Lesson 21: An Exercise in Changing Scales

Student Outcomes

- Given a scale drawing, students produce a scale drawing of a different scale.
- Students recognize that the scale drawing of a different scale is a scale drawing of the original scale drawing.
- For the scale drawing of a different scale, students compute the scale factor for the original scale drawing.

Classwork

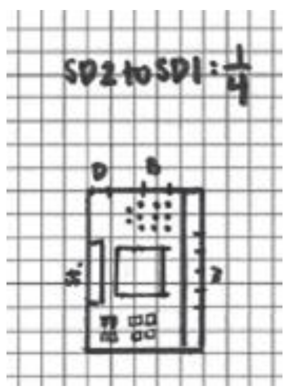
How does your scale drawing change when a new scale factor is presented?

Exploratory Challenge (20 minutes): A New Scale Factor

MP.1 & MP.2

Exploratory Challenge: A New Scale Factor

The school plans to publish your work on the dream classroom in the next newsletter. Unfortunately, in order to fit the drawing on the page in the magazine, it must be $\frac{1}{4}$ its current length. Create a new drawing ($SD2$) in which all of the lengths are $\frac{1}{4}$ those in the original scale drawing ($SD1$) from Lesson 20.



An example is included for students unable to create $SD1$ at the end of Lesson 20. Pose the following questions:

- Would the new scale create a larger or smaller scale drawing as compared to the original drawing?
  - *It would be smaller because $\frac{1}{4}$ is smaller than one.*
- How would you use the scale factor between $SD1$ to $SD2$ to calculate the new scale drawing lengths without having to get the actual measurement first?
  - *Take the original scale drawing lengths and multiply them by $\frac{1}{4}$ to find the new scale lengths.*

Once the students have finished creating $SD2$, ask students to prove to the architect that $SD2$ is actually a scale drawing of the original room.

- How can we go about proving that the new scale drawing ($SD2$) is actually a scale drawing of the original room?
  - *The scale lengths of $SD2$ have to be proportional to the actual lengths. We need to find the constant of proportionality, the scale factor.*
- How do we find the new scale factor?
  - *Divide one of the new scale lengths by its corresponding actual length.*
- If the actual measurement was not known, how could we find it?
  - *Calculate the actual length by using the scale factor on the original drawing. Multiply the scale length of the original drawing by the original scale factor.*

Exercise (20 minutes)

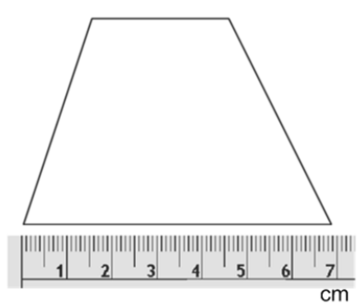
Write different scale factors on cards, which the students will choose: $\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, \frac{3}{2}, 2, 3, 4$. They will then create a new scale drawing and calculate the scale factor between their drawing and the original trapezoid in the student material.

After completing parts (a)–(c) independently, have all of the students who were working with enlargements move to the right side of the room and those with reductions to the left. Have students first discuss in smaller groups on their side of the room and then come together as a class to discuss the following:

- Compare your answers to part (a). What can you conclude?
  - *All of the enlargements had a scale factor that was greater than 1. The reductions have a scale factor between zero and 1.*
- What methods did you use to answer part (c)?
  - *The scale factor between $SD2$ (student drawn trapezoid) and the original figure can be determined by multiplying the scale factor of $SD1$ (scale drawing given in the materials) to the original figure by the scale factor of $SD2$ to $SD1$.*

Exercise

The picture shows an enlargement or reduction of a scale drawing of a trapezoid.



Using the scale factor written on the card you chose, draw your new scale drawing with correctly calculated measurements.

Answers may vary depending on the card. One sample response could be, 1 cm, $1\frac{5}{6}$ cm, $2\frac{1}{3}$ cm, $1\frac{2}{3}$ cm.

a. What is the scale factor between the original scale drawing and the one you drew?

$\frac{1}{3}$

b. The longest base length of the actual trapezoid is 10 cm. What is the scale factor between the original scale drawing and the actual trapezoid?

$\frac{7}{10}$

c. What is the scale factor between the new scale drawing you drew and the actual trapezoid?

$\frac{2\frac{1}{3}}{10} = \frac{\frac{7}{3}}{10} = \frac{7}{3} \times \frac{1}{10} = \frac{7}{30}$

Changing Scale Factors:

- To produce a scale drawing at a different scale, you must determine the new scale factor. The new scale factor is found by dividing the different (new drawing) scale factor by the original scale factor.
- To find each new length, you can multiply each length in the original scale drawing by this new scale factor.

