Lesson 16: Applying the Properties of Operations to Multiply and Divide Rational Numbers

Student Outcomes

- Students use properties of operations to multiply and divide rational numbers without the use of a calculator. They use the commutative and associative properties of multiplication to generate equivalent expressions. They use the distributive property of multiplication over addition to create equivalent expressions, representing the sum of two quantities with a common factor as a product, and vice-versa.
- Students recognize that any problem involving multiplication and division can be written as a problem involving only multiplication.
- Students determine the sign of an expression that contains products and quotients by checking whether the number of negative terms is even or odd.

Classwork

Fluency Exercise (6 minutes): Integer Division

Photocopy the attached 2-page fluency-building exercises so that each student receives a copy. Allow students one minute to complete Side A. Before beginning, inform students that they may not skip over questions, and that they must move in order. After one minute, discuss the answers. Before moving on to Side B, elicit strategies from those students who were able to accurately complete many problems on Side A. Administer Side B in the same fashion, and review the answers. Refer to the Sprints and Sprint Delivery Script sections in the Module Overview for directions to administer a Sprint.

Example 1 (5 minutes): Using the Commutative and Associative Properties to Efficiently Multiply Rational Numbers

Present the question below and have students share their thoughts. Answers will vary, but accept all answers at this point.

- How can we evaluate the expression below? Will different strategies result in different answers? Why or why not?

$$-6 \times 2 \times (-2) \times (-5) \times (-3)$$

Example 1: Using the Commutative and Associative Properties to Efficiently Multiply Rational Numbers

a. **Evaluate the expression below.**

$$-6 \times 2 \times (-2) \times (-5) \times (-3)$$

$-6 \times 2 \times (-2) \times (-5) \times (-3)$

$-12 \times (-2) \times (-5) \times (-3)$

$24 \times (-5) \times (-3)$

$-120 \times (-3)$

360

$-6 \times 2 \times (-2) \times (-5) \times (-3)$

$-6 \times 2 \times 10 \times (-3)$ *Associative property of multiplication*

$-6 \times 2 \times (-3) \times 10$ *Commutative property of multiplication*

$-6 \times (-6) \times 10$ *Associative property of multiplication*

36×10

360

Students experiment with different strategies from their discussion to evaluate the product of integers. After time to work, student groups share their strategies and solutions. Students and teacher discuss the properties (commutative and associative) that allow us to manipulate expressions.

b. **What types of strategies were used to evaluate the expressions?**

The strategies used were order of operations, rearranging the terms using the commutative property, and multiplying the terms in various orders using the associative property.

c. **Can you identify the benefits of choosing one strategy versus another?**

Multiplying the terms allowed me to combine factors in more manageable ways, such as multiplying $(-2) \times (-5)$ *to get* 10*. Multiplying other numbers by* 10 *is very easy.*

d. **What is the sign of the product and how was the sign determined?**

The product is a positive value. When calculating the product of the first two factors, the answer will be negative because when the factors have opposite signs, the result is a negative product. Two negative values multiplied together yield a positive product. When a negative value is multiplied by a positive product, the sign of the product again changes to a negative value. When this negative product is multiplied by the last (fourth) negative value, the sign of the product again changes to a positive value.

Exercise 1 (3 minutes)

Exercise 1

Find an efficient strategy to evaluate the expression and complete the necessary work.

Methods will vary.

$-1 \times (-3) \times 10 \times (-2) \times 2$ *Associative property*

$-1 \times (-3) \times 10 \times (-4)$

$3 \times 10 \times (-4)$

$3 \times (-4) \times 10$ *Commutative multiplication*

-12×10

-120

Discussion

- What aspects of the expression did you consider when choosing a strategy for evaluating this expression?
  - *Answers will vary.*
- What is the sign of the product, and how was the sign determined?
  - *The sign of the product is negative. If we follow the method above, the first two factors result in a negative product because they have opposite signs. The next two factors we used to calculate the product were both negative, so the result was a positive product. The next product we calculated was negative because the factors had opposite signs. Finally, the final two factors also had opposite signs, so the final product was negative.*
- How else could we have evaluated this problem?
  - *Answers may vary, but two possible answers are provided.*
  - *We could have solved this problem by following order of operations or multiplying the factors left to right.*

Exercises 2–4 (6 minutes)

- Is order of operations an efficient strategy to multiply the expression below? Why or why not?

$$4 \times \frac{1}{3} \times (-8) \times 9 \times \left(-\frac{1}{2}\right)$$

After discussion, student groups choose a strategy to evaluate the expression:

Exercise 2

Find an efficient strategy to evaluate the expression and complete the necessary work.

