Lesson 7: Addition and Subtraction of Rational Numbers

Student Outcomes

- Students recognize that the rules for adding and subtracting integers apply to rational numbers.
- Given a number line, students use arrows to model rational numbers where the length of the arrow is the absolute value of the rational number and the sign of the rational number is determined by the direction of the arrow with respect to the number line.
- Students locate the sum $p + q$ of two rational numbers on a number line by placing the tail of the arrow for q at p and locating $p + q$ at the head of the arrow. They create an arrow for the difference $p - q$ by first rewriting the difference as a sum, $p + (-q)$, and then locating the sum.

Classwork

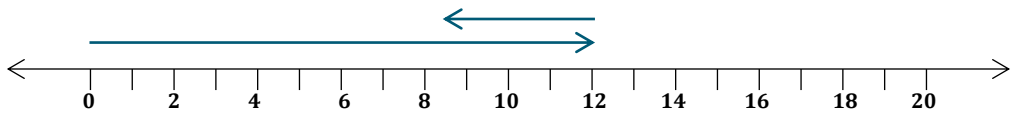
Exercise 1 (5 minutes)

MP.4 Students answer the following question independently as the teacher circulates around the room providing guidance and feedback as needed. Students focus on how to represent the answer using both an equation and a number line diagram.

Exercise 1

Suppose a 7th grader's birthday is today, and she is 12 years old. How old was she $3\frac{1}{2}$ years ago? Write an equation and use a number line to model your answer.

$12 + \left(-3\frac{1}{2}\right) = 8\frac{1}{2}$ *or* $12 - 3\frac{1}{2} = 8\frac{1}{2}$



Example 1 (5 minutes): Representing Sums of Rational Numbers on a Number Line

MP.6 Teacher leads a whole group instruction illustrating the sum of $12 + \left(-3\frac{1}{2}\right)$ on a number line. Elicit student responses to assist in creating the steps. Students record the steps and diagram.

Example 1: Representing Sums of Rational Numbers on a Number Line

a. **Place the tail of the arrow on 12.**

b. **The length of the arrow is the absolute value of $-3\frac{1}{2}$, $\left|-3\frac{1}{2}\right| = 3\frac{1}{2}$.**

c. **The direction of the arrow is to the *left* since you are adding a negative number to 12.**

Scaffolding:

- Laminate an index card with the steps for Examples 1 and 2 and the number line diagram so that students can easily refer to it.

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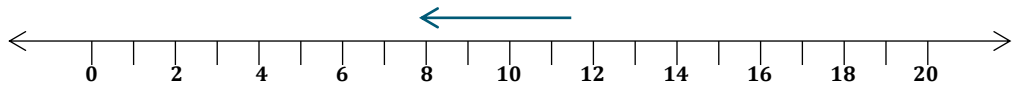
Draw the number line model in the space below.

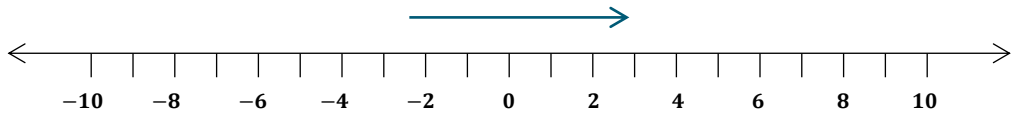


$12 + \left(-3\frac{1}{2}\right) = 8\frac{1}{2}$ *or* $12 - 3\frac{1}{2} = 8\frac{1}{2}$

Exercise 2 (3 minutes)

Exercise 2

Find the following sum using a number line diagram. $-2\frac{1}{2} + 5$.



$\left(-2\frac{1}{2}\right) + 5 = 2\frac{1}{2}$

Example 2 (5 minutes): Representing Differences of Rational Numbers on a Number Line

Teacher leads a whole group instruction illustrating how to find the difference of $1 - 2\frac{1}{4}$ on a number line. Elicit student responses to assist in creating the steps. Students record the steps and diagram.

Scaffolding:

- Have students subdivide their number line intervals into $\frac{1}{2}$'s and $\frac{1}{4}$'s.

