Lesson 13: Converting Between Fractions and Decimals Using Equivalent Fractions

Student Outcomes

- Students understand that the context of a real-life situation often determines whether a rational number should be represented as a fraction or decimal.
- Students understand that decimals specify points on the number line by repeatedly subdividing intervals into tenths (*deci*- means one-tenth).
- Students convert positive decimals to fractions and fractions to decimals when the denominator is a product of only factors of either 2 or 5.

Classwork

Opening Exercise (4 minutes)

As was seen in Lesson 12, when dividing many integers the result is a non-integer quotient. These types of numbers are evident in the real world. For an Opening Exercise, direct students as they enter the room to provide responses to each of two questions posted on poster paper (questions listed below) using sticky notes.

The two questions to post are as follows:

1. What are some examples from the real world where decimals are used?

   Possible answers: Money, metric system, etc.

2. What are some examples from the real world where fractions are used?

   Possible answers: Some measurement (carpentry, cooking, etc.)

Discuss appropriate responses as a class; then, ask the following questions aloud:

- Have you ever seen a recipe call for 2.7 cups of flour? Why or why not?
  - *Measuring cups for cooking are generally labeled with $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, etc., for recipes requiring measurements in fractional cups.*
- How do you think people would react if a local gas station posted the price of gasoline as $3\frac{3}{7}$ dollars per gallon? Why?
  - *Dollars are never measured as $\frac{1}{6}$ or $\frac{1}{3}$ of a dollar; dollars are measured in decimal form using tenths and hundredths.*

Example 1 (1 minute): Representations of Rational Numbers in the Real World

Students describe in their own words why they need to know how to represent rational numbers in different ways.

Example 1: Representations of Rational Numbers in the Real World

Following the Opening Exercise and class discussion, describe why we need to know how to represent rational numbers in different ways.

Different situations in the real world require different representations of rational numbers. Because of common usage in life outside of the classroom, we may automatically know that a quarter of a dollar is the same as 25 cents, or a "quarter," but for people who are used to measuring money in only decimals, a quarter of a dollar might not make much sense.

Example 2 (10 minutes): Using Place Values to Write (Terminating) Decimals as Equivalent Fractions

MP.2

Students use the place value of the right-most decimal place in a terminating decimal to rewrite a positive rational number as an equivalent fraction.

Scaffolding:

- Provide or create a place-value chart to aid those who do not remember their place values or for ELL students who are unfamiliar with the vocabulary.

Example 2: Using Place Values to Write (Terminating) Decimals as Equivalent Fractions

a. **What is the value of the number 2.25? How can this number be written as a fraction or mixed number?**

Two and twenty-five hundredths or $2\frac{25}{100}$

- How do we rewrite this fraction (or any fraction) in its simplest form?
  - *If a factor is common to both the numerator and denominator of a fraction, the fraction can be simplified, resulting in a fraction whose numerator and denominator only have a common factor of* 1 *(the numerator and denominator are relatively prime).*

b. **Rewrite the fraction in its simplest form showing all steps that you use.**

$$\frac{25}{100} = \frac{25}{4\times 25} = \frac{1}{4} \quad \rightarrow \quad 2\frac{25}{100} = 2\frac{1}{4}$$

c. **What is the value of the number 2.025? How can this number be written as a mixed number?**

Two and twenty-five thousandths, or $2\frac{25}{1{,}000}$

d. **Rewrite the fraction in its simplest form showing all steps that you use.**

$$\frac{25}{1{,}000} = \frac{25}{100\times 10} \qquad \frac{25}{4\times 25\times 10} = \frac{1}{40} \quad \rightarrow \quad 2\frac{25}{1{,}000} = 2\frac{1}{40}$$

Scaffolding:

- Have students create a graphic organizer to relate the different representations of rational numbers, including fraction, decimals, and words. Pictures may also be used if applicable.

Exercise 1 (5 minutes)

Exercise 1

Use place value to convert each terminating decimal to a fraction. Then rewrite each fraction in its simplest form.

a. 0.218

$\frac{218}{1{,}000} = \frac{109\times 2}{500\times 2} = \frac{109}{500} \rightarrow 0.218 = \frac{109}{500}$

b. 0.16

$\frac{16}{100} = \frac{4\times 4}{4\times 25} = \frac{4}{25} \rightarrow 0.16 = \frac{4}{25}$

c. 2.72

$\frac{72}{100} = \frac{4\times 18}{4\times 25} = \frac{18}{25} \rightarrow 2.72 = 2\frac{18}{25}$

d. 0.0005

$\frac{5}{10{,}000} = \frac{5\times 1}{5\times 2{,}000} = \frac{1}{2{,}000} \rightarrow 0.0005 = \frac{1}{2{,}000}$

