



Lesson 6: The Distance Between Two Rational Numbers

Student Outcomes

- Students justify the distance formula for rational numbers on a number line: If p and q are rational numbers on a number line, then the distance between p and q is $|p - q|$.
- Students know the definition of subtraction in terms of addition (i.e., $a - b = c$ means that $b + c = a$) and use the definition of subtraction to justify the distance formula.
- Students solve word problems involving changes in distance or temperature.

Classwork

Exercise 1 (4 minutes)

Students are in groups of 2; one person is Person A, and the other is Person B. Using a number line, each person independently counts the number of units that make up the distance between the two numbers listed in his assigned column. (Teacher may want to provide students with larger number lines.)

Exercise 1

Use the number line to answer each of the following.

Person A	Person B
What is the distance between -4 and 5 ? 9 	What is the distance between 5 and -4 ? 9
What is the distance between -5 and -3 ? 2 	What is the distance between -3 and -5 ? 2
What is the distance between 7 and -1 ? 8 	What is the distance between -1 and 7 ? 8

After 3 minutes, partners share their answers and determine that their distances are the same because their endpoints are the same.

Note: A common mistake is that students count the first number as one. Another common mistake is that students describe the distance as negative.

Scaffolding:

- Students may find it easier to see the distance if they use a highlighter on the number line and highlight the distance between the two numbers.

Discussion (5 minutes)

Follow-up Discussion

- What was the distance between -4 and 5 ?
 - 9
- What was the distance between 5 and -4 ?
 - 9
- Were you and your partner’s answers the same for the second and third problems as well?
 - *Yes.*
- Why did you both get the same answers for all three problems?
 - *Because the endpoints were the same, so the distance between them is the same.*
- Take a minute to share with your partner a destination that you or your family usually travel to, for example, a relative’s house or the location of an activity you attend each week. Assuming you take the same route to and from the location, what is the approximate distance in miles that you travel each way?
 - *Answers will vary, but students should recognize that the distance is the same to and from the location and should state the same number of miles whether traveling from home to the location, or from the location back home.*
- In life, at any given moment, will we always be able to use a number line to find the distance between two rational numbers? Is it the most efficient way to calculate distance between the two points?
 - *No.*
- What represents the distance between a number and zero on the number line?
 - *Absolute value*
- If the distance between 5 and 0 can be calculated using $|5 - 0|$, or $|5|$, do you think we might be able to calculate the distance between -4 and 5 using absolute value? Take a minute to see if it works.
 - *Yes; $|-4 - 5| = |-9| = 9$ and $|5 - (-4)| = |9| = 9$, which is the answer we found in Exercise 1.*
- Will this work for the other two distances we looked at in Exercise 1? Take a minute to test it out.
 - *Yes.*

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Exercise 2 (5 minutes)

Students now work independently using the formula to find the distance between each of the two given endpoints. They should verify their answer by using a number line model.

Scaffolding:

- Consider having students determine the distance on the number line first, and then use the formula to verify (rather than the other way around).

Exercise 2

Use the number line to answer each of the following questions.

a. What is the distance between 0 and -8 ?

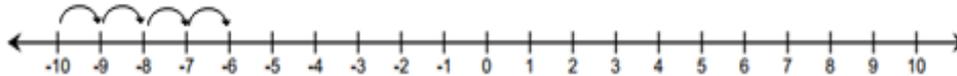
$|0 - (-8)| = |0 + 8| = |8| = 8$

b. What is the distance between -2 and $-1\frac{1}{2}$?



$$\left| -2 - \left(-1\frac{1}{2}\right) \right| = \left| -2 + 1\frac{1}{2} \right| = \left| -\frac{1}{2} \right| = \frac{1}{2}$$

c. What is the distance between -6 and -10 ?



$$\left| -6 - (-10) \right| = \left| -6 + 10 \right| = \left| 4 \right| = 4$$

Scaffolding:

- Before going over answers as a class, have students share their findings with their learning partner. They should justify their findings in words and be ready to share with the class.

Example 1 (3 minutes): Formula for the Distance Between Two Rational Numbers

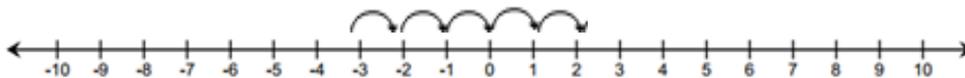
If p and q are rational numbers on a number line, then the distance between p and q is $|p - q|$. It does not matter which endpoint we call p or which endpoint we call q . Distance is always positive.

Example 1: Formula for the Distance Between Two Rational Numbers

Find the distance between -3 and 2 .

Step 1: Start on an endpoint.

Step 2: Count the number of units from the endpoint you started on to the other endpoint.



Using a formula,

$$\left| -3 - 2 \right| = \left| -3 + -2 \right| = \left| -5 \right| = 5 \quad \text{OR} \quad \left| 2 - (-3) \right| = \left| 2 + 3 \right| = \left| 5 \right| = 5$$

For two rational numbers p and q , the distance between p and q is $|p - q|$.

