A.APR.C.5	1	D
28	MC	4.2	A.APR.C.4	2	A
29	MC	4.3	A.APR.B.3	1	C
30	MC	4.3	A.APR.B.3	1	C
31	MC	4.4	A.APR.B.2	1	D
32	MC	4.4	A.APR.B.2	2	A
33	MC	4.4	A.APR.D.6	2	B
34	CR	4.6	F.IF.B.4-2	1	—
35	MC	4.6	F.IF.C.7	1	C
36	CR	4.7	A.APR.B.3	3	—
37	MC	4.8	N.CN.C.9	1	A
38	MC	4.9	A.APR.B.3	1	A
39	MC	4.9	A.APR.B.3	1	C
40	MC	4.9	A.APR.B.2	1	D
41	MC	4.9	A.APR.B.2	2	A
42	CR	4.9	A.APR.B.2, A.APR.B.3, F.BF.B.3-2	2	—
43	CR	4.9	A.APR.B.2, A.APR.B.3	2	—
44	MC	4.9	A.APR.B.2	2	D
45	MC	4.9	A.APR.B.3	1	C
46	CR	4.9	A.APR.B.3	2	—
47	CR	4.11	A.APR.B.2, A.APR.B.3, F.BF.B.3-2	3	—
48	CR	4.11	A.APR.B.2, A.APR.B.3, F.BF.B.3-2	3	—
49	CR	4.11	A.APR.B.3, F.BF.B.3-2	3	—

SELECTED RESPONSE

#	Question Type	Learning Target	Nevada Academic Content Standard(s)	DOK	Key
50	CR	4.12	A.APR.B.3, F.IF.7e	3	—
51	MC	5.2	A.REI.A.2	1	C
52	MC	5.3	F.IF.C.7b-2	2	A
53	MC	5.3	F.IF.C.7b-2, A.REI.A.2	2	A
54	MC	5.4	F.IF.C.7b-2	2	C
55	MC	5.5	A.REI.A.2	1	C
56	MC	5.5	A.REI.A.2	1	C
57	MC	5.5	A.REI.A.2	1	C
58	MC	5.5	A.REI.A.2	1	D
59	MC	5.5	A.REI.A.2	2	D
60	MC	5.6	F.BF.B.4a-2	1	C

CCSD CLARK COUNTY SCHOOL DISTRICT

CONSTRUCTED RESPONSE

7. This question assesses the student's understanding of a quadratic function written in vertex form.

$y = a(x-h)^2 + k$ where the vertex has the coordinates $V(h,k)$

a) The leading coefficient is $a = -1$, so the parabola opens downward and $V(6,99)$
   The student must spend $t = 6$ hours to achieve the maximum score.
b) The maximum score is 99 because $s(6) = 99$.
c) If the student does no homework, $t = 0$, the score would be $s(0) = -36 + 99 = 63$.

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8. There are three methods: using the quadratic formula, completing the square or by substituting $(-3+i)$ in the quadratic equation.

Completing the square method:

$x^2 + 6x + 10 = 0$

$x^2 + 6x = -10$

$x^2 + 6x + 9 = -10 + 9$

$(x+3)^2 = -1$

$(x+3)^2 = i^2$

$so\,(x+3) = i$

$or\,(x+3) = -i$

So $x = -3 + i$ is a root of the quadratic equation.

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18. Consider the function $f(x) = x^2 - 2x - 48$.

a) Determine the roots of the function. Show your work.

   Rewrite the quadratic function in factored form $f(x) = (x-8)(x+6)$. Setting each factor equal to zero, the roots are 8 and -6.

b) The vertex of $g(x)$ is $(3,30)$. Write the function rule for g in vertex form.

   $g(x) = a(x-3)^2 + 30$ because $g(x) = a(x-h)^2 + k$ where (h,k) is the vertex.

c) Explain how $f(x)$ transformed to become $g(x)$.

   In vertex form, $f(x) = (x-1)^2 - 49$ (by completing the square). There has been a horizontal shift 2 units to the right and vertical shift 79 units up.

21. This question assesses the student's ability to determine the number of solutions to a quadratic equation without solving the equation.

a) If b = 0 there could be 0,1, or 2 solutions.

b) If c ≤ 0 there are 2 real solutions.

c) If c > 0 there could be 0, 1, or 2 solutions.

d) In order for the solution to have imaginary parts $b^2 - 4ac < 0$.

