Lesson 7: Unit Rate as the Constant of Proportionality

Student Outcomes

- Students identify the same value relating the measures of x and the measures of y in a proportional relationship as the constant of proportionality and recognize it as the unit rate in the context of a given situation.
- Students find and interpret the constant of proportionality within the contexts of problems.

Classwork

Example 1 (20 minutes): National Forest Deer Population in Danger?

Scaffolding:

Use a map of a national forest in your area so that students who are not familiar with square miles can view a model.

Begin this lesson by presenting the following situation: Wildlife conservationists are concerned that the deer population might not be constant across the National Forest. The scientists found that there were 144 deer in a 16 square mile area of the forest. In another part of the forest, conservationists counted 117 deer in a 13 square mile area. Yet a third conservationist counted 24 deer in a 216 square mile plot of the forest. Do conservationists need to be worried?

Questions for Discussion: Guide students to complete necessary information on student handout.

Example 1: National Forest Deer Population in Danger?

Wildlife conservationists are concerned that the deer population might not be constant across the National Forest. The scientists found that there were 144 deer in a 16 square mile area of the forest. In another part of the forest, conservationists counted 117 deer in a 13 square mile area. Yet a third conservationist counted 24 deer in a 216 square mile plot of the forest. Do conservationists need to be worried?

a. Why does it matter if the deer population is not constant in a certain area of the national forest?

Have students generate as many theories as possible (e.g., food supply, overpopulation, damage to land, etc.).

b. What is the population density of deer per square mile?

See chart below.

MP.

Encourage students to make a chart to organize the data from the problem and then explicitly model finding the constant of proportionality. Students have already found unit rate in earlier lessons but have not identified it as the constant of proportionality. Remember that the constant of proportionality is also like a scalar and will be used in an equation as the constant.

- When we find the number of deer per 1 square mile, what is this called?
  - *Unit rate.*
- When we look at the relationship between square miles and number of deer in the table below, how do we know if the relationship is proportional?
  - *The square miles is always multiplied by the same value, 9 in this case.*

Table:

Square Miles	*Number of Deer*	
16	*144*	$\frac{144}{16} = 9$
13	*117*	$\frac{117}{13} = 9$
24	*216*	$\frac{216}{24} = 9$

- We call this constant (or same) value the "constant of proportionality".
- So deer per square mile is 9, and the constant of proportionality is 9. Is that a coincidence or will that always be the case: that the unit rate and the constant of proportionality are the same?

Allow for comments or observations but leave a lingering question for now.

- We could add the unit rate to the table so that we have 1 square mile in the first column and 9 in the second column? (Add this to table for students to see). Does that help to guide your decision about the relationship between unit rate and COP? We will see if your hypothesis holds true as we move through more examples.

The Unit Rate of deer per 1 square mile is ___*9*___.

Constant of Proportionality: *k = 9*

Meaning of Constant of Proportionality in this problem: *There are 9 deer for every 1 square mile of forest.*
(Could be completed later after formalizing this concept in a few more examples)

c. Use the unit rate of deer per square mile to determine how many deer are there for every 207 square miles.

$$9(207) = 1863$$

d. Use the unit rate to determine the number of square miles in which you would find 486 deer?

$$\frac{486}{9} = 54$$

Based upon the discussion of the questions above, answer the question: Do conservationists need to be worried? Be sure to support your answer with mathematical reasoning about rate and unit rate.

Review vocabulary box with students.

Vocabulary:

A constant specifies a unique number.

A variable is a letter that represents a number.

If a proportional relationship is described by the set of ordered pairs that satisfies the equation $y = kx$, where k is a positive constant, then k is called the constant of proportionality. It is the value that describes the multiplicative relationship between two quantities, x and y. The (x, y) pairs represent all the pairs of values that make the equation true.

Note: In a given situation, it would be reasonable to assign any variable as a placeholder for the given quantities. For example, a set of ordered pairs (t, d) would be all the points that satisfy the equation $d = rt$, where r is

Remind students that in the example with the deer population, we are looking for deer per square mile, so the number of square miles could be defined as x, and the number of deer could be defined as y, so the unit rate in deer per square mile is $\frac{144}{16}$, or 9. The constant of proportionality, k, is 9. The meaning in the context of Example 1 is: There are 9 deer for every 1 square mile of forest.

Discussion

- How are the constant of proportionality and the unit rate alike?
  - *They are the same. They both represent the same number that is the value of the ratio of y to x.*

Example 2 (18 minutes): You Need WHAT???

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Brandon came home from school and informed his mother that he had volunteered to make cookies for his entire grade level. He needed 3 cookies for each of the 96 students in 7th grade. Unfortunately, he needed the cookies for an event at school on the very next day! Brandon and his mother determined that they can fit 36 cookies on two cookie sheets.
Encourage students to make a chart to organize the data from the problem.

a. Is the number of cookies proportional to the number of sheets used in baking? Create a table that shows data for the number of sheets needed for the total number of cookies needed.

Table:

# of cookie sheets	*# of cookies baked*	
2	*36*	$\frac{36}{2} = 18$
4	*72*	$\frac{72}{4} = 18$

10	*180*	$\frac{180}{10} = 18$
16	*288*	$\frac{288}{16} = 18$

Scaffolding:

For students who need more challenge, have them create a problem in which the constant rate is a fraction.