Methods will vary.

$\mathbf{4 \times \frac{1}{3} \times (-8) \times 9 \times \left(-\frac{1}{2}\right)}$

$\mathbf{4 \times \left[\left(-\frac{1}{2}\right) \times (-8)\right] \times 9 \times \frac{1}{3}}$ *Commutative multiplication*

$\mathbf{4 \times 4 \times \left[9 \times \frac{1}{3}\right]}$ *Associative property*

$\mathbf{4 \times 4 \times [3]}$ *Associative property*

$\mathbf{4 \times 12}$ *Associative property*

$\mathbf{48}$

Exercise 3

What terms did you combine first and why?

I multiplied the $-\frac{1}{2} \times -8$ *and* $\frac{1}{3} \times 9$ *because their products are integers; this eliminated the fractions.*

MP.8

Exercise 4

Refer to the example and exercises. Do you see an easy way to determine the sign of the product first?

The product of two negative integers yields a positive product. If there is an even number of negative factors, then each negative value can be paired with another negative value yielding a positive product. This means that all factors become positive values and, therefore, have a positive product.

For example: $\underbrace{(-1)\times(-1)}_{1}\times\underbrace{(-1)\times(-1)}_{1}\times\underbrace{(-1)\times(-1)}_{1} = 1$

If there are an odd number of negative factors, then all except one can be paired with another negative. This leaves us with a product of a positive value and a negative value, which is negative.

For example: $(-1)\times(-1)\times(-1)\times(-1)\times(-1)\times(-1)\times(-1)$

$\mathbf{1} \times \mathbf{1} \times \mathbf{1} \times (-1)$

$\mathbf{1} \times (-1) = -1$

Example 2 (4 minutes): Using the Distributive Property to Multiply Rational Numbers

- What is a mixed number?
  - *A mixed number is the sum of a whole number and a fraction.*
- What does the opposite of a mixed number look like?
  - *The opposite of a sum is equal to the sum of its opposites.*

Scaffolding:

- Remind students that "the opposite of a sum is equivalent to the sum of its opposites."

Example 2: Using the Distributive Property to Multiply Rational Numbers

Rewrite the mixed number as a sum; then, multiply using the distributive property.

$-6\times\left(5\frac{1}{3}\right)$

$-6\times\left(5+\frac{1}{3}\right)$

$\underbrace{(-6\times 5)}_{-30}+\underbrace{\left(-6\times\frac{1}{3}\right)}_{(-2)}$ ***Distributive property***

-32

- Did the distributive property make this problem easier to evaluate? How so?
  - *Answers will vary, but most students will think that distributive property did make the problem easier to solve.*

Exercise 5 (3 minutes)

Exercise 5

Multiply the expression using the distributive property.

$9 \times \left(-3\frac{1}{2}\right)$

$9 \times \left(-3 \quad + \quad \left(-\frac{1}{2}\right)\right)$

$\underbrace{(9 \times (-3))}_{-27} + \underbrace{\left(9 \times \left(-\frac{1}{2}\right)\right)}_{\left(-4\frac{1}{2}\right)}$

$-27 \quad + \quad \left(-4\frac{1}{2}\right)$

$-31\frac{1}{2}$

Example 3 (4 minutes): Using the Distributive Property to Multiply Rational Numbers

Teacher and students together complete the given expression with justification.

Example 3: Using the Distributive Property to Multiply Rational Numbers

Evaluate using the distributive property.