Example 2: Representing Differences of Rational Numbers on a Number Line

a. **Rewrite the difference $1 - 2\frac{1}{4}$ as a sum: $1 + \left(-2\frac{1}{4}\right)$.**

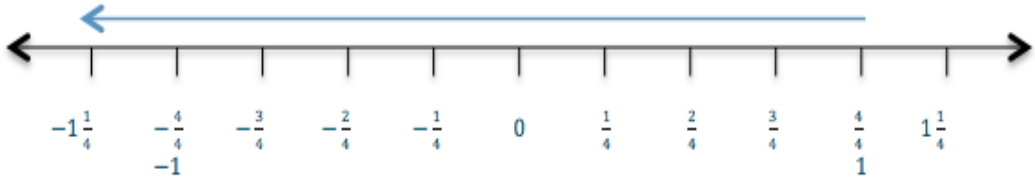
<u>Now follow the steps to represent the sum</u>:

b. **Place the tail of the arrow on 1.**

c. **The length of the arrow is the absolute value of $-2\frac{1}{4}$; $\left|-2\frac{1}{4}\right| = 2\frac{1}{4}$.**

d. **The direction of the arrow is to the *left* since you are adding a negative number to 1.**

Draw the number line model in the space below.



$1 + \left(-2\frac{1}{4}\right) = -1\frac{1}{4}$

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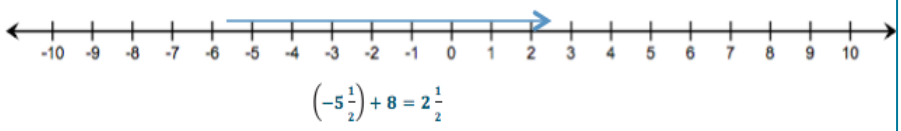
Exercise 3 (3 minutes)

Exercise 3

Find the following difference, and represent it on a number line. $-5\frac{1}{2} - (-8)$.



Scaffolding:

- Ask students to explain and justify what they drew to check for understanding.
- Ask probing questions such as "Why does your arrow go to the right?"

Exercise 4 (10 minutes)

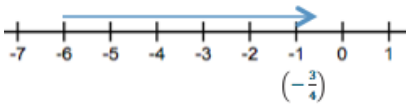
Next, students work independently in Exercise 4 to create a number line model to represent each sum or difference. After 5–7 minutes, students are selected to share their responses and work with the class.

Exercise 4

Find the following sums and differences using a number line model.

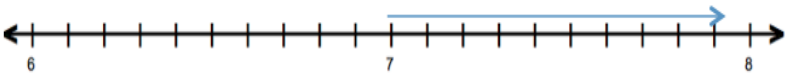
a. $-6 + 5\frac{1}{4}$

$-6 + 5\frac{1}{4} = -\frac{3}{4}$



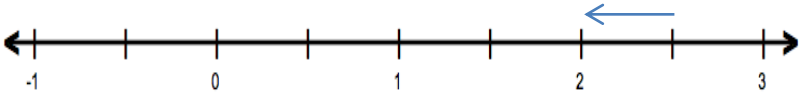
b. $7 - (-0.9)$

$7 + (0.9) = 7.9$



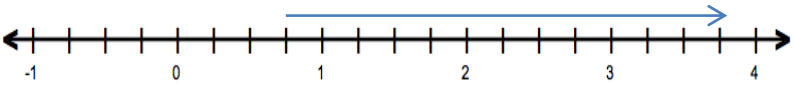
c. $2.5 + \left(-\frac{1}{2}\right)$

$2.5 + (-0.5) = 2$



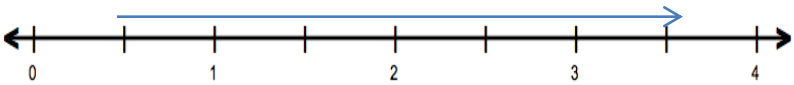
d. $-\frac{1}{4} + 4$

$-\frac{1}{4} + 4 = 3\frac{3}{4}$



e. $\frac{1}{2} - (-3)$

$\frac{1}{2} + 3 = 3\frac{1}{2}$



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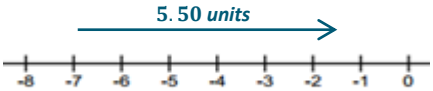
Exercise 5 (6 minutes)

Exercise 5

Create an equation and number line diagram to model each answer.