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23. The height of Carl, the human cannonball, is given by $h(t) = -16t^2 + 56t + 40$ where *h* is in feet and *t* is in seconds after the launch.

a) What was his height at the launch?

At the launch, $t = 0$, so $h(0) = -16 \cdot 0^2 + 56 \cdot 0 + 40$, the height is 40 feet.

b) What is his maximum height?

The maximum height is the y coordinate of the vertex, y_v.

$$x_v = \frac{-b}{2a} = \frac{-56}{2 \cdot (-16)} = \frac{7}{4} = 1.75$$
$$y_v = h(1.75) = -16 \cdot 1.75^2 + 56 \cdot 1.75 + 40 = 89$$

c) How long before he lands in the safety net, 8 feet above the ground?

The net is 8 feet above the ground so $h(t) = 8$

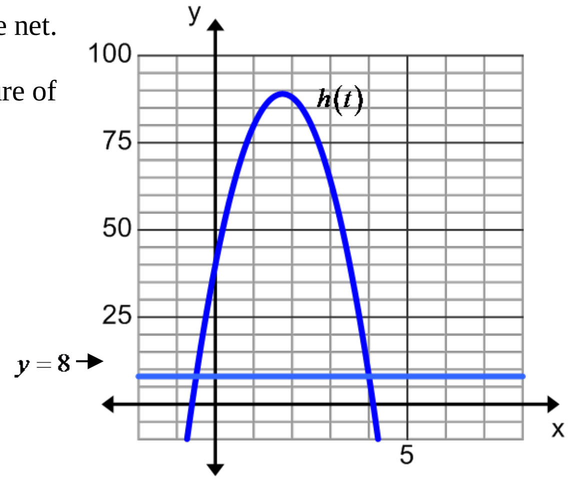
$$-16t^2 + 56t + 40 = 8$$
$$-16t^2 + 56t + 32 = 0$$

Factor out -8: $-8(2t^2 - 7t - 4) = 0$

Factor completely: $-8(2t+1)(t-4) = 0$ This quadratic equation has one positive ($t = 4$) and one negative ($t = -\frac{1}{2}$) root, but since *t* represents time we will consider only the positive root $t = 4$.

Therefore it will take 4 seconds before it lands in the net.

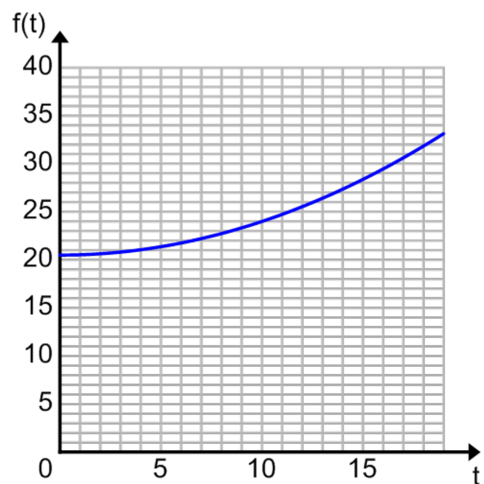
This could also be found using the intersection feature of a graphing calculator. Graph $h(t)$ and y = 8.



26. The amount of fuel F (in billions of gallons) used by trucks from 1990 through 2009 can be approximated by the function $F = f(t) = 20.5 + 0.035t^2$ where $t = 0$ represents 1990.

a) Describe the transformation of the common function $f(t) = t^2$. Then sketch the graph over the interval $0 \le t \le 19$.

Vertical shrink by a factor of 0.035 and a vertical shift of 20.5 units up.



b) Find and interpret $\frac{f(19) - f(0)}{19 - 0}$.

$$\frac{f(19) - f(0)}{19 - 0} = \frac{33.135 - 20.5}{19} = \frac{12.635}{19} = 0.665$$

On average 0.665 billion(665 million) of gallons of fuel is used per year by trucks from 1990 to 2009.

c) Rewrite the function so that $t = 0$ represents 2000. Explain how you got your answer.
Move $f(t)$ ten units to the left. Let's call the new function $w(t)$. $w(t) = f(t+10) = 0.035(t+10)^2 + 20.5$
In f(t), the year 2000 is represented by t=10.