- What do you notice about the denominators of fractions that represent each decimal place?
  - *The denominators are all powers of* 10.
- What are the prime factors of 10? 100? $1{,}000$?
  - $10 = 2 \times 5$ $\quad 100 = 2^2 \times 5^2$ $\quad 1{,}000 = 2^3 \times 5^3$
    $10^1 = 2 \times 5$ $\quad 10^2 = 2^2 \times 5^2$ $\quad 10^3 = 2^3 \times 5^3$
- What prime factors make up the powers of 10?
  - *The powers of* 10 *contain only the factors* 2 *and* 5 *and in each case the number of factors of* 2 *and* 5 *are equal to the number of factors of* 10.
- How can the prime factorization of the powers of ten be used to write fractions in decimal form?
  - *Find an equivalent fraction whose denominator is a power of ten, then write the decimal representation using place values.*

Example 3 (10 minutes): Converting Fractions to Decimals—Fractions with Denominators Having Factors of only 2 or 5

Discuss the meaning of the term *decimal* as it is derived from the Latin word *decimus,* meaning *one-tenth*.

- What is the meaning of *one-tenth?* Provide real-world examples where *tenths* are regularly used.
  - *If a unit has been divided into ten equal-sized pieces, then one-tenth is the value of one of those ten pieces. A dime is one-tenth of a dollar; a penny is one-tenth of a dime.*

Scaffolding:

- The prefix *deci-* is also used in the metric system of measurement in which its meaning is one-tenth of a unit.

Students use equivalent fractions whose denominators include only the factors 2 and 5 to write decimal representations of rational numbers.

Example 3: Converting Fractions to Decimals—Fractions with Denominators Having Factors of only 2 or 5

a. **What are *decimals*?**

Decimals specify points on the number line by repeatedly subdividing intervals into tenths. If a unit is divided into ten equal-sized pieces, one piece would be one-tenth of that unit.

b. **Use the meaning of *decimal* to relate decimal place values.**

Each place value in a decimal is $\frac{1}{10}$ of the value of the place to its left. This means that the denominators of the fractions that represent each decimal place value must be powers of ten.

c. **Write the number $\frac{3}{100}$ as a decimal. Describe your process.**

The decimal form is 0.03. The fraction includes a power of ten, 100, as its denominator. The value of the second decimal place is $\frac{1}{100}$, so $\frac{3}{100}$ in decimal form is 0.03.

- How could we obtain an equivalent fraction to $\frac{3}{20}$ with a power of ten in the denominator?
  - *If there was another factor of 5 in the denominator, then we would have an equal number of 2's and 5's resulting in power of ten. If we multiply the fraction by $\frac{5}{5}$ (or 1), we get an equivalent fraction with a power of ten in its denominator.*

d. **Write the number $\frac{3}{20}$ as a decimal. Describe your process.**

The fractional form is $\frac{3}{20} = \frac{3}{2^2\times 5}$. The denominator lacks a factor of 5 to be a power of ten. To arrive at the decimal form I multiply the fractional form by $\frac{5}{5}$ to arrive at $\frac{3}{2^2\times 5}\times\frac{5}{5} = \frac{3\times 5}{2^2\times 5^2} = \frac{15}{100}$, and $\frac{15}{100} = 0.15$.

e. **Write the number $\frac{10}{25}$ as a decimal. Describe your process.**

The fractional form is $\frac{10}{25} = \frac{2\times 5}{5\times 5}$; and, since $\frac{5}{5} = 1$, then $\frac{2\times 5}{5\times 5} = \frac{2}{5}$. The denominator lacks a factor of 2 to be a power of ten. To arrive at the decimal form I multiply the fractional form by $\frac{2}{2}$ to arrive at $\frac{2}{5}\times\frac{2}{2} = \frac{4}{10}$, and $\frac{4}{10} = 0.4$

f. **Write the number $\frac{8}{40}$ as a decimal. Describe your process.**

The fractional form is $\frac{8}{40} = \frac{2^3}{2^3\times 5}$. There are factors of 2 in the numerator and denominator that will cancel. If I leave one factor of two in the denominator, it will be 10 (a power of ten).

$$\frac{2^3}{2^3 \times 5} = \frac{2^2 \times 2}{2^2 \times 2 \times 5} = \frac{2}{2 \times 5} = \frac{2}{10} = 0.2$$

Exercise 2 (5 minutes)

Students convert fractions to decimal form using equivalent fractions.

