Lesson 16: Relating Scale Drawings to Ratios and Rates

Student Outcomes

- Students understand that a scale drawing is either the reduction or the enlargement of a two-dimensional picture.
- Students compare the scale drawing picture with the original picture and determine if the scale drawing is a reduction or an enlargement.
- Students match points and figures in one picture with points and figures in the other picture.

Classwork

Intro Activity (3 minutes): Can You Guess the Image?

Use the attached Intro Activity pages to show students a series of images to see if they can guess what is pictured and then identify whether it is a reduction or enlargement of the original image. The purpose of this activity is for students to get an understanding of the terms reduction and enlargement. The scale drawings produced in grade 7 will focus on creating a scale drawing from a two-dimensional picture. Teachers could also post alternate images of choice on a projector or interactive whiteboard where only one portion is revealed. Students need to guess the object and whether it is a reduction or enlargement of the actual object.

> *Scaffolding:*
> In each intro problem, the image could be re-sized on a digital display for students to see the enlarged version of this original picture.

Give students 2 minutes to try to guess each image in the student pages and share responses (see responses to the right). Then show the full size images and have them decide whether the images in the student pages are reductions or enlargements compared to what is now being shown.

Responses for attached images and points for discussion follow.

- It is picture of a subway map. Was the cropped photo that was just seen a reduction or an enlargement of the original picture below? How do you know?
  - *It is a reduction since it was a scaled down picture of a map of a subway. If you compare the length from one end of a track to the other end, you can see that the cropped photo has a shorter length as compared to the original photo. Any corresponding length could be compared.*
- It is a fingerprint. Was the cropped photo that was just seen a reduction or an enlargement of the original picture below? How do you know?
  - *It is an enlargement since it was from a picture of a fingerprint. If you compare the length of one of the swirls to the actual fingerprint picture, you can see that the cropped photo has a longer length compared to the original fingerprint picture.*

Intro Activity: Can You Guess the Image?

1.

2.

This is a reduction of a subway map.

This is an enlarged picture of a fingerprint.

Example 1 (3 minutes)

For each scale drawing, have students identify if it is a reduction or an enlargement to the actual object in real life or to the given original picture. Discuss:

MP.2

- What are possible uses for enlarged drawings/pictures?
    - *Enlarged drawings are good to observe details such as textures and parts that are hard to see to the naked eye. In art, enlargements are used in murals or portraits.*
- What are the possible purposes of reduced drawings/pictures?
    - *Reductions are purposeful to get a general idea of a picture/object. These scale drawings can fit in folders, books, wallets, etc.*

Read over the "Key Idea" with the class and introduce the term "scale drawing". Emphasize the importance of scale drawings being reductions or enlargements of two-dimensional drawings, not of actual objects.

Example 1

For the following problems, (a) is the actual picture and (b) is the scale drawing. Is the scale drawing an enlargement or a reduction of the actual picture?

1.   a.   b.

2.   a.   0 1 2 3 4 5 6 7 8 9 10

       b.   0 1 2 3 4 5 6 7 8 9 10

Enlargement   *Reduction*

> **Key Idea:**
>
> **Scale Drawing: a reduced or enlarged two-dimensional drawing of an original two-dimensional drawing.**

Example 2 (7 minutes)

Complete this activity together as a class. Discuss:

- Why doesn't point V correspond with point R?
  - *Although both points are on the bottom right hand corner, they are positioned on two different ends of the path. Point V only corresponds to Point W.*
- What must we consider before identifying correspond points?
  - *We have to make sure we are looking at the maps in the same direction. Then we can see that this is a one-to-one correspondence because they are scale drawings of each other and each point corresponds to one specific point on the other map.*

Example 2

Derek's family took a day trip to a modern public garden. Derek looked at his map of the park that was a reduction of the map located at the garden entrance. The dots represent the placement of rare plants. The diagram below is the top-view as Derek held his map while looking at the posted map.
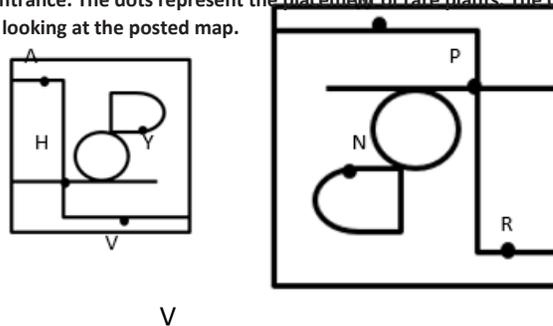
What are the corresponding points of the scale drawings of the maps?