If you want the year 2000 to be located at t=0, f(t) has to be moved horizontally 10 units to the left.

d) Use the model from part (c) to predict the amount of fuel used by trucks in 2015. Does your answer seem reasonable? Explain.
$w(15) = 0.035(15+10)^2 + 20.5 = 42.375$
So, 42.375 billions of gallons of fuel will be used in the year 2015. This is reasonable because fuel consumption is increasing.

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34. This problem assesses the student's ability to analyze a graph without the equation.
a) True: By applying the Leading Coefficient Test, students could determine that the leading coefficient is positive
because both ends of the graph rise.

b) False: because the graph goes through the point (-3, 0), the multiplicity is odd.

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36. Use the information in the table.

Interval	Value of f(x)
$(-\infty, -2)$	Negative
$(-2, 1)$	Positive
$(1, 4)$	Negative
$(4, \infty)$	Positive

a) What are the three real zeros of the polynomial function *f*?
The three real zeros of the polynomial function $f(x)$ are $(-2,0)(1,0)(4,0)$

b) What can be said about the behavior of the graph of *f* at $x = 0$?
$f(0) > 0$

c) What is the least possible degree of *f*? Explain. Can the degree of *f* ever be even? Explain.
The least possible degree of the polynomial is three. If the curve is tangent to the *x*-axis the multiplicity for that zero would be 2, so since the curve crosses the *x*-axis three times (because it changes sign 4 times), the only degree for the polynomial would be an odd number.

42. $p(x)=3x^5+13x^4+19x^3+17x^2+16x+4$

a) Show that $p(-2)$ is a root.

You could substitute $x=-2$, if $p(-2)$ = 0, then it is a root.

Based on the Remainder Theorem you could use synthetic division to show that you get a remainder of zero.

-2	3	13	19	17	16	4
		-6	-14	-10	-14	-4
	3	7	5	7	2	0

b) Factor $p(x)$ completely.

With the result from part a, that -2 is a root, we know that $(x+2)$ is a factor. Using the Possible Rational Zero Theorem, list all the possible rational zeros for $3x^4+7x^3+5x^2+7x+2$ which is the remainder after synthetically dividing $p(x)$ by -2. Possible rational zeros are:

$$\frac{\pm 1 \quad \pm 2}{\pm 1 \quad \pm 3}=\pm 1,\pm\frac{1}{3}\pm 2,\pm\frac{2}{3}$$

Synthetic division show that -2 is a zero, so the multiplicity for -2 becomes 2.

-2	3	7	5	7	2
		-6	-2	-6	-2
	3	1	3	1	0

So $p(x)=(x+2)^2(3x^3+x^2+3x+1)$

We can now factor $3x^3+x^2+3x+1$ by grouping:

$3x^3+x^2+3x+1$

$x^2(3x+1)+(3x+1)$

$(3x+1)(x^2+1)$

Finally, the complete factorization is $p(x)=(x+2)^2(3x+1)(x^2+1)$

c) If $f(x)=p(x-3)$, what are the real roots of $f(x)$?

$f(x)=p(x-3)=(x-3+2)^2\left[3(x-3)+1\right]\left[(x-3)^2+1\right]=(x-1)^2(3x-8)\left[(x-3)^2+1\right]$ so the real roots of $f(x)$ are 1 and $\frac{8}{3}$.

43. Given the polynomial $p(x)=x^4+3x^3+12x-16$

a) Show that $p(2i)$ is a root.

$p(2i)=(2i)^4+3(2i)^3+12(2i)-16=16-24i+24i-16=0$,so $2i$ is a zero of the polynomial.

b) What other root must also be a root of $p(x)$? Explain.

If a polynomial with real coefficients has a complex zero, then its complex conjugate will be a zero. Therefore, since 2*i* was a zero, its conjugate, $-2i$, will also be a zero.
Important: Complex conjugates are generally not negatives of each other. For the general complex number a+b*i* the complex conjugate is a-b*i*. In this problem the real part, i.e. the "a", of 2*i* is zero. In other words, 2i, written as complex number with real and imaginary parts is 0 + 2*i*. Its conjugate is $0-2i$ or just $-2i$. In this case the conjugates turn out to be negatives of each other, 2*i* and -2*i*. Only when the "a" is zero does the conjugate work out to be the negative of the original number!
Since -2i and 2i are zeros we can divide by $(x-(-2i))(x-2i)=(x^2+4)$ knowing that it will divide evenly.

$$
\begin{array}{r|l}
 & x^2+3x-4 \\
x^2+4 & x^4+3x^3 \qquad +12x-16 \\
 & -x^4 \qquad -4x^2 \\
 & +3x^3-4x^2+12x-16 \\
 & -3x^3 \qquad -12x \\
 & -4x^2 \qquad -16 \\
 & +4x^2 \qquad +16 \\
 & 0
\end{array}
$$

c) Factor $p(x)$ completely.

