📝 # Lesson 17: The Unit Rate as the Scale Factor

Student Outcomes

- Students recognize that the enlarged or reduced distances in a scale drawing are proportional to the corresponding distances in the original picture.
- Students recognize the scale factor to be the constant of proportionality.
- Given a picture or description of geometric figures, students make a scale drawing with a given scale factor.

Classwork

Example 1 (7 minutes): Jake's Icon

After reading the prompt with the class, discuss the following questions:

- What type of scale drawing is the sticker?
  - *It is an enlargement or a magnification of the original sketch.*
- What is the importance of proportionality for Jake?
  - *If the image is not proportional, it looks less professional. The image on the sticker will be distorted.*
- How could we go about checking for proportionality of these two images? (Have students record steps in their student materials.)
  - *Measure corresponding lengths and check to see if they all have the same constant of proportionality.*

As a class, label points correspondingly on the original sketch, and then on the sticker sketch. Use inches to measure the distance between the points and record on a table.

> *Scaffolding:*
> - Give the measurements of the original image lengths for the table prior to beginning Example 1.
> - Challenge students by giving problems that use different units of measurement and have them compare the scale factors.

Example 1: Jake's Icon

Jake created a simple game on his computer and shared it with his friends to play. They were instantly hooked, and the popularity of his game spread so quickly that Jake wanted to create a distinctive icon so that players could easily identify his game. He drew a simple sketch. From the sketch, he created stickers to promote his game, but Jake wasn't quite sure if the stickers were proportional to his original sketch.

Original Sketch:

Sticker:

Original	Sticker
1 *in.*	2 *in.*
$\frac{3}{4}$ *in.*	$1\frac{1}{2}$ *in.*
1 *in.*	2 *in.*
$\frac{7}{8}$ *in.*	$1\frac{3}{4}$ *in.*

> **Steps to check for proportionality for scale drawing and original object or picture:**
>
> 1. *Measure the lengths of scale drawing. Record the data on the table.*
> 2. *Measure the corresponding lengths on the actual object or picture. Record the data on the table.*
> 3. *Check for the constant of proportionality.*

> **Key Idea:**
>
> The <u>scale factor</u> can be calculated from the ratio of any length in the scale drawing to its corresponding length in the actual picture. The scale factor corresponds to the unit rate and the constant of proportionality.
>
> Scaling by factors *greater than* 1 enlarge the segment, and scaling by factors *less than* 1, reduce the segment.

- What relationship do you see between the measurements?
  - *The corresponding lengths are proportional.*
- Is the sticker proportional to the original sketch?
  - *Yes, the sticker lengths are twice as long as the lengths in the original sketch.*
- How do you know?
  - *The unit rate,* 2*, is the same for the corresponding measurements.*
- What is this called?
  - *Constant of proportionality*

Introduce the term *scale factor* and review the key idea box with students.

- Is the new figure larger or smaller than the original?
  - *Larger.*
- What is the scale factor for the sticker? How do you know?
  - *The scale factor is two because the scale factor is the same as the constant of proportionality. It is the ratio of a length in the scale drawing to the corresponding length in the actual picture, which is* 2 *to* 1*. The enlargement is represented by a number greater than* 1*.*
- Each of the corresponding lengths is how many times larger?
  - *Two times*
- What can you predict about an image that has a scale factor of 3?
  - *The lengths of the scaled image will be three times as long as the lengths of the original image.*

Exercise 1 (5 minutes): App Icon

Give students time to measure the lengths (in inches) of the app icon that correspond to the lengths measured in Example 1, and record the data on tables with partners.

- What was the relationship between the sticker and the original sketch?
  - *The sticker is larger than the original.*
- What was the constant of proportionality, or scale factor, for this relationship?
  - *2*

- What is the relationship between the icon and the original sketch?
  - *The icon is smaller than the original sketch.*
- What was the constant of proportionality, or scale factor, for this relationship?
  - *$\frac{1}{2}$*
- How do we determine the scale factor?
  - *Measure the lengths of the app icon and the corresponding lengths of the original sketch and record the data. Using the data, determine the constant of proportionality.*
- What does the scale factor indicate?
  - *A scale factor less than* 1 *indicates a reduction from the original picture, and a scale factor greater than* 1 *indicates a magnification or enlargement from the original picture.*