Point A to: _Point R_   Point V to: _Point W_   Point H to: _Point P_   Point Y to: _Point N_


Exercise 1 (10 minutes)

In this Exercise, the size of the units on the grid are enlarged then reduced to produce two different scale drawings with lengths that are proportional to one another. Guide students to notice that the number of units of length is staying the same, but the size of each unit changes from one drawing to another due to the shrinking and enlarging of the grid. This allows for students to create a scale drawing without having to complete the computation required in finding equivalent ratios (which will be done later in Topic D).

- How will we make the enlarged robot? Will we need to adjust the number of units?
  - *No, since the grid is enlarged (thus changing the size of each unit), we can draw the new robot using the same number of units for each given length.*
- What is the importance of matching corresponding points and figures from the actual picture to the scale drawing (the mural piece)?
  - *The scale drawing will not be proportional and the picture will be distorted.*
- How can you check the accuracy of the proportions?
  - *You can count the squares and check that the points match.*

Exercise 1

Create scale drawings of your own modern nesting robots using the grids provided.
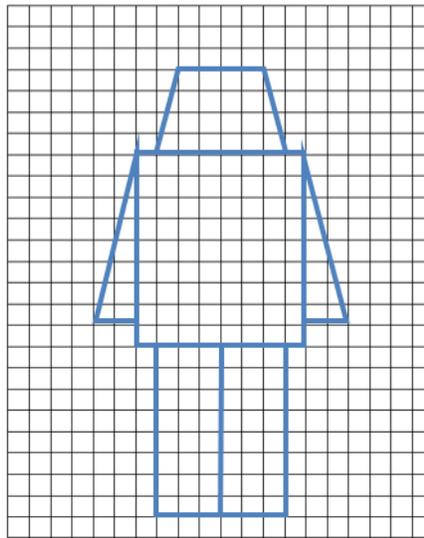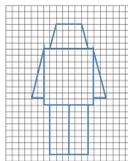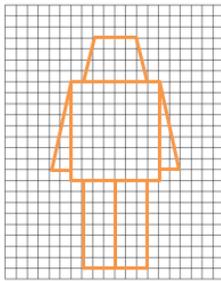
Example 3 (7 minutes)

Work on the problem as a class and fill in the table together. Discuss as students record important points in the "Notes" section:

- Is the second image a reduction or enlargement of the first image? How do you know?
  - *It is a reduction because the second image is smaller than the first, original image.*
- What do you notice about the information on the table?
  - *The pairs of corresponding lengths are all proportional.*
- Does a constant of proportionality exist? How do you know?
  - *Yes, it does because there is a constant value to get from each length to its corresponding length.*
- What is the constant of proportionality, and why is it important in scale drawings?
  - *The constant of proportionality is –, and it needs to exist for images to be considered scale drawings. If not, then there would be a lack of proportionality, and the images would not be a scaled up or down version of the original image.*

	Height	Length
Original Drawing	*18 units*	*6 units*
Second Drawing	*9 units*	*3 units*

Notes:

Exercise 2 (7 minutes)

Have students work in pairs to fill out the table and answer questions. Direct students to the vertical and horizontal lengths of the legs. Reconvene as a class to discuss answers to the given questions and the following:

- Why is it difficult to determine if the second image is a reduction or an enlargement of the first image?
  - *The second image is not a scale drawing of the first image, so even though it is bigger, it is not a true reduction or enlargement of the first image.*
- What must one check before determining if one image is a scale drawing of another?
  - *The corresponding lengths must all be proportional to each other. If only one pair is proportional and another is not, then the images cannot be identified as scale drawings of one another.*

Exercise 2

Luca drew and cut out small right triangle for a mosaic piece he was creating for art class. His mother really took a liking and asked if he could create a larger one for their living room and Luca made a second template for his triangle pieces.