After step (b), $p(x)=(x-2i)(x+2i)(x^2+3x-4)$. x^2+3x-4 will factor into $(x+4)(x-1)$ so the complete factorization of $p(x)$ is: $p(x)=(x-2i)(x+2i)(x+4)(x-1)$.

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46. This polynomial function has at least one rational root.
$p(x)=x^4+kx^2+9$

a) What are all the possible integer values of *k*? Show your work or explain how you know.
Based on the Rational Zero Theorem we can list the possible rational zeros:

$\frac{\pm1 \quad \pm3 \quad \pm9}{\pm1}=\pm1,\ \pm3,\ \pm9$

$p(\pm1)=0 \Leftrightarrow 10+k=0 \Leftrightarrow k=-10$

$p(\pm3)=0 \Leftrightarrow 90+9k=0 \Leftrightarrow k=-10$

$p(\pm9)=0 \Leftrightarrow 6570+9k=0 \Leftrightarrow k=-81.111...$

The last value of *k* does not work because it is not an integer, so $k=-10$.

b) What are all the possible real roots of the function? Show your work or explain how you know.

$p(x) = x^4 - 10x^2 + 9 = \left(x^2 - 9\right)\left(x^2 - 1\right) = (x+3)(x-3)(x+1)(x-1)$, so the real roots of the function are $\pm 3, \pm 1$.

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47. Consider $p(x) = 2x^4 - x^3 - 11x^2 + 5x + 5$.

a) Show that $x = \sqrt{5}$ and $x = -\sqrt{5}$ are zeros of *p(x).*

Let's define what a root or zero of a polynomial is. We say that $x = a$ is a root or zero of a polynomial *p(x)* if *p(a)* = 0.

$$p\left(\sqrt{5}\right) = 2\left(\sqrt{5}\right)^4 - \left(\sqrt{5}\right)^3 - 11\left(\sqrt{5}\right)^2 + 5\left(\sqrt{5}\right) + 5$$
$$p\left(\sqrt{5}\right) = 50 - 5\sqrt{5} - 55 + 5\sqrt{5} + 5$$
$$p\left(\sqrt{5}\right) = 0$$

$$p\left(-\sqrt{5}\right) = 2\left(-\sqrt{5}\right)^4 - \left(-\sqrt{5}\right)^3 - 11\left(-\sqrt{5}\right)^2 + 5\left(-\sqrt{5}\right) + 5$$
$$p\left(-\sqrt{5}\right) = 50 + 5\sqrt{5} - 55 - 5\sqrt{5} + 5$$
$$p\left(-\sqrt{5}\right) = 0$$

b) Completely factor *p(x)* where all the coefficients are rational numbers.

The point of the Factor Theorem is the reverse of the Remainder Theorem: If you synthetically divide a polynomial by $x = a$ and get a zero remainder, then not only is $x = a$ a zero of the polynomial (courtesy of the Remainder Theorem), but $x - a$ is also a factor of the polynomial (courtesy of the Factor Theorem). So both $\left(x - \sqrt{5}\right)$ and $\left(x + \sqrt{5}\right)$ are factors of *p(x),* which implies that $\left(x^2 - 5\right)$ is a factor of *p(x).* Long division yields a quotient of $\left(2x^2 - x - 1\right)$. The complete factorization of *p(x)* is $p(x) = \left(x - \sqrt{5}\right)\left(x + \sqrt{5}\right)(2x+1)(x-1)$.

c) $h(x)$ is $p(x)$ translated 4 units right and 2 units up. What is the equation of $h(x)$?

$$h(x) = p(x-4) + 2 = \left(x - 4 - \sqrt{5}\right)\left(x - 4 + \sqrt{5}\right)(2x-7)(x-5) + 2$$

48. Consider $p(x) = x^4 - 2.5x^3 - 7.5x^2 + 15x + 9$

a) Show that $x = \pm\sqrt{6}$ are roots of $p(x)$, then write $p(x)$ as the appropriate factorizations at this point.