$16 \times \left(-\frac{3}{8}\right) + 16 \times \frac{1}{4}$

$16\left(-\frac{3}{8} + \frac{1}{4}\right)$ *Distributive property*

$16\left(-\frac{3}{8} + \frac{2}{8}\right)$ *Equivalent fractions*

$16\left(-\frac{1}{8}\right)$

-2

Example 4 (4 minutes): Using the Multiplicative Inverse to Rewrite Division as Multiplication

- How is this expression different from the previous examples, and what can we do to make it more manageable?
  - *This expression involves division by fractions, and we know that dividing by a number is equivalent to multiplying by its multiplicative inverse (reciprocal); so, we can rewrite the entire expression as multiplication.*

Example 4: Using the Multiplicative Inverse to Rewrite Division as Multiplication

Rewrite the expression as only multiplication and evaluate.

$1 \div \frac{2}{3} \times (-8) \times 3 \div \left(-\frac{1}{2}\right)$

$1 \times \frac{3}{2} \times (-8) \times 3 \times (-2)$ *Multiplicative inverse*

$1 \times \left[(-2) \times \left(\frac{3}{2}\right)\right] \times (-8) \times 3$ *Commutative multiplication*

$1 \times [\ -3\ ] \times (-8) \times 3$ *Associative property*

$-3 \times (-8) \times 3$

$-3 \times 3 \times (-8)$ *Commutative multiplication*

$-9 \times (-8)$

72

Exercise 6 (4 minutes)

Students in groups evaluate the following expression using the multiplicative inverse property. Methods will vary.

Exercise 6

$4.2 \times \left(-\frac{1}{3}\right) \div \frac{1}{6} \times (-10)$

$4.2 \times \left(-\frac{1}{3}\right) \times \frac{6}{1} \times (-10)$ *Multiplicative inverse*

$4.2 \times (-10) \times \left(-\frac{1}{3}\right) \times 6$ *Commutative multiplication*

$-42 \times \left(-\frac{1}{3}\right) \times 6$

14×6

84

Have student groups present their solutions to the class, describe the properties used, and explain the reasoning that supports their choices.

Closing (2 minutes)

- How do we determine the sign of expressions that include several products and quotients?
  - *We can determine the sign of the product or quotient by counting the number of negative terms. An even number of negative terms results in a positive answer. On the other hand, an odd number of negative terms results in a negative answer.*
- Name a property of operations, and describe how it is helpful when multiplying and dividing rational numbers.
  - *Answers will vary, but students should discuss the associative, commutative, and distributive properties.*

Lesson Summary

Multiplying and dividing using strictly order of operations is not always efficient. The properties of multiplication allow us to manipulate expressions by rearranging and regrouping factors that are easier to compute. Where division is involved, we can easily rewrite division as multiplication to allow the use of these properties. The signs of expressions with products and quotients can be easily determined by checking whether the number of negative terms is even or odd.

Exit Ticket (4 minutes)

Name ____ Date____________________

Lesson 16: Applying the Properties of Operations to Multiply and Divide Rational Numbers

Exit Ticket

1. Evaluate the expression below using the properties of operations.

$$18 \div \left(-\frac{2}{3}\right) \times 4 \div (-7) \times (-3) \div \left(\frac{1}{4}\right)$$

2. a. Given the expression below, what will the sign of the product be? Justify your answer.

$$-4 \times \left(-\frac{8}{9}\right) \times 2.78 \times \left(1\frac{1}{3}\right) \times \left(-\frac{2}{5}\right) \times (-6.2) \times (-0.2873) \times \left(3\frac{1}{11}\right) \times A$$

b. Give a value for A that would result in a positive value for the expression.

c. Give a value for A that would result in a negative value for the expression.

Exit Ticket Sample Solutions

1. Evaluate the expression below using the properties of operations.

$$18 \div \left(-\frac{2}{3}\right) \times 4 \div (-7) \times (-3) \div \left(\frac{1}{4}\right)$$

$$18 \times \left(-\frac{3}{2}\right) \times 4 \times \left(-\frac{1}{7}\right) \times (-3) \times \left(\frac{4}{1}\right)$$

$$-27 \times 4 \times \left(-\frac{1}{7}\right) \times (-3) \times \left(\frac{4}{1}\right)$$

$$-\frac{1296}{7}$$

Answer: $185\frac{1}{7}$ *or* $185.\overline{142857}$

2. a. Given the expression below, what will the sign of the product be? Justify your answer.