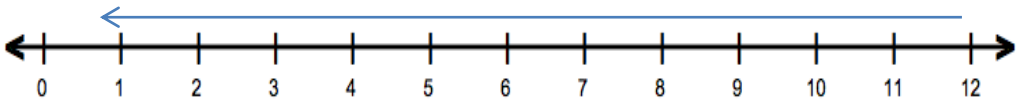
a. **Samantha owes her father $\$7$. She just got paid $\$5.50$ for babysitting. If she gives that money to her dad, how much will she still owe him?**

$-7 + 5.50 = -1.50$. *She still owes him* $\$1.50$.



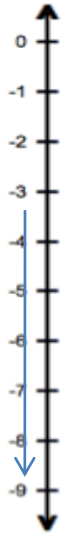
b. **At the start of a trip, a car's gas tank contains 12 gallons of gasoline. During the trip, the car consumes $10\frac{1}{8}$ gallons of gasoline. How much gasoline is left in the tank?**

$12 + \left(-10\frac{1}{8}\right) = 1\frac{7}{8}$ *or* $12 - 10\frac{1}{8} = 1\frac{7}{8}$, $1\frac{7}{8}$ *gallons*



c. **A fish was swimming $3\frac{1}{2}$ feet below the water's surface at $7:00$ a.m. Four hours later, the fish was at a depth that is $5\frac{1}{4}$ feet below where it was at $7:00$ a.m. What rational number represents the position of the fish with respect to the water's surface at $11:00$ a.m.?**

$-3\frac{1}{2} + \left(-5\frac{1}{4}\right) = -8\frac{3}{4}$. *The fish is* $8\frac{3}{4}$ *feet below the water's surface.*



Follow-Up Discussion

For Exercise 5(a) discuss with students how the mathematical answer of -1.50 means Samantha owes her father $\$1.50$ and that we do not say she owes her father $-\$1.50$.

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Closing (3 minutes)

- What challenges do you face when using the number line model to add non-integer rational numbers?
  - *Answers will vary.*
- When using a number line to model $8-(-2.1)$, how many units do we move from 8 and in what direction? Where is the tail of the arrow, and where is the head? What does your arrow represent?
  - *First, we would change the expression to an addition expression, $8+2.1$. The tail of the arrow would start at 8, the first addend. The arrow would be 2.1 units long and pointing to the right, which would mean the arrow would end on 10.1. The arrow represents the second addend.*

Lesson Summary

The rules for adding and subtracting integers apply to all rational numbers.

The sum of two rational numbers (e.g., $-1+4.3$) can be found on the number line by placing the tail of an arrow at -1 and locating the head of the arrow 4.3 units to the right to arrive at the sum, which is 3.3.

To model the difference of two rational numbers on a number line (e.g., $-5.7-3$), first rewrite the difference as a sum, $-5.7+(-3)$, and then follow the steps for locating a sum. Place a single arrow with its tail at -5.7 and the head of the arrow 3 units to the left to arrive at -8.7.

Exit Ticket (5 minutes)

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Name _____ Date_____

Lesson 7: Addition and Subtraction of Rational Numbers

Exit Ticket

At the beginning of the summer, the water level of a pond is 2 feet below its normal level. After an unusually dry summer, the water level of the pond dropped another $1\frac{1}{3}$ feet.

1. Use a number line diagram to model the pond's current water level in relation to its normal water level.

2. Write an equation to show how far above or below the normal water level the pond is at the end of the summer.

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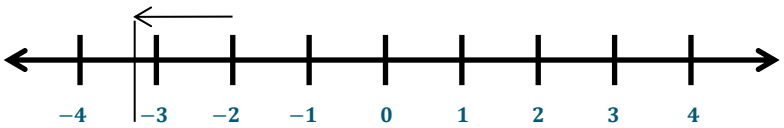
Exit Ticket Sample Solutions

At the beginning of the summer, the water level of a pond is 2 feet below its normal level. After an unusually dry summer, the water level of the pond dropped another $1\frac{1}{3}$ feet.