Exercise 1: App Icon
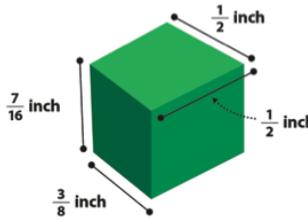
Original	App Icon
1 *in.*	$\frac{1}{2}$ *in.*
$\frac{3}{4}$ *in.*	$\frac{3}{8}$ *in.*
1 *in.*	$\frac{1}{2}$ *in.*
$\frac{7}{8}$ *in.*	$\frac{7}{16}$ *in.*

Example 2 (7 minutes)

Begin this example by giving the scale factor, 3. Demonstrate how to make a scale drawing using the scale factor. Use a table or an equation to show how you computed your actual lengths. Note that the original image of the flag should be 1 inch by $1\frac{1}{2}$ inches.

- Is this a reduction or an enlargement?
  - *An enlargement*
- How could you determine that it was an enlargement even before seeing the drawing?
  - *A scale factor greater than one represents an enlargement.*
- Can you predict what the lengths of the scale drawing will be?
  - *Yes, they will be* 3 *times as large as the actual picture.*
- What steps were used to create this scale drawing?
  - *Measure lengths of the original drawing and record onto a table. Multiply by* 3 *to compute the scale drawing lengths. Record and draw.*
- How can you double check your work?
  - *Divide the scale lengths by* 3 *to see if they match actual lengths.*

Example 2

Use a scale factor of 3 to create a scale drawing of the picture below.

Picture of the flag of Colombia:

A. $1\frac{1}{2}\text{ in.} \times 3 = 4\frac{1}{2}\text{ in.}$

B. $\frac{1}{2}\text{ in.} \times 3 = 1\frac{1}{2}\text{ in.}$

C. $\frac{1}{4}\text{ in.} \times 3 = \frac{3}{4}\text{ in.}$

D. $\frac{1}{4}\text{ in.} \times 3 = \frac{3}{4}\text{ in.}$

Exercise 2 (6 minutes)

Have students work with partners to create a scale drawing of the original picture of the flag from Example 2 but now applying a scale factor of $\frac{1}{2}$.

- Is this a reduction or an enlargement?
  - *This is a reduction because the scale factor is less than one.*
- What steps were used to create this scale drawing?
  - *Compute the scale drawing lengths by multiplying by $\frac{1}{2}$ or dividing by 2. Record. Measure the new segments with a ruler and draw.*

Exercise 2

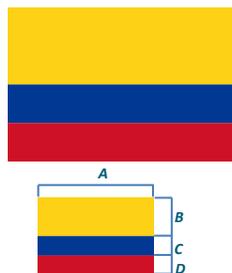**Scale Factor $= \frac{1}{2}$**   **Picture of the flag of Colombia:**

Sketch and notes:

A. $1\frac{1}{2}\text{ in.} \times \frac{1}{2} = \frac{3}{4}\text{ in.}$

B. $\frac{1}{2}\text{ in.} \times \frac{1}{2} = \frac{1}{4}\text{ in.}$

C. $\frac{1}{4}\text{ in.} \times \frac{1}{2} = \frac{1}{8}\text{ in.}$

D. $\frac{1}{4}\text{ in.} \times \frac{1}{2} = \frac{1}{8}\text{ in.}$

Example 3 (5 minutes)

After reading the prompt with the class, discuss the following questions:

- What is the shape of the portrait?
  - *Square*

- Will the resulting picture be a reduction or a magnification?
  - *It will be a reduction because the phone picture is smaller than the original portrait. Also, the scale factor is less than one, so this indicates a reduction.*

- One student calculated the length to be 2 inches while another student's response was $\frac{1}{6}$ of a foot. Which answer is more reasonable?
  - *Although both students are correct,* 2 *inches is more reasonable for the purpose of measuring and drawing.*

- What will the scale drawing look like?
  - *The scale drawing should be a square measuring* 2 *inches by* 2 *inches.*

Example 3

Your family recently had a family portrait taken. Your aunt asks you to take a picture of the portrait using your phone and send it to her. If the original portrait is 3 feet by 3 feet, and the scale factor is $\frac{1}{18}$, draw the scale drawing that would be the size of the portrait on your phone.