$$-4 \times \left(-\frac{8}{9}\right) \times 2.78 \times \left(1\frac{1}{3}\right) \times \left(-\frac{2}{5}\right) \times (-6.2) \times (-0.2873) \times \left(3\frac{1}{11}\right) \times A$$

There are five negative values in the expression. Because the product of two numbers with the same sign yield a positive product, pairs of negative factors have positive products. Given an odd number of negative factors, all but one can be paired into positive products. The remaining negative factor causes the product of the terms without A to be a negative value. If the value of A is negative, then the pair of negative factors forms a positive product. If the value of A is positive, the product of the two factors with opposite signs yields a negative product.

b. Give a value for A that would result in a positive value for the expression.

Answers will vary, but the answer must be negative. -2

c. Give a value for A that would result in a negative value for the expression.

Answers will vary, but the answer must be positive. 3.6

Problem Set Sample Solutions

1. Evaluate the expression $-2.2 \times (-2) \div \left(-\frac{1}{4}\right) \times 5$

a. Using the order of operations only.

$4.4 \div \left(-\frac{1}{4}\right) \times 5$

-17.6×5

-88

b. Using the properties and methods used in Lesson 16.

$-2.2 \times (-2) \times (-4) \times 5$

$-2.2 \times (-2) \times 5 \times (-4)$

$-2.2 \times (-10) \times (-4)$

$22 \times (-4)$

-88

c. **If you were asked to evaluate another expression, which method would you use, (a) or (b), and why?**

Answers will vary; however, most students should have found method (b) to be more efficient.

2. **Evaluate the expressions using the distributive property.**

a. $\left(2\frac{1}{4}\right) \times (-8)$

$2 \times (-8) + \frac{1}{4} \times (-8)$

$-16 + (-2)$

-18

b. $\frac{2}{3}(-7) + \frac{2}{3}(-5)$

$\frac{2}{3}(-7 + (-5))$

$\frac{2}{3}(-12)$

-8

3. **Mia evaluated the expression below but got an incorrect answer. Find Mia's error(s), find the correct value of the expression, and explain how Mia could have avoided her error(s).**

$0.38 \times 3 \div \left(-\frac{1}{20}\right) \times 5 \div (-8)$

$0.38 \times 5 \times \left(\frac{1}{20}\right) \times 3 \times (-8)$

$0.38 \times \left(\frac{1}{4}\right) \times 3 \times (-8)$

$0.38 \times \left(\frac{1}{4}\right) \times (-24)$

$0.38 \times (-6)$

-2.28

Mia made two mistakes in the second line; first, she dropped the negative symbol from $-\frac{1}{20}$ when she changed division to multiplication. The correct term should be (-20) because dividing a number is equivalent to multiplying its multiplicative inverse (or reciprocal). Mia's second error occurred when she changed division to multiplication at the end of the expression; she changed only the operation, not the number. The term should be $\left(-\frac{1}{8}\right)$. The correct value of the expressions is $14\frac{1}{4}$, or 14.25.

Mia could have avoided part of her error if she had determined the sign of the product first. There are two negative values being multiplied, so her answer should have been a positive value.

Number Correct: ______

Integer Division – Round 1

Directions: Determine the quotient of the integers, and write it in the column to the right.

1.	$4 \div 1$	
2.	$4 \div (-1)$	
3.	$-4 \div (-1)$	
4.	$-4 \div 1$	
5.	$6 \div 2$	
6.	$-6 \div (-2)$	
7.	$-6 \div 2$	
8.	$6 \div -2$	
9.	$8 \div (-4)$	
10.	$-8 \div (-4)$	
11.	$-8 \div 4$	
12.	$8 \div 4$	
13.	$9 \div (-3)$	
14.	$-9 \div 3$	
15.	$-10 \div 5$	
16.	$10 \div (-2)$	
17.	$-10 \div (-2)$	
18.	$-10 \div (-5)$	
19.	$-14 \div 7$	
20.	$14 \div (-2)$	
21.	$-14 \div (-2)$	
22.	$-14 \div (-7)$	