Steps:

- Find each scale factor.
- Divide the new scale factor by the original scale factor.
- Divide the given length by the new scale factor (the quotient from the prior step).

Closing (5 minutes)

- Why might you want to produce a scale drawing of a different scale?
  - *To produce multiple formats of a drawing (e.g., different sized papers for a blueprint)*
- How do you produce another scale drawing given the original scale drawing and a different scale?
  - *Take the lengths of the original scale drawing and multiply by the different scale. Measure and draw out the new scale drawing.*
- How can you tell if a new scale drawing is a scale drawing of the original figure?
  - *If the new scale drawing ($SD2$) is a scale drawing of $SD1$, then it is a scale drawing of the original figure with a different scale.*
- How can the scale factor of the new drawing to the original figure be determined?
  - *Take the scale length of the new scale drawing and divide it by the actual length of the original figure.*

Lesson Summary

Variations of Scale Drawings with different scale factors are scale drawings of an original scale drawing.

From a scale drawing at a different scale, the scale factor for the original scale drawing can be computed without information of the actual object, figure, or picture.

- **For example, if *scale drawing one* has a scale factor of $\frac{1}{24}$ and *scale drawing two* has a scale factor of $\frac{1}{72}$, then the scale factor relating *scale drawing two* to *scale drawing one* is**

$$\frac{1}{72} \text{ to } \frac{1}{24} = \frac{\frac{1}{72}}{\frac{1}{24}} = \frac{1}{72} \cdot \frac{24}{1} = \frac{1}{3}.$$

Scale drawing two* has lengths that are $\frac{1}{3}$ the size of the lengths of *scale drawing one.

Problem Set Sample Solutions

1. **Jake reads the following problem: If the original scale factor for a scale drawing of a square swimming pool is $\frac{1}{90}$, and the length of the original drawing measured to be 8 inches, what is the length on the new scale drawing if the scale factor of the new scale drawing length to actual length is $\frac{1}{144}$?**

   He works out the problem:

   $8 \div \frac{1}{90} = 720$ **inches.**

   $720 \times \frac{1}{144} = 5$ **inches.**

   Is he correct? Explain why or why not.

   Jake is correct. He took the original scale drawing length and divided by the original scale factor to get the actual length, 720 inches. To get the new scale drawing length, he takes the actual length, 720, and multiplies by the new scale factor, $\frac{1}{144}$, to get 5 inches.

2. **What is the scale factor of the new scale drawing to the original scale drawing ($SD2$ to $SD1$)?**

   $$\frac{\frac{1}{144}}{\frac{1}{90}} = \frac{5}{8}$$

3. **Using the scale, if the length of the pool measures 10 cm on the new scale drawing:**

   a. **Using the scale factor from Problem 1, $\frac{1}{144}$, find the actual length of the pool in meters?**

      $14.40\ m$

   b. **What is the surface area of the floor of the actual pool? Rounded to the nearest tenth.**

      $14.4\ m \times 14.4\ m$

      $207.36\ m^2 = 207.4\ m^2$

c. **If the pool has a constant depth of 1.5 meters, what is the volume of the pool? Rounded to the nearest tenth**

$14.4\ m \times 14.4\ m \times 1.5\ m$

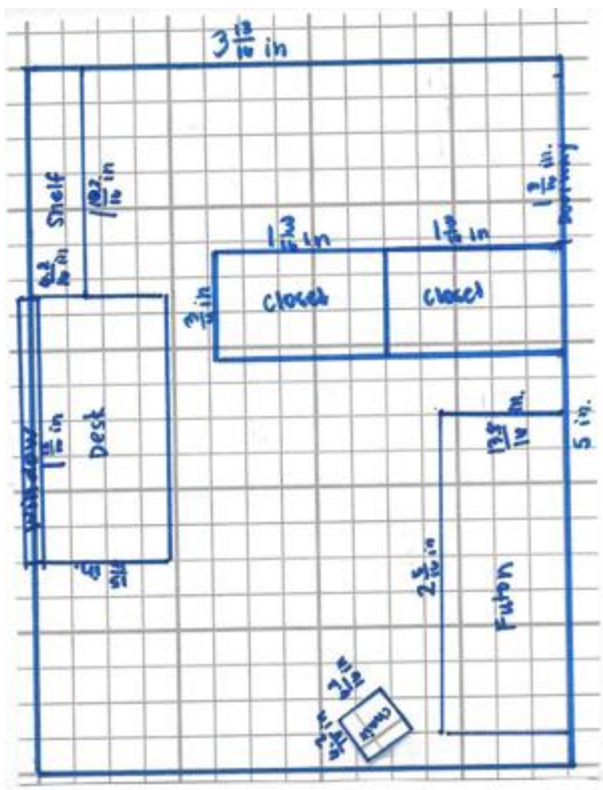
$311.04\ m^3 = 311.0\ m^3$

d. **If 1 cubic meter of water is equal to 264.2 gallons, how much water will the pool contain when completely filled? Rounded to the nearest unit.**

$82,176.768 = 82,177$ *gallons.*

4. **Complete a new scale drawing of your dream room from the Problem Set in Lesson 20 by either reducing by $\frac{1}{4}$ or enlarging it by 4.**

Scale drawings will vary.