Exercise 2

Convert each fraction to a decimal using an equivalent fraction.

a. $\frac{3}{16} =$

$\frac{3}{16} = \frac{3}{2^4} \rightarrow \frac{3 \times 5^4}{2^4 \times 5^4} = \frac{1,875}{10,000} \rightarrow$

$\frac{1,875}{10,000} = 0.1875$

b. $\frac{7}{5} =$

$\frac{7}{5} \rightarrow \frac{7 \times 2}{5 \times 2} = \frac{14}{10} \rightarrow \frac{14}{10} = 1\frac{4}{10} = 1.4$

c. $\frac{11}{32} =$

$\frac{11}{32} = \frac{11}{2^5} \rightarrow \frac{11 \times 5^5}{2^5 \times 5^5} = \frac{34,375}{100,000} \rightarrow$

$\frac{34,375}{100,000} = 0.34375$

d. $\frac{35}{50} =$

$\frac{35}{50} = \frac{5 \times 7}{5^2 \times 2} \rightarrow \frac{7}{5 \times 2} = \frac{7}{10} \rightarrow \frac{7}{10} = 0.7$

Closing (5 minutes)

The closing questions reinforce the important ideas in the lesson.

- When asked to write a decimal value as a fraction (or mixed number), how do we determine the value of the denominator?
  - *The place value of the right-most decimal place shares the same denominator as an equivalent fraction representing the decimal.*
- If the denominator of a fraction in its simplest form has four factors of 2 and seven factors of 5, describe two different ways in which a power of ten can be obtained in the denominator.
  - *Three factors of 2 could be multiplied in to obtain an equivalent fraction, or three factors of 5 could be divided out to obtain a different equivalent fraction.*
- Consider for Lesson 14: Do you think it is possible to write a fraction whose denominator has factors other than 2 and 5 as a decimal?

Lesson Summary

Any terminating decimal can be converted to a fraction using place value (e.g., 0.35 is thirty-five hundredths or $\frac{35}{100}$). A fraction whose denominator includes only factors of 2 and 5 can be converted to a decimal by writing the denominator as a power of ten.

Exit Ticket (5 minutes)

Name ____ Date____________________

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Exit Ticket

1. Write 3.0035 as a fraction. Explain your process.

2. This week is just one of 40 weeks that you spend in the classroom this school year. Convert the fraction $\frac{1}{40}$ to decimal form.

Exit Ticket Sample Solutions

1. Write 3.0035 as a fraction. Explain your process.

   The left-most decimal place is the ten-thousandths place, so the number in fractional form would be $3\frac{35}{10{,}000}$. There are common factors of 5 in the numerator and denominator and dividing both by these results in the fraction $3\frac{7}{2{,}000}$.

2. This week is just one of 40 weeks that you spend in the classroom this school year. Convert the fraction $\frac{1}{40}$ to decimal form.

   $$\frac{1}{40} = \frac{1}{2^3 \times 5} \times \frac{5^2}{5^2}$$
   $$\frac{5^2}{2^3 \times 5^3} = \frac{25}{1{,}000} = 0.025$$

Scaffolding:

- Extend Exit Ticket Problem 2 by asking students to represent this week as a percentage of the school year.

  Answer: 2.5%

Problem Set Sample Solutions

1. Convert each terminating decimal to a fraction in its simplest form.

   a. 0.4

   $0.4 = \frac{2}{5}$

   b. 0.16

   $0.16 = \frac{4}{25}$

   c. 0.625

   $0.625 = \frac{5}{8}$

   d. 0.08

   $0.08 = \frac{2}{25}$

   e. 0.012

   $0.012 = \frac{3}{250}$

2. Convert each fraction or mixed number to a decimal using an equivalent fraction.

   a. $\frac{4}{5}$

   $\frac{4}{5} = 0.8$

   b. $\frac{3}{40}$

   $\frac{3}{40} = 0.075$

   c. $\frac{8}{200}$

   $\frac{8}{200} = 0.04$

   d. $3\frac{5}{16}$

   $3\frac{5}{16} = 3.3125$

3. Tanja is converting a fraction into a decimal by finding an equivalent fraction that has a power of 10 in the denominator. Sara looks at the last step in Tanja's work (shown below) and says that she cannot go any further. Is Sara correct? If she is, explain why. If Sara is incorrect, complete the remaining steps.

   $$\frac{72}{480} = \frac{2^3 \cdot 3^2}{2^5 \cdot 3 \cdot 5}$$

   Tanja can finish the conversion since there is a factor pair of 3's in the numerator and denominator that can be divided out with a quotient of 1.

   Remaining Steps:

$$\frac{720}{480} = \frac{2^3 \cdot 3^2}{2^5 \cdot 3 \cdot 5} = \frac{3}{2^2 \cdot 5}\left(\frac{5}{5}\right) = \frac{3 \cdot 5}{2^2 \cdot 5^2} = \frac{15}{100}$$

Answer: 0.15