Example 2 (5 minutes): Change in Elevation vs. Distance

Whole-group instruction; students record examples in their student materials.

Example 2: Change in Elevation vs. Distance

Distance is positive. Change in elevation or temperature may be positive or negative depending on whether it is increasing or decreasing (going up or down).

- a. A hiker starts hiking at the beginning of a trail at a point which is 200 feet below sea level. He hikes to a location on the trail that is 580 feet above sea level and stops for lunch.

- i. What is the vertical distance between 200 feet below sea level and 580 feet above sea level?

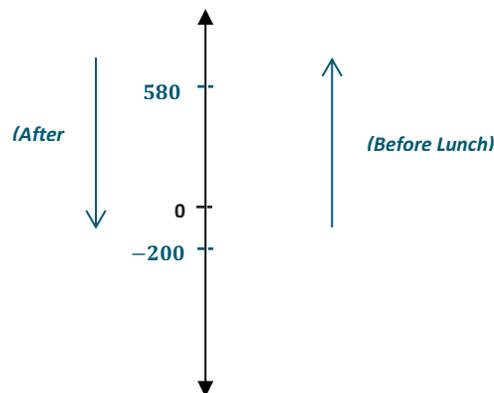
$$|-200 - 580| = |-200 + (-580)| = |-780| = 780$$

The vertical distance is 780 feet.

- ii. How should we interpret 780 feet in the context of this problem?

The hiker hiked 780 feet from a point below sea level to a point above sea level.

- b. After lunch, the hiker hiked back down the trail from the point of elevation, which is 580 feet above sea level, to the beginning of the trail which is 200 feet below sea level.



- i. What is the vertical distance between 580 feet above sea level and 200 feet below sea level?

$$|580 - (-200)| = |580 + 200| = |780| = 780$$

The vertical distance is 780 feet.

- ii. What is the change in elevation?

780 feet

Exercise 3 (4 minutes)

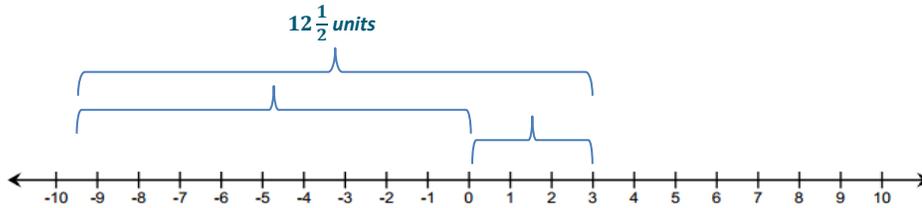
Students work with a partner to come up with a solution to the following exercise. They use the distance formula to verify their answers but may first need to use a number line model to arrive at the two numbers for their solutions.

Note: A student may be tempted to use zero as one of the numbers; if that is the case, ask the student if zero is negative or positive.

Exercise 3

The distance between a negative number and a positive number is $12\frac{1}{2}$. What are the numbers?

Answers will vary; a possible solution is $-9\frac{1}{2}$ and 3. $|-9\frac{1}{2} - 3| = |-9\frac{1}{2} + (-3)| = |-12\frac{1}{2}| = 12\frac{1}{2}$.



$|-9\frac{1}{2}| = 9\frac{1}{2}$ $|3| = 3$

- Consider the different solutions students came up with, and notice that for each case if we take the absolute value of each of the endpoints, their sum is $12\frac{1}{2}$. Is that the case for Exercises 2 (a)–(c)? Why not?
 - No, that is not always the case. If you have a positive number and a negative number as endpoints, they are separated by zero. And since absolute value is a number's distance from zero, taking the absolute value of each endpoint and adding them together will give you the same results as using the formula, $|p - q|$. However, if the signs of each endpoint are the same, you cannot calculate the distance between the two endpoints by finding the sum of their absolute values.

Scaffolding:

- For parts (b)–(e), visual learners will benefit from using the number line to break down the distance into two sections, from zero to each given number.

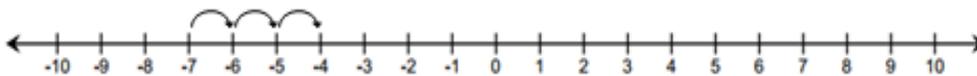
Exercise 4 (10 minutes)

Exercise 4

Use the distance formula to find each answer. Support your answer using a number line diagram.

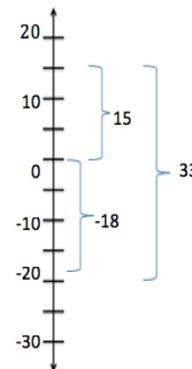
- a. Find the distance between -7 and -4 .

$|-7 - (-4)| = |-7 + 4| = |-3| = 3$ units



- b. Find the change in temperature if the temperature rises from -18°F to 15°F (use a vertical number line).

