



## Lesson 14: Multi-Step Ratio Problems

### Student Outcomes

- Students solve multi-step ratio problems including fractional markdowns, markups, commissions, fees, etc.

### Lesson Notes

In this lesson, students solve multi-step ratio problems including fractional markdowns, fractional commissions, fees, and discounts. Problems with similar context that apply percent concepts will be introduced in Modules 2 and 4.

### Classwork

#### Example 1 (12 minutes): Bargains

Begin this lesson by discussing advertising. Share with students that businesses will create an advertisement that will encourage consumers to come to their business in order to purchase their products. Many businesses subscribe to the idea that if a consumer thinks that he or she is saving money, then the consumer will be more motivated to purchase the product.

MP.1

Have students verbalize how they would determine the sale prices with a discount rate of  $\frac{1}{2}$  off the original price of the shirt,  $\frac{1}{3}$  off the original price of pants, and  $\frac{1}{4}$  off the original price of the shoes.

Students should provide an idea that is similar to this: discount price = original price – rate times the original price.

#### Scaffolding:

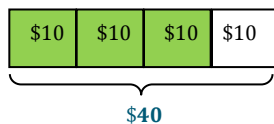
- Consumer:** person buying items
- Remind students that “of” in mathematics is an operational word for multiply.
- Note that students may find the amount of the discount and forget to subtract it from the original price.

#### Example 1: Bargains

Peter’s Pants Palace advertises the following sale: Shirts are  $\frac{1}{2}$  off the original price; pants are  $\frac{1}{3}$  off the original price, and shoes are  $\frac{1}{4}$  off the original price.

- a. If a pair of shoes costs \$40, what is the sales price?

#### Method 1: Tape Diagram



After  $\frac{1}{4}$  of the price is taken off the original price, the discounted price is \$30.

#### Method 2: Subtracting $\frac{1}{4}$ of the price from the original price

$$40 - \frac{1}{4}(40)$$

$$40 - 10$$

$$\$30$$

#### Method 3: Finding the fractional part of the price being paid by subtracting $\frac{1}{4}$ of the price from 1 whole

$$\left(1 - \frac{1}{4}\right) 40$$

$$\left(\frac{3}{4}\right) 40$$

$$\$30$$

b. At Peter’s Pants Palace, a pair of pants usually sells for \$33.00. What is the sale price of Peter’s pants?

**Method 1: Tape Diagram**



$$\begin{aligned} \$33 \div 3 &= \$11 \\ 2(\$11) &= \$22 \end{aligned}$$

**Method 2: Use the given rate of discount, multiply by the price, and then subtract from the original price.**

$$33 - \frac{1}{3}(33) = 33 - 11 = \$22$$

The consumer pays  $\frac{2}{3}$  of the original price.

**Method 3: Subtract the rate from 1 whole, then multiply that rate by the original price.**

$$1 - \frac{1}{3} = \frac{2}{3}$$

$$\frac{2}{3}(33) = \$22.00$$

Use questioning to guide students to develop the methods above. Students do not need to use all three methods, but should have a working understanding of how and why they work in this problem.

### Example 2 (3 minutes): Big Al’s Used Cars

Have students generate an equation that would find the commission for the salesperson.

**Example 2: Big Al’s Used Cars**

A used car salesperson receives a commission of  $\frac{1}{12}$  of the sales price of the car for each car he sells. What would the sales commission be on a car that sold for \$21,999?

$$\text{Commission} = 21,999 \left( \frac{1}{12} \right) = \$1833.25$$

### Example 3 (8 minutes): Tax Time

**Example 3: Tax Time**

As part of a marketing plan, some businesses mark up their prices before they advertise a sales event. Some companies use this practice as a way to entice customers into the store without sacrificing their profits.

A furniture store wants to host a sales event to improve its profit margin and to reduce its tax liability before its inventory is taxed at the end of the year.

How much profit will the business make on the sale of a couch that is marked up by  $\frac{1}{3}$  and then sold at a  $\frac{1}{5}$  off discount if the original price is \$2,400?

$$\text{Markup: } \$2,400 + \$2,400 \left( \frac{1}{3} \right) = \$3,200 \text{ or } \$2,400 \left( 1 \frac{1}{3} \right) = \$3,200$$

$$\text{Markdown: } \$3,200 - \$3,200 \left( \frac{1}{5} \right) = \$2,560 \text{ or } \$3,200 \left( \frac{4}{5} \right) = \$2,560$$

$$\text{Profit} = \text{sales price} - \text{original price} = \$2,560 - \$2,400 = \$160.00$$

**Example 4 (7 minutes): Born to Ride**

Explain that a whole plus the fractional increase will give  $1 + \frac{1}{5} = \frac{6}{5}$  of the original price.

**Example 4: Born to Ride**

A motorcycle dealer paid a certain price for a motorcycle and marked it up by  $\frac{1}{5}$  of the price he paid. Later, he sold it for \$14,000. What is the original price?