Sketch and notes:

3×12 *in.* $= 36$ *in.*

36 *in.* $\times \frac{1}{18} = 2$ *in.*

2 *in.*

2 *in.*
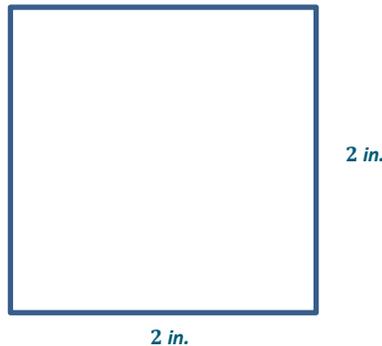
Exercise 3 (5 minutes)

Read the problem aloud, and ask students to solve the problem with another student.

- What is the diameter of the window in the sketch of the model house?
  - *2 inches*

Exercise 3

John is building his daughter a doll house that is a miniature model of their house. The front of their house has a circular window with a diameter of 5 feet. If the scale factor for the model house is $\frac{1}{30}$, make a sketch of the circular doll house window.

$5 \times 12 \ in. = 60 \ in.$

$60 \ in. \times \frac{1}{30} = 2 \ in.$

2 in.
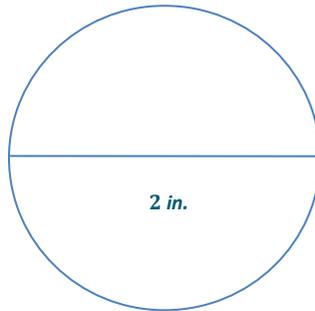
Closing (5 minutes)

- How is the constant of proportionality represented in scale drawings?
  - *Scale factor*
- Explain how to calculate scale factor.
  - *Measure the actual picture lengths and the scale drawing lengths. Write the values as a ratio of the length of the scale drawing length to the length of the actual picture.*
- What operation(s) is (are) used to create scale drawings?
  - *After the lengths of the actual picture are measured and recorded, multiply each length by the scale factor to find the corresponding scale drawing lengths. Measure and draw.*

Exit Ticket (5 minutes)

Name _____   Date_____

Lesson 17: The Unit Rate as the Scale Factor

Exit Ticket

A rectangular pool in your friend's yard is 150 ft. × 400 ft. Create a scale drawing with a scale factor of $\frac{1}{600}$. Use a table or an equation to show how you computed the scale drawing lengths.

Exit Ticket Sample Solutions

A rectangular pool in your friend's yard is 150 ft. $\times$ 400 ft. Create a scale drawing with a scale factor of $\frac{1}{600}$. Use a table or an equation to show how you computed the scale drawing lengths.

Actual Length	Scale Length
150 ft.	150 ft. multiplied by $\frac{1}{600} = \frac{1}{4}$ ft., or 3 in.
400 ft.	400 ft. multiplied by $\frac{1}{600} = \frac{2}{3}$ ft., or 8 in.

8 in.

3 in.
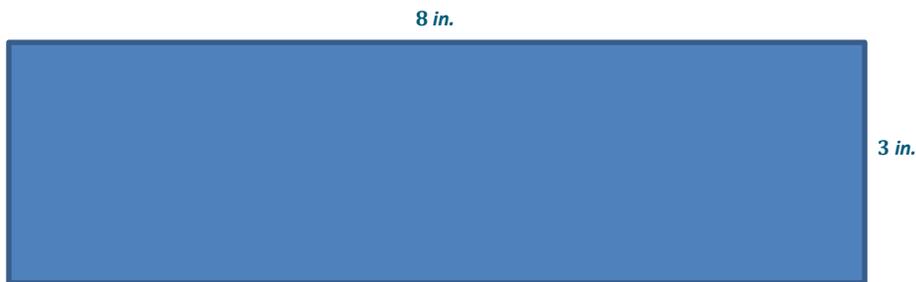
Problem Set Sample Solutions

1. **Giovanni went to Los Angeles, California for the summer to visit his cousins. He used a map of bus routes to get from the airport to his cousin's house. The distance from the airport to his cousin's house is 56 km. On his map, the distance was 4 cm. What is the scale factor?**

    The scale factor is $\frac{1}{1,400,000}$. I had to change km to cm or cm to km or both to meters in order to determine the scale factor.