Lengths of the original image	*5 units*	*3 units*
Lengths of the second image	*15 units*	*10 units*

a. **Does a constant of proportionality exist? If so, what is it? If not, explain.**

No, because the ratios of corresponding side lengths are not equal, or proportional to each other

b. **Is Luca's enlarged mosaic a scale drawing of the first image? Explain why or why not.**

No, the enlarged mosaic image is not a scale drawing of the first image. We know this because the images do not have all side lengths proportional to each other; there is no constant of proportionality.

Closing (3 minutes)

- What is a scale drawing?
  - *It is a drawing that is a reduction or enlargement of an actual picture.*
- What is an enlargement? What is a reduction?
  - *A drawing that is larger in scale than its original picture. A drawing that is smaller in scale than its original picture.*
- What is the importance of matching points and figures from one picture/drawing to the next?
  - *The corresponding lines, points and figures need to match because otherwise the scale drawing will be distorted and not proportional throughout.*
- How do scale drawings related to rates and ratios?
  - *The corresponding lengths between scale drawings and original images are equivalent ratios.*

Lesson Summary:

Scale Drawing: A drawing in which all lengths between points or figures in the drawing are reduced or enlarged proportional to the lengths in the actual picture. A constant of proportionality exists between corresponding lengths of the two images.

Reduction: The lengths in the scale drawing are smaller than those in the actual object or picture.

Enlargement/Magnification: The lengths in the scale drawing are larger than those in the actual object or picture.

One-to-one Correspondence: Each point in one figure corresponds to one and only one point in the second figure.

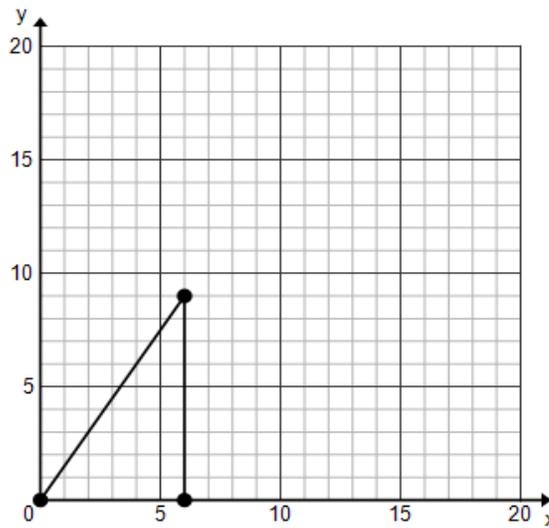## Exit Ticket (5 minutes)

Name _____   Date_____

Lesson 16: Relating Scale Drawings to Ratios and Rates

Exit Ticket

Use the following figure on the graph for problems 1 and 2.

1.

    a.   If the original lengths are multiplied by 2, what are the new coordinates?

    b.   Use the table to organize lengths.

Actual Picture Lengths (in units)		
New Picture Lengths (in units)		

    c.   Is the new picture a reduction or an enlargement?

    d.   What is the constant of proportionality?

2.

    a.   If the original lengths are multiplied by **–** what are the new coordinates?

    b.   Use the table to organize the lengths.

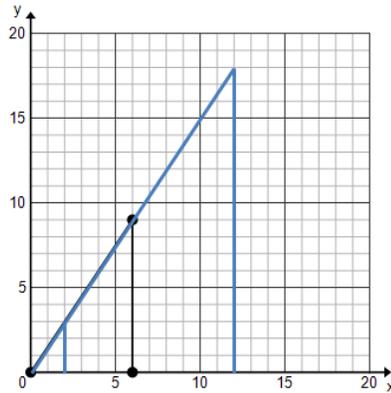| Actual Picture Lengths (in units)		
New Picture Lengths (in units)		

    c.   Is the new picture a reduction or an enlargement?

    d.   What is the constant of proportionality?

Exit Ticket Sample Solutions

The following solutions indicate an understanding of the objectives of this lesson:

Use the following figure on the graph for problems 1 and 2.