1. Use a number line diagram to model the pond's current water level in relation to its normal water level.

Move $1\frac{1}{3}$ units to the left of -2. $-3\frac{1}{3}$



2. Write an equation to show how far above or below the normal water level the pond is at the end of the summer.

$-2-1\frac{1}{3}=-3\frac{1}{3}$ *or* $-2+\left(-1\frac{1}{3}\right)=-3\frac{1}{3}$

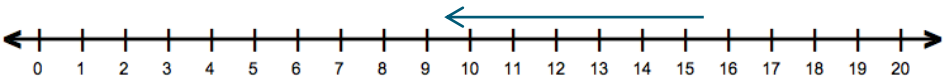
Problem Set Sample Solutions

Represent each of the following problems using both a number line diagram and an equation.

1. A bird that was perched atop a $15\frac{1}{2}$-foot tree dives down six feet to a branch below. How far above the ground is the bird's new location?

$15\frac{1}{2}+(-6)=9\frac{1}{2}$ *or* $15\frac{1}{2}-6=9\frac{1}{2}$

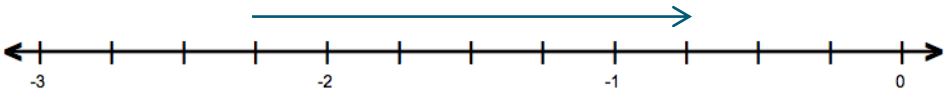
The bird is $9\frac{1}{2}$ feet above the ground.



2. Mariah had owed her grandfather $\$2.25$ but was recently able to pay him back $\$1.50$. How much does Mariah currently owe her grandfather?

$-2.25+1.50=-0.75$

Mariah owes her grandfather 75 cents.



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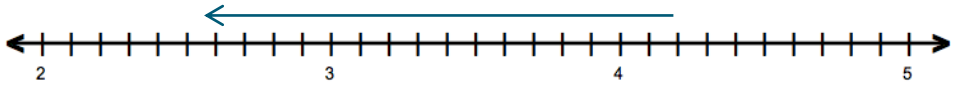
3. Jake is hiking a trail that leads to the top of a canyon. The trail is 4.2 miles long, and Jake plans to stop for lunch after he completes 1.6 miles. How far from the top of the canyon will Jake be when he stops for lunch?

$4.2 - 1.6 = 2.6$

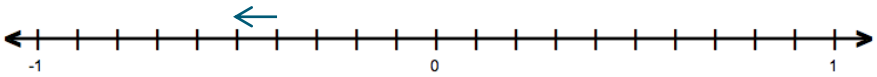
Jake will be 2.6 miles from the top of the canyon.



4. Sonji and her friend Rachel are competing in a running race. When Sonji is 0.4 miles from the finish line, she notices that her friend Rachel has fallen. If Sonji runs one tenth of a mile back to help her friend, how far will she be from the finish line?

$-0.4 + (-0.1) = -0.5$ *or* $-0.4 - 0.1 = -0.5$

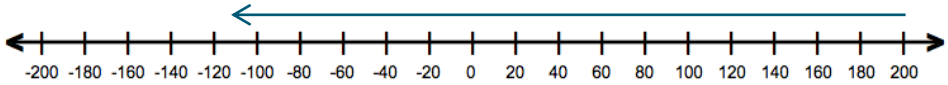
Sonji will be 0.5 miles from the finish line.



5. Mr. Henderson did not realize his checking account had a balance of $\$200$ when he used his debit card for a $\$317.25$ purchase. What is his checking account balance after the purchase?

$200 + (-317.25) = -117.25$ *or* $200 - 317.25 = -117.25$

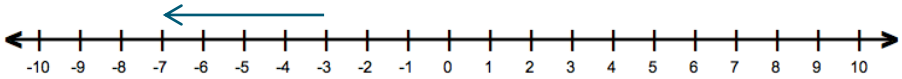
Mr. Henderson's checking account balance will be $-\$117.25$.



6. If the temperature is $-3°\text{F}$ at $10:00$ p.m. and the temperature falls four degrees overnight, what is the resulting temperature?

$-3 - 4 = -3 + (-4) = -7$

The resulting temperature is $-7°F$.



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