$$p(\sqrt{6}) = \sqrt{\left(\sqrt{6}\right)^4} - 2.5\left(\sqrt{6}\right)^3 - 7.5\left(\sqrt{6}\right)^2 + 15\left(\sqrt{6}\right) + 9 =$$
$$= 36 - 15\sqrt{6} - 45 + 15\sqrt{6} + 9 = 0$$
$$p(-\sqrt{6}) = \sqrt{\left(-\sqrt{6}\right)^4} - 2.5\left(-\sqrt{6}\right)^3 - 7.5\left(-\sqrt{6}\right)^2 + 15\left(-\sqrt{6}\right) + 9 =$$
$$= 36 + 15\sqrt{6} - 45 - 15\sqrt{6} + 9 = 0$$

Since both $\sqrt{6}$ *and* $-\sqrt{6}$ are zeros, $\left(x - \sqrt{6}\right)$ *and* $\left(x + \sqrt{6}\right)$ are factors of the given polynomial so $\left(x^2 - 6\right)$ is a factor. Let's use long division to find the other factor.

$$\begin{array}{r|l} & x^2 - 2.5x - 1.5 \\ x^2 - 6 & x^4 - 2.5x^3 - 7.5x^2 + 15x + 9 \\ & -x^4 \qquad\qquad +6x^2 \\ & -2.5x^3 - 1.5x^2 + 15x + 9 \\ & +2.5x^3 \qquad\qquad -15x \\ & -1.5x^2 \qquad +9 \\ & +1.5x^2 \qquad -9 \\ & 0 \end{array}$$

So at this point the factorization is $p(x) = \left(x + \sqrt{6}\right)\left(x - \sqrt{6}\right)\left(x^2 - 2.5x - 1.5\right)$

b) Factor $p(x)$ completely.

$p(x) = \left(x + \sqrt{6}\right)\left(x - \sqrt{6}\right)\left(x^2 - 2.5x - 1.5\right)$; $\left(x^2 - 2.5x - 1.5\right)$ will factor into $(x-3)(x+0.5)$. So $p(x)$ factored completely: $p(x) = \left(x - \sqrt{6}\right)\left(x + \sqrt{6}\right)(x-3)(x+0.5)$

c) Let $q(x) = p(4x)$. List out the roots of $q(x)$.

$q(x) = p(4x) = \left(4x - \sqrt{6}\right)\left(4x + \sqrt{6}\right)(4x-3)(4x+0.5)$

So the roots of $q(x)$ are $\frac{\sqrt{6}}{4}, -\frac{\sqrt{6}}{4}, \frac{3}{4}, -\frac{1}{8}$

d) Let $f(x)$ be $p(x)$ vertically stretched by 2, translated 2 units to the right and 4 units up. Wrtie out the algebraic relationship between $f(x)$ and $p(x)$.

$f(x) = 2p(x-2) + 4$

49. Consider the function $f(x)=3x^3-9x^2-3x+9$.

a) Use the leading coefficient and degree of $f(x)$ to describe the end behavior.

The leading term is $3x^3$ so as x increases without bound $f(x)$ increases without bound and as x decreases without bound $f(x)$ decreases without bound.

$\lim\limits_{x\to\infty} f(x)\to\infty$ and $\lim\limits_{x\to-\infty} f(x)\to-\infty$

b) Write the rule for the function $g(x)=f(-x)$, and describe the transformation.

$g(x)=f(-x)=3(-x)^3-9(-x)^2-3(-x)+9=-3x^3-9x^2+3x+9$

The transformation here is a reflection with respect to the *y*-axis since the input values were replaced with their opposites.

c) Describe the end behavior of $g(x)$. How does the end behavior of $g(x)$ relate to the transformation of $f(x)$?

Since $g(x)$ is a reflection of $f(x)$, as x increases without bound, $g(x)$ decreases without bound and as x decreases without bound, $g(x)$ increases without bound.