*Let  $x =$  the original price*

$$x + \frac{1}{5}x = 14,000$$

$$\frac{6}{5}x = 14,000$$

$$\left(\frac{5}{6}\right)\left(\frac{6}{5}x\right) = (14,000)\left(\frac{5}{6}\right)$$

$$x = 14,000\left(\frac{5}{6}\right)$$

$$x = \$11,666.67$$

**Closing (5 minutes)**

- Name at least two methods used to find the solution to a fractional markdown problem.
  - Find the fractional part of the markdown, and subtract it from the original price.
  - Use a tape diagram to determine the value each part represents, and then subtract the fractional part from the whole.
- Compare and contrast a commission and a discount price?
  - The commission and the discount price are both fractional parts of the whole. The difference between them is that commission is found by multiplying the commission rate times the sale, while the discount is the difference between 1 and the fractional discount multiplied by the original price.

**Lesson Summary**

- Discount price = original price – rate  $\times$  original price      **OR**       $(1 - \text{rate}) \times$  original price
- Commission = rate  $\times$  total sales amount
- Markup price = original price + rate  $\times$  original price      **OR**       $(1 + \text{rate}) \times$  original price

**Exit Ticket (5 minutes)**



Name \_\_\_\_\_

Date \_\_\_\_\_

## Lesson 14: Multi-Step Ratio Problems

### Exit Ticket

1. A bicycle shop advertised all mountain bikes priced at a  $\frac{1}{3}$  discount.
  - a. What is the amount of the discount if the bicycle originally costs \$327?
  - b. What is the discount price of the bicycle?
  - c. Explain how you found your solution to part (b).
2. A hand-held digital music player was marked down by  $\frac{1}{4}$  of the original price.
  - a. If the sales price is \$128.00, what is the original price?
  - b. If the item was marked up by  $\frac{1}{2}$  before it was placed on the sales floor, what was the price that the store paid for the digital player?
  - c. What is the difference between the discount price and the price that the store paid for the digital player?



## Exit Ticket Sample Solutions

1. A bicycle shop advertised all mountain bikes priced at a  $\frac{1}{3}$  discount.
- What is the amount of the discount if the bicycle originally costs \$327?  

$$\frac{1}{3}(\$327) = \$109 \text{ discount}$$
  - What is the discount price of the bicycle?  

$$\frac{2}{3}(\$327) = \$218 \text{ discount price. Methods will vary.}$$
  - Explain how you found your solution to part (b).  
*Answers will vary.*
2. A hand-held digital music player was marked down by  $\frac{1}{4}$  of the original price.
- If the sales price is \$128.00, what is the original price?  

$$x - \frac{1}{4}x = 128$$

$$\frac{3}{4}x = 128$$

$$x = \$170.67$$
  - If the item was marked up by  $\frac{1}{2}$  before it was placed on the sales floor, what was the price that the store paid for the digital player?  

$$x + \frac{1}{2}x = 170.67$$

$$\frac{3}{2}x = 170.67$$

$$x = \$113.78$$
  - What is the difference between the discount price and the price that the store paid for the digital player?  

$$\$128 - \$113.78 = \$14.22$$

## Problem Set Sample Solutions

1. A salesperson will earn a commission equal to  $\frac{1}{32}$  of the total sales. What is the commission earned on sales totaling \$24,000?  

$$\left(\frac{1}{32}\right)\$24,000 = \$750$$



2. DeMarkus says that a store overcharged him on the price of the video game he bought. He thought that the price was marked  $\frac{1}{4}$  off the original price, but it was really  $\frac{1}{4}$  off the original price. He misread the advertisement. If the original price of the game was \$48, what is the difference between the price that DeMarkus thought he should pay and the price that the store charged him?

$$\frac{1}{4} \text{ of } \$48 = \$12 \text{ (the price DeMarkus thought he should pay); } \frac{1}{4} \text{ off } \$48 = \$36; \text{ Difference between prices } \\ \$36 - \$12 = \$24$$

3. What is the cost of a \$1,200 washing machine after a discount of  $\frac{1}{5}$  the original price?

$$\left(1 - \frac{1}{5}\right) 1200 = \$960 \text{ or } 1200 - \frac{1}{5}(1200) = \$960$$

4. If a store advertised a sale that gave customers a  $\frac{1}{4}$  discount, what is the fractional part of the original price that the customer will pay?

$$1 - \frac{1}{4} = \frac{3}{4} \text{ of original price}$$

5. Mark bought an electronic tablet on sale for  $\frac{1}{4}$  off the original price of \$825.00. He also wanted to use a coupon for  $\frac{1}{5}$  off the sales price. Before taxes, how much did Mark pay for the tablet?

$$\$825 \left(\frac{3}{4}\right) = \$618.75, \text{ then } \$618.75 \left(\frac{4}{5}\right) = \$495$$

6. A car dealer paid a certain price for a car and marked it up by  $\frac{7}{5}$  of the price he paid. Later he sold it for \$24,000. What is the original price?

$$x + \frac{7}{5}x = \$24,000, \frac{12}{5}x = \$24,000, x = \$10,000$$

7. Joanna ran a mile in physical education class. After resting for one hour, her heart rate was 60 beats per minute. If her heart rate decreased by  $\frac{2}{5}$ , what was her heart rate immediately after she ran the mile?

$$x - \frac{2}{5}x = 60, \frac{3}{5}x = 60, x = 100 \text{ beats per minute}$$