23.	$-16 \div (-4)$	
24.	$16 \div (-2)$	
25.	$-16 \div 4$	
26.	$-20 \div 4$	
27.	$-20 \div (-4)$	
28.	$-28 \div 4$	
29.	$28 \div (-7)$	
30.	$-28 \div (-7)$	
31.	$-40 \div (-5)$	
32.	$56 \div (-7)$	
33.	$96 \div (-3)$	
34.	$-121 \div (-11)$	
35.	$169 \div (-13)$	
36.	$-175 \div 25$	
37.	$1 \div 4$	
38.	$-1 \div 4$	
39.	$-1 \div (-4)$	
40.	$-3 \div (-4)$	
41.	$-5 \div 20$	
42.	$6 \div (-18)$	
43.	$-24 \div 48$	
44.	$-16 \div 64$	

Integer Division – Round 1 [KEY]

Directions: Determine the quotient of the integers, and write it in the column to the right.

1.	$4 \div 1$	$\mathbf{4}$
2.	$4 \div (-1)$	$\mathbf{-4}$
3.	$-4 \div (-1)$	$\mathbf{4}$
4.	$-4 \div 1$	$\mathbf{-4}$
5.	$6 \div 2$	$\mathbf{3}$
6.	$-6 \div (-2)$	$\mathbf{3}$
7.	$-6 \div 2$	$\mathbf{-3}$
8.	$6 \div -2$	$\mathbf{-3}$
9.	$8 \div (-4)$	$\mathbf{-2}$
10.	$-8 \div (-4)$	$\mathbf{2}$
11.	$-8 \div 4$	$\mathbf{-2}$
12.	$8 \div 4$	$\mathbf{2}$
13.	$9 \div (-3)$	$\mathbf{-3}$
14.	$-9 \div 3$	$\mathbf{-3}$
15.	$-10 \div 5$	$\mathbf{-2}$
16.	$10 \div (-2)$	$\mathbf{-5}$
17.	$-10 \div (-2)$	$\mathbf{5}$
18.	$-10 \div (-5)$	$\mathbf{2}$
19.	$-14 \div 7$	$\mathbf{-2}$
20.	$14 \div (-2)$	$\mathbf{-7}$
21.	$-14 \div (-2)$	$\mathbf{7}$
22.	$-14 \div (-7)$	$\mathbf{2}$

23.	$-16 \div (-4)$	$\mathbf{4}$
24.	$16 \div (-2)$	$\mathbf{-8}$
25.	$-16 \div 4$	$\mathbf{-4}$
26.	$-20 \div 4$	$\mathbf{-5}$
27.	$-20 \div (-4)$	$\mathbf{5}$
28.	$-28 \div 4$	$\mathbf{-7}$
29.	$28 \div (-7)$	$\mathbf{-4}$
30.	$-28 \div (-7)$	$\mathbf{4}$
31.	$-40 \div (-5)$	$\mathbf{8}$
32.	$56 \div (-7)$	$\mathbf{-8}$
33.	$96 \div (-3)$	$\mathbf{-32}$
34.	$-121 \div (-11)$	$\mathbf{11}$
35.	$169 \div (-13)$	$\mathbf{-13}$
36.	$-175 \div 25$	$\mathbf{-7}$
37.	$1 \div 4$	$\frac{1}{4}$
38.	$-1 \div 4$	$-\frac{1}{4}$
39.	$-1 \div (-4)$	$\frac{1}{4}$
40.	$-3 \div (-4)$	$\frac{3}{4}$
41.	$-5 \div 20$	$-\frac{5}{20}$ *or* $-\frac{1}{4}$
42.	$6 \div (-18)$	$-\frac{6}{18}$ *or* $-\frac{1}{3}$
43.	$-24 \div 48$	$\mathbf{-2}$
44.	$-16 \div 64$	$-\frac{16}{64}$ *or* $-\frac{1}{4}$

Number Correct: ______

Integer Division – Round 2

Improvement: ______

Directions: Determine the quotient of the integers, and write it in the column to the right.