$|15 - (-18)| = |15 + 18| = |33| = 33^\circ\text{F}$

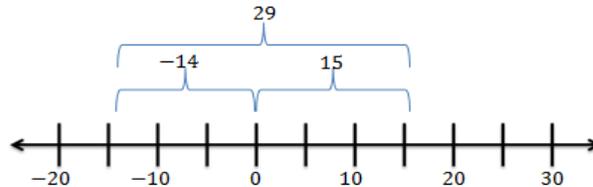


c. Would your answer for part (b) be different if the temperature dropped from 15°F to −18°F? Explain.

Yes. The distance between 15 and −18 on a number line is the same as the distance between −18 and 15 because the endpoints are the same, but the temperature would change by −33°F because it is going from a higher value to a lower value.

d. Beryl is the first person to finish a 5K race and is standing 15 feet beyond the finish line. Another runner, Jeremy, is currently trying to finish the race and has approximately 14 feet before he reaches the finish line. What is the minimum possible distance between Beryl and Jeremy?

$$|-14 - 15| = |-14 + (-15)| = |-29| = 29 \text{ feet}$$



e. What is the change in elevation from 140 feet above sea level to 40 feet below sea level? Explain.

180 feet. I used the distance formula: $|140 - (-40)| = 180$ and a vertical number line to show 180 feet between the two locations. But since you are moving from above sea level to below sea level, the change is negative.

Closing (3 minutes)

- How can we use a number line to find the distance between two rational numbers?
 - We can count the number of units in between the two numbers.
- What does it mean to find the absolute value of a number?
 - You are finding the distance between that number and zero.
- Is it possible to use absolute value to find the distance between a number, p , and another number, q , that is not zero? If so, how?
 - Yes, instead of using $|p - 0|$ you would use $|p - q|$.
- Is distance always positive? Is change always positive?
 - Distance is always positive, but change can be positive or negative.

Lesson Summary

- To find the distance between two rational numbers on a number line, you can count the number of units between the numbers.
- Using a formula, the distance between rational numbers, p and q , is $|p - q|$.
- Distance is always positive.
- Change may be positive or negative. For instance, there is a -4° change when the temperature goes from 7° to 3° .

Exit Ticket (6 minutes)



Name _____

Date _____

Lesson 6: The Distance Between Two Rational Numbers

Exit Ticket

Two 7th grade students, Monique and Matt, both solved the following math problem:

If the temperature drops from 7°F to -17°F , by how much did the temperature *decrease*?

The students came up with different answers. Monique said the answer is 24°F , and Matt said the answer is 10°F . Who is correct? Explain, and support your written response with the use of a formula and a vertical number line diagram.

Exit Ticket Sample Solutions

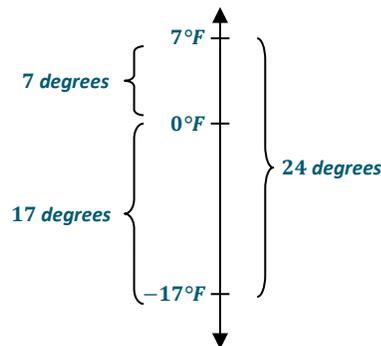
Two 7th grade students, Monique and Matt, both solved the following math problem:

If the temperature drops from 7°F to -17°F, by how much did the temperature decrease?

The students came up with different answers. Monique said the answer is 24°F, and Matt said the answer is 10°F. Who is correct? Explain, and support your written response with the use of a formula and a vertical number line diagram.

Monique is correct. If you use the distance formula, you take the absolute value of the difference between 7 and -17 and that equals 24. Using a number line diagram you can count the number of units between 7 and -17 to get 24.

$|7 - (-17)| = |7 + 17| = |24| = 24$. There was a 24°F drop in the temperature.



Problem Set Sample Solutions

1. $|-19 - 12| = |-19 + (-12)| = |-31| = 31$

2. $|19 - (-12)| = |19 + 12| = |31| = 31$

3. $|10 - (-43)| = |10 + 43| = |53| = 53$

4. $|-10 - 43| = |-10 + (-43)| = |-53| = 53$

5. $|-1 - (-16)| = |-1 + 16| = |15| = 15$

6. $|1 - 16| = |1 + (-16)| = |-15| = 15$

7. $|0 - (-9)| = |0 + 9| = |9| = 9$

8. $|0 - 9| = |0 + (-9)| = |-9| = 9$

9. $|-14.5 - 13| = |-14.5 + (-13)| = |-27.5| = 27.5$

10. $|14.5 - (-13)| = |14.5 + 13| = |27.5| = 27.5$

11. Describe any patterns you see in the answers to the problems in the left- and right-hand columns. Why do you think this pattern exists?

Each problem in the right-hand column has the same answer as the problem across from it in the left-hand column. That is because you are finding the distance between the opposite numbers as compared to the first column. The difference between the opposite numbers is opposite the difference between the original numbers. The absolute values of opposite numbers are the same.

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