2. **Nicole is running for school president. Her best friend designed her campaign poster, which measured 3 feet by 2 feet. Nicole liked the poster so much, she reproduced the artwork on rectangular buttons that measured 2 inches by $1\frac{1}{3}$ inches. What is the scale factor?**

    The scale factor is $\frac{2}{3}$.

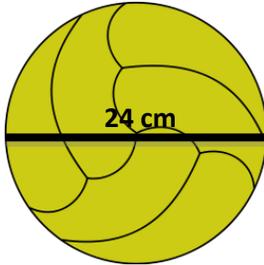3. **Find the scale factor using the given scale drawings and measurements below.**

    Scale Factor: $\frac{5}{3}$ _____

    Actual   **Scale Drawing**
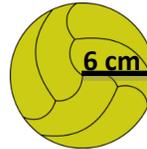
    3 cm   5 cm
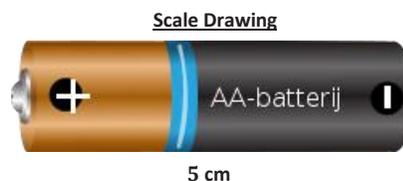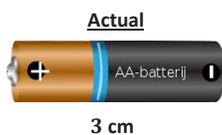
4.   Find the scale factor using the given scale drawings and measurements below.

Scale Factor: ___ $\frac{1}{2}$ ___ **Compare diameter to diameter or radius to radius*

<div align="center">

Actual Picture   **Scale Drawing**

</div>

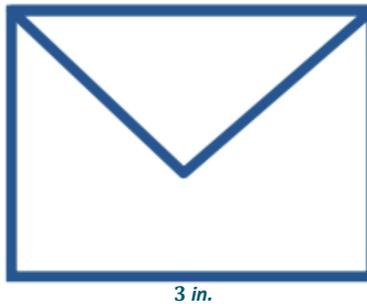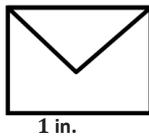24 cm   6 cm

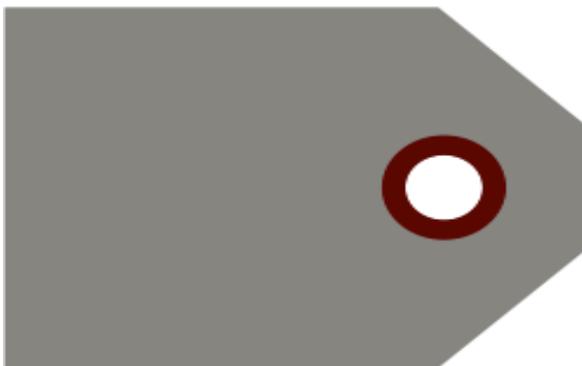5.   Using the given scale factor, create a scale drawing from the actual pictures in centimeters:

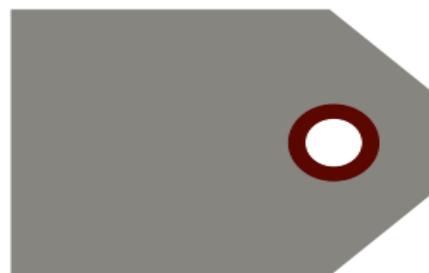   a.   Scale factor: 3

   Small Picture : 1 in.

   Large Picture: 3 in.

1 in.

3 in.

   b.   Scale factor: $\frac{3}{4}$

Actual Drawing Measures: 4 in.   *Scale Drawing Measures: 3 in.*

6.   Hayden likes building radio-controlled sailboats with her father. One of the sails, shaped like a right triangle, has side lengths measuring 6 inches, 8 inches, and 10 inches. To log her activity, Hayden creates and collects drawings of all the boats she and her father built together. Using the scale factor of $\frac{1}{4}$, create a scale drawing of the sail.

A triangle with sides 1.5 *inches,* 2 *inches, and* 2.5 *inches is drawn.*

Scaffolding:

- Extension: Students can enlarge an image they want to draw or paint by drawing a grid using a ruler over their reference picture and drawing a grid of equal ratio on their work surface. Direct students to focus on one square at a time until the image is complete. Have students compute the scale factor for the drawing.