Equivalent Fraction Computations

$\frac{1}{13} \approx \frac{1}{16}$	$\frac{2}{13} \approx \frac{3}{16}$	$\frac{3}{13} \approx \frac{4}{16}$	$\frac{4}{13} \approx \frac{5}{16}$	$\frac{5}{13} \approx \frac{6}{16}$	$\frac{6}{13} \approx \frac{7}{16}$	$\frac{7}{13} \approx \frac{9}{16}$	$\frac{8}{13} \approx \frac{10}{16}$	$\frac{9}{13} \approx \frac{11}{16}$	$\frac{10}{13} \approx \frac{12}{16}$	$\frac{11}{13} \approx \frac{14}{16}$	$\frac{12}{13} \approx \frac{15}{16}$
$16 \times \frac{1}{13} = \frac{16}{13} \approx 1.2$	$16 \times \frac{2}{13} = \frac{32}{13} \approx 2.5$	$16 \times \frac{3}{13} = \frac{48}{13} \approx 3.7$	$16 \times \frac{4}{13} = \frac{64}{13} \approx 4.9$	$16 \times \frac{5}{13} = \frac{80}{13} \approx 6.2$	$16 \times \frac{6}{13} = \frac{96}{13} \approx 7.4$	$16 \times \frac{7}{13} = \frac{112}{13} \approx 8.6$	$16 \times \frac{8}{13} = \frac{128}{13} \approx 9.8$	$16 \times \frac{9}{13} = \frac{144}{13} \approx 11.0$	$16 \times \frac{10}{13} = \frac{160}{13} \approx 12.3$	$16 \times \frac{11}{13} = \frac{176}{13} \approx 13.5$	$16 \times \frac{12}{13} = \frac{192}{13} \approx 14.8$

Conversions (inches)

	Entire Room	Windows	Doors	Desk	Seating	Storage	Bed	Shelf	Side Table
Scale Drawing Length	*See above*	$2\frac{1}{2} \times \frac{10}{13} = \frac{5}{2} \times \frac{10}{13} = \frac{50}{26} = 1\frac{12}{13} \approx 1\frac{15}{16}$	$1\frac{1}{2} \times \frac{10}{13} = \frac{3}{2} \times \frac{10}{13} = \frac{15}{13} = 1\frac{2}{13} \approx 1\frac{3}{16}$	*Same as window measurement* $\approx 1\frac{15}{16}$	$\frac{1}{2} \times \frac{10}{13} = \frac{10}{26} = \frac{5}{13} \approx \frac{6}{16}$	*Same as door measurement* $\approx 1\frac{3}{16}$	$3 \times \frac{10}{13} = \frac{30}{13} = 2\frac{4}{13} \approx 2\frac{5}{16}$	$2\frac{1}{8} \times \frac{10}{13} = \frac{17}{8} \times \frac{10}{13} = \frac{170}{104} = 1\frac{66}{104} \approx 1\frac{10}{16}$	$\frac{3}{4} \times \frac{10}{13} = \frac{30}{52} = \frac{15}{26} \approx \frac{9}{16}$
Scale Drawing Width	$5 \times \frac{10}{13} = \frac{50}{13} = 3\frac{11}{13} \approx 3\frac{14}{16}$	/	/	$1\frac{1}{4} \times \frac{10}{13} = \frac{5}{4} \times \frac{10}{13} = \frac{50}{52} = \frac{25}{26} \approx \frac{15}{16}$	*See above* $\approx \frac{6}{16}$	$1 \times \frac{10}{13} = \frac{10}{13} \approx \frac{12}{16}$	$1\frac{1}{8} \times \frac{10}{13} = \frac{9}{8} \times \frac{10}{13} = \frac{90}{104} \approx 14.16$	*See seating measurement* $\approx \frac{6}{16}$	*See above* $\approx \frac{6}{16}$

SD1 Example for students who were unable to create their own from Lesson 20

SCALE FACTOR: $\frac{1}{120}$