The unit rate is *18*.

The constant of proportionality is *18*.

Meaning of Constant of Proportionality in this problem: *There are 18 cookies per 1 sheet*.

b. It took 2 hours to bake 8 sheets of cookies. If Brandon and his mother begin baking at 4:00 pm, when will they finish baking the cookies?

96 students (3 cookies each) = 288 cookies

$\frac{288\ cookies}{18\ cookies\ per\ sheet}$ *= 16 sheets of cookies*

If it takes 2 hours to bake 8 sheets, it will take 4 hours to bake 16 sheets of cookies. They will finish baking at 8:00 pm.

Example 3: French Class Cooking

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Suzette and Margo want to prepare crepes for all of the students in their French class. A recipe makes 20 crepes with a certain amount of flour, milk, and 2 eggs. The girls know that they already have plenty of flour and milk but need to determine the number of eggs needed to make 50 crepes because they are not sure they have enough eggs for the recipe.

a. Considering the amount of eggs necessary to make the crepes, what is the constant of proportionality?

$\frac{2\ eggs}{20\ crepes} = \frac{1\ egg}{10\ crepes}$ *; The constant of proportionality is* $\frac{1}{10}$*.*

b. What does the constant or proportionality mean in the context of this problem?

One egg is needed to make 10 *crepes.*

c. How many eggs will be needed for 50 crepes?

$50\left(\frac{1}{10}\right) = 5$ *; Five eggs will be needed to make* 50 *crepes.*

Closing Question (2 minutes)

- What is another name for the constant that relates the measures of two quantities?

- *Constant of Proportionality*
- How is the Constant of Proportionality related to the unit rate?
  - *They are the same.*

Lesson Summary

If a proportional relationship is described by the set of ordered pairs that satisfies the equation $y = kx$, where k is a positive constant, then k is called the *constant of proportionality*.

Exit Ticket (5 minutes)

Name ____ Date____________________

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Exit Ticket

Susan and John are buying cold drinks for a neighborhood picnic. Each person is expected to drink one can of soda. Susan says that if you multiply the unit price for a can of soda by the number of people attending the picnic, you will be able to determine the total cost of the soda. John says that if you divide the cost of a 12-pack of soda by the number of sodas, you will be able to determine the total cost of the sodas. Who is right and why?

Exit Ticket Sample Solutions

The following solutions indicate an understanding of the objectives of this lesson:

Susan and John are buying cold drinks for a neighborhood picnic. Each person is expected to drink one can of soda. Susan says that if you multiply the unit price for a can of soda by the number of people attending the picnic, you will be able to determine the total cost of the soda. John says that if you divide the cost of a 12-pack of soda by the number of sodas, you will be able to determine the total cost of the sodas. Who is right and why?

Susan is correct. The table below shows that if you multiply by the unit price, say 0.50, by the number of people, say 12, you will determine the total cost of the soda. I created a table to model the proportional relationship. I used a unit price of 0.50 to make the comparison.

Susan

# of people	2	3	4	12
Total cost of soda (in $)	1	1.50	2	6

I used the same values to compare to John. $\frac{\text{John total cost}}{12\text{ people}}$ *= ?*

If the total cost is $\$24$ *, then* $\frac{24}{12}=2$. *This would mean that $2 will be needed for the* 12 *pack which is not correct. This amount would represent the unit price or cost for 1 soda.*

Problem Set Sample Solutions

For each of the following problems, define the constant of proportionality to answer the follow-up question.

1. **Bananas are $0.59/pound.**
   a. **What is the constant of proportionality, k?**

      K = 0.59

   b. **How much do 25 pounds of bananas cost?**

$$25(0.59) = \$14.75$$

2. The dry cleaning fee for 3 pairs of pants is $18.

a. What is the constant of proportionality?

$\frac{18}{3} = 6$ *so K=6*

b. How much will the dry cleaner charge for 11 pairs of pants?

6(11) = $66

3. For every $5 that Micah saves, his parents give him $10.

a. What is the constant of proportionality?

$\frac{10}{5} = 2$ *so k=2*

b. If Micah saves $150, how much money will his parents give him?

2(150) = $300

4. Each school year, the 7th graders who study Life Science participate in a special field trip to the city zoo. In 2010, the school paid $1260 for 84 students to enter the zoo. In 2011, the school paid $1050 for 70 students to enter the zoo. In 2012, the school paid $1395 for 93 students to enter the zoo.

a. Is the price the school pays each year in entrance fees proportional to the number of students entering the zoo?

# of students	*Price*	
84	*1,260*	$\frac{1,260}{84} = 15$
70	*1,050*	$\frac{1,050}{70} = 15$
93	*1,395*	$\frac{1,395}{93} = 15$

YES

b. Explain why or why not?

Because the ratio of the entrance fee paid per student was the same

$\frac{1,260}{84} = 15$

c. Identify the constant of proportionality and explain what it means in the context of this situation.

K = 15. the price per student

d. What would the school pay if 120 students entered the zoo?

120(15) = $1,800

e. How many students would enter the zoo if the school paid $1,425?

$\frac{1,425}{15} = 95\ students$