1.	$5 \div 1$		23.	$-18 \div (-9)$	
2.	$5 \div (-1)$		24.	$18 \div (-2)$	
3.	$-5 \div (-1)$		25.	$-18 \div 9$	
4.	$-5 \div 1$		26.	$-24 \div 4$	
5.	$6 \div 3$		27.	$-24 \div (-4)$	
6.	$-6 \div (-3)$		28.	$-24 \div 6$	
7.	$-6 \div 3$		29.	$30 \div (-6)$	
8.	$6 \div -3$		30.	$-30 \div (-5)$	
9.	$8 \div (-2)$		31.	$-48 \div (-6)$	
10.	$-8 \div (-2)$		32.	$64 \div (-4)$	
11.	$-8 \div 2$		33.	$105 \div (-7)$	
12.	$8 \div 2$		34.	$-144 \div (-12)$	
13.	$-9 \div (-3)$		35.	$196 \div (-14)$	
14.	$9 \div 3$		36.	$-225 \div 25$	
15.	$-12 \div 6$		37.	$2 \div 4$	
16.	$12 \div (-2)$		38.	$-2 \div 4$	
17.	$-12 \div (-2)$		39.	$-2 \div (-4)$	
18.	$-12 \div (-6)$		40.	$-4 \div (-8)$	
19.	$-16 \div 8$		41.	$-5 \div 40$	
20.	$16 \div (-2)$		42.	$6 \div (-42)$	
21.	$-16 \div (-2)$		43.	$-25 \div 75$	
22.	$-16 \div (-8)$		44.	$-18 \div 108$	

Integer Division – Round 2 [KEY]

Directions: Determine the quotient of the integers, and write it in the column to the right.

1.	$5 \div 1$	**5**	23.	$-18 \div (-9)$	**2**
2.	$5 \div (-1)$	**-5**	24.	$18 \div (-2)$	**-9**
3.	$-5 \div (-1)$	**5**	25.	$-18 \div 9$	**-2**
4.	$-5 \div 1$	**-5**	26.	$-24 \div 4$	**-6**
5.	$6 \div 3$	**2**	27.	$-24 \div (-4)$	**6**
6.	$-6 \div (-3)$	**2**	28.	$-24 \div 6$	**-4**
7.	$-6 \div 3$	**-2**	29.	$30 \div (-6)$	**-5**
8.	$6 \div -3$	**-2**	30.	$-30 \div (-5)$	**6**
9.	$8 \div (-2)$	**-4**	31.	$-48 \div (-6)$	**8**
10.	$-8 \div (-2)$	**4**	32.	$64 \div (-4)$	**-16**
11.	$-8 \div 2$	**-4**	33.	$105 \div (-7)$	**-15**
12.	$8 \div 2$	**4**	34.	$-144 \div (-12)$	**12**
13.	$-9 \div (-3)$	**3**	35.	$196 \div (-14)$	**-14**
14.	$9 \div 3$	**3**	36.	$-225 \div 25$	**-9**
15.	$-12 \div 6$	**-2**	37.	$2 \div 4$	**$\frac{2}{4}$ *or* $\frac{1}{2}$**
16.	$12 \div (-2)$	**-6**	38.	$-2 \div 4$	**$-\frac{2}{4}$ *or* $-\frac{1}{2}$**
17.	$-12 \div (-2)$	**6**	39.	$-2 \div (-4)$	**$\frac{2}{4}$ *or* $\frac{1}{2}$**
18.	$-12 \div (-6)$	**2**	40.	$-4 \div (-8)$	**$-\frac{4}{8}$ *or* $-\frac{1}{2}$**
19.	$-16 \div 8$	**-2**	41.	$-5 \div 40$	**$-\frac{5}{40}$ *or* $-\frac{1}{8}$**
20.	$16 \div (-2)$	**-8**	42.	$6 \div (-42)$	**$-\frac{6}{42}$ *or* $-\frac{1}{7}$**
21.	$-16 \div (-2)$	**8**	43.	$-25 \div 75$	**$-\frac{25}{75}$ *or* $-\frac{1}{3}$**
22.	$-16 \div (-8)$	**8**	44.	$-18 \div 108$	**$-\frac{18}{108}$ *or* $-\frac{1}{6}$**