$\lim\limits_{x\to\infty} g(x)\to-\infty$ and $\lim\limits_{x\to-\infty} g(x)\to\infty$

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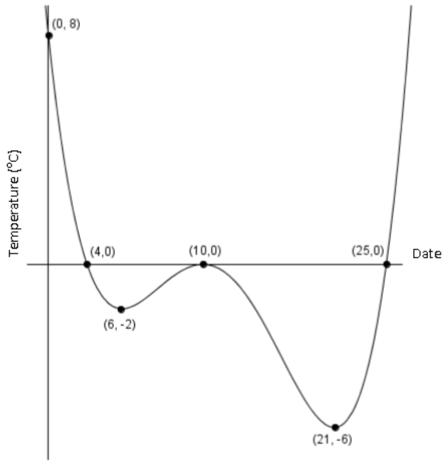
50. The town of Frostburg experienced a bit of a heat wave during January of this year. The graph below shows the curve of best fit that represents the low temperature of every day in January.

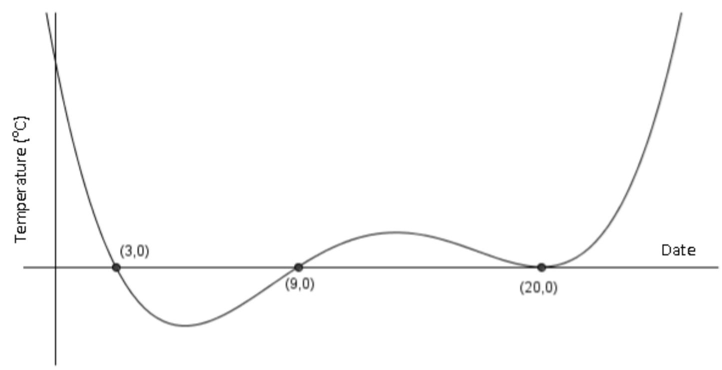


A newspaper journalist is writing a story on the weather and needs to report some information. He needs a bit of guidance with interpreting the graph.

1) Write a few sentences describing the key characteristics of the graphs as it relates to the context of the problem. Be sure to include domain, range, intervals where the function increases and decreases, *x* and *y* intercepts, and any other important information

There aren't many temperature changes in the month of January. First day of January brings us a temperature of 8 degrees but it will stay above zero for four days only. Starting with January 4th the temperature will drop below 0 and will continue in the negative zone for another three days until January 24th.The lowest of the month will be on the 21st when it will be -6 degrees. For the first 6 days of the month the temperature is decreasing, reaching -2 degrees on January 6th, then it will rise for the next 4 days, reaching 0 degrees on January 10th, then will drop again until the 21st when is going to reach -6 degrees the lowest of the month. After this date the temperature will only increase and after the 25th we will experience only positive values.

The graph below shows the curve of best fit that represents the low temperature of every day in February.



2) Three different models have been proposed that could be used to determine the temperature for a particular date in February. The models are given below:

Model 1: $y = ax^2 + bx + c$

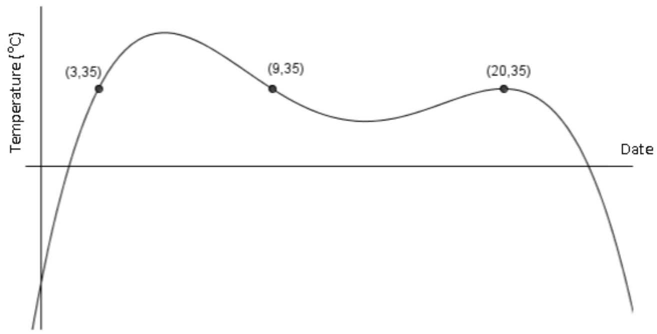
Model 2: $y = a(x-3)(x-9)(x-20)$

Model 3: $y = a(x+3)(x+9)(x+20)^2$

Which model would best describe the low temperatures for February? Explain why you chose that model.

Model 2 because at $x = 20$ the graph is tangent to the *x*-axis so the multiplicity of the zero must be an even number. At $x = 3$ **and** $x = 9$ the graph crosses the *x*-axis so their multiplicity is an odd number.

The weather in July showed a related pattern to the weather in February. The curve of best fit for July is shown below:



3) Explain the relationship between the graph for February and the graph for July. Use that relationship to create an equation for the temperatures in July.

Let's name the function that describes the temperature in February *f*(*x*) and the one in July *g*(*x*). Looking at the two graphs, *g*(*x*) is a reflection of *f*(*x*) with respect to the *x*-axis, followed by a vertical shift 35 up, so the equation that connects the two functions is: $g(x) = -f(x) + 35$

Since $f(x) = a(x-3)(x-9)(x-20)^2$ then $g(x) = -a(x-3)(x-9)(x-20)^2 + 35$