1.

    a.   **If the original lengths are multiplied by 2, what are the new coordinates?**

        (0,0), (12,18), (12,0)

    b.   **Use the table to organize lengths (the vertical and horizontal legs).**

Actual Picture Lengths (in units)	*6 units*	*9 units*
New Picture Lengths (in units)	*12 units*	*18 units*

    c.   **Is the new drawing a reduction or an enlargement?**

        Enlargement

    d.   **What is the constant of proportionality?**

        2

2.

    a.   **If the original lengths are multiplied by – what are the new coordinates?**

        (0,0), (2,3), (2,0)

    b.   **Use the table to organize lengths (the vertical and horizontal legs).**

Actual Picture Lengths (in units)	*6 units*	*9 units*
New Picture Lengths (in units)	*2 units*	*3 units*

---

c.   **Is the new drawing a reduction or an enlargement?**

   Reduction

d.   **What is the constant of proportionality?**

   –

Problem Set Sample Solutions

For Problems 1–3, identify if it the scale drawing is a reduction or enlargement of the actual picture.
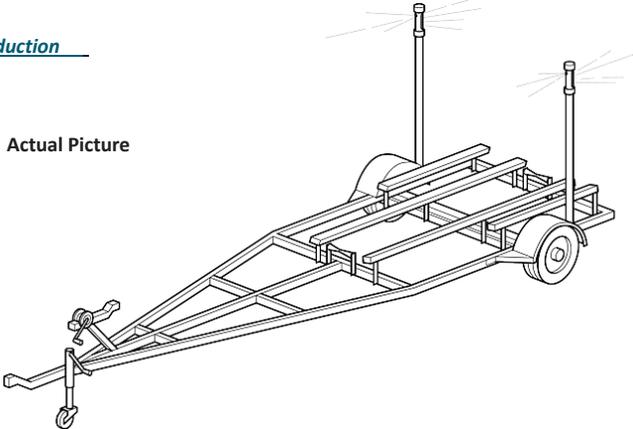
1.   *Enlargement*
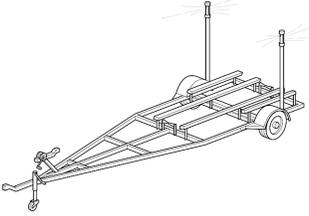
a.   **Actual Picture**   b.   **Scale Drawing**

2.   *Reduction*

a.   **Actual Picture**

b.   **Scale Drawing**

3.   _Enlargement_

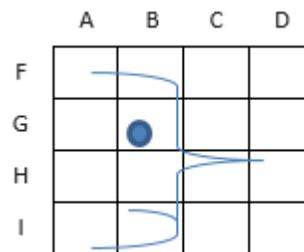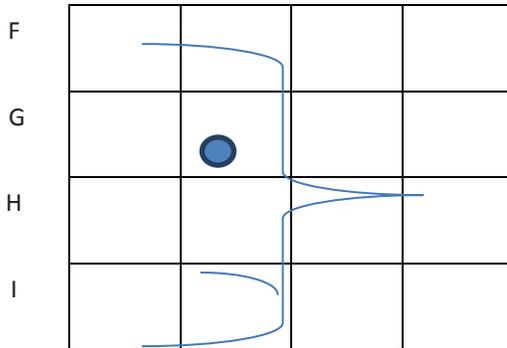a.   **Actual Picture**   b.   **Scale Drawing**

4.   **Using the grid and the abstract picture of a face, answer the following questions:**

a.   **On the grid, where is the eye?**

   Intersection BG

b.   **What is located in DH?**

   Tip of the nose

c.   **In what part of the square BI is the chin located?**

   Bottom right corner

5. Use the graph provided to decide if the rectangular cakes are scale drawings of each other.

    Cake 1: (5,3), (5,5), (11,3), (11, 5)

    Cake 2: (1,6), (1, 12), (13,12), (13, 6)

    How do you know?

    These images are not scale drawings of each other because the short length of cake 2 is three times longer than cake 1, but the longer length of cake 2 is only twice as long as cake 1. Both should either be twice as long or three times as long to have one-to-one correspondence and to be scale drawings of each other.