



## Lesson 4: Efficiently Adding Integers and Other Rational Numbers

### Student Outcomes

- Students understand the rules for adding rational numbers:
  - Add rational numbers with the same sign by adding the absolute values and using the common sign.
  - Add rational numbers with opposite signs by subtracting the smaller absolute value from the larger absolute value and using the sign of the number with the larger absolute value.
- Students justify the rules using arrows and a number line or by using the Integer Game. They extend their findings to begin to include sums of rational numbers.

### Classwork

#### Exercise 1 (5 minutes): Hands Up, Pair Up!

MP.3 Students review concepts from Lessons 1 through 3 by playing the Kagan Strategy Game, “Hands Up, Pair Up!” (Refer to the description at the end of this lesson.)<sup>1</sup> During play, students should critique each other’s questions when necessary. They should use accurate vocabulary learned so far in this module when explaining and defending their answers. The following are possible student questions:

- When playing the Integer Game, you have 3 cards in your hand with a sum of  $-15$ . Then, you draw a  $(-5)$  card. Using addition, how would you write an equation to represent your score?
- What is the absolute value of 15?
- What is the sum of  $-4 + (-10)$ ?
- In what direction does the arrow point on a number line for a negative number?
- What is an additive inverse of 5? What is the additive inverse of  $-9$ ? What is the additive inverse of a number?

#### Scaffolding:

- Provide some pre-made index cards for learners who struggle forming a question with limited time.
- Ask students to refer to anchor posters for support during the game.

<sup>1</sup> Allow students 1–2 minutes to think of a question and record it on an index card. Write the answer to the question on the back. Ask the class to stand up, each person with one hand in the air. Students will find partners and greet each other with a high-five. Once a pair is formed, partners will take turns asking each other their questions. After both partners have asked and answered each other’s questions, they will switch cards. Both partners will again raise their hands to signify they are ready for a new partner and repeat the exercise. Allow enough time for each student to partner with 2–3 different people.

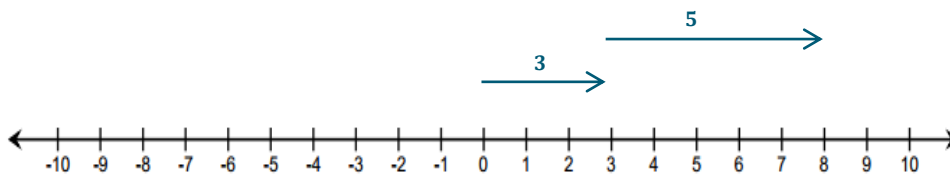
**Example 1 (5 minutes): Rule for Adding Integers with Same Signs**

The teacher leads the whole class to find the sum of  $3 + 5$ .

- In the Integer Game, I would combine 5 and 3 to give me 8.

**Example 1: Rule for Adding Integers with Same Sign**

a. Represent the sum of  $3 + 5$  using arrows on the number line.



$3 + 5 = 8$

i. How long is the arrow that represents 3?

*3 units*

ii. What direction does it point?

*Right/up*

iii. How long is the arrow that represents 5?

*5 units*

iv. What direction does it point?

*Right/up*

v. What is the sum?

*8*

vi. If you were to represent the sum using an arrow, how long would the arrow be and what direction would it point?

*The arrow would be 8 units long and point to the right (or up on a vertical number line).*

vii. What is the relationship between the arrow representing the number on the number line and the absolute value of the number?

*The length of an arrow representing a number is equal to the absolute value of the number.*

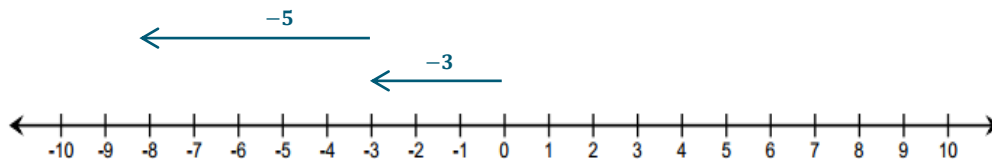
**Scaffolding:**

- Provide pre-made number lines for use throughout the lesson.
- Introduce questions one at a time using projection technology to support non-auditory learners.
- Use polling software throughout the lesson to gauge the entire class's understanding.

- viii. Do you think that adding two positive numbers will always give you a greater positive number? Why?

*Yes, because the absolute values are positive, so the sum will be a greater positive. On a number line, adding them would move you further away from 0 (to the right or above) on a number line.*

- b. Represent the sum of  $-3 + (-5)$  using arrows that represent  $-3$  and  $-5$  on the number line. From part (a), use the same questions to elicit feedback. In the Integer Game, I would combine  $-3$  and  $-5$  to give me  $-8$ .



$$-3 + (-5) = -8$$

- i. How long is the arrow that represents  $-3$ ?

*3 units*

- ii. What direction does it point?

*Left/down*

- iii. How long is the arrow that represents  $-5$ ?

*5 units*

- iv. What direction does it point?

*Left/down*

- v. What is the sum?

*-8*

- vi. If you were to represent the sum using an arrow, how long would the arrow be and what direction would it point?

*The arrow would be 8 units long and point to the left (or down on a vertical number line).*

- vii. Do you think that adding two negative numbers will always give you a smaller negative number? Why?

*Yes, because the absolute values of negative numbers are positive, so the sum will be a greater positive. However, the opposite of a greater positive is a smaller negative. On a number line, adding two negative numbers would move you further away from 0 (to the left or below) on a number line.*

- c. What do both examples have in common?

*The length of the arrow representing the sum of two numbers with the same sign is the same as the sum of the absolute values of both numbers.*

The teacher writes the rule for adding integers with the same sign.

*RULE: Add rational numbers with the same sign by adding the absolute values and using the common sign.*

### Exercise 2 (5 minutes)

Students work in pairs while solving practice problems.

#### Exercise 2

a. Decide whether the sum will be positive or negative without actually calculating the sum.

i.  $-4 + (-2)$  Negative

ii.  $5 + 9$  Positive

iii.  $-6 + (-3)$  Negative

iv.  $-1 + (-11)$  Negative

v.  $3 + 5 + 7$  Positive

vi.  $-20 + (-15)$  Negative

b. Find the sum.

i.  $15 + 7$   
22

ii.  $-4 + (-16)$   
-20

iii.  $-18 + (-64)$   
-82

iv.  $-205 + (-123)$   
-328

#### Scaffolding:

- Create anchor posters when introducing integer addition rules. (i.e., Adding Same Sign and Adding Opposite Signs)
- Use a gallery wall to post examples and generate student discussion.

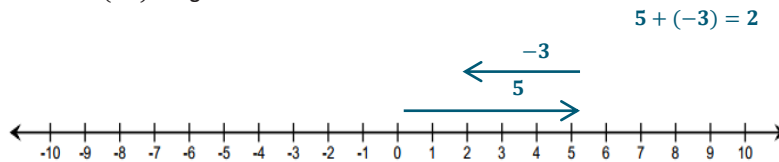
### Example 2 (7 minutes): Rule for Adding Opposite Signs

The teacher leads the whole class to find the sum of  $5 + (-3)$ .

- In the Integer Game, I would combine 5 and  $-3$  to give me 2.

**Example 2: Rule for Adding Opposite Signs**

a. Represent  $5 + (-3)$  using arrows on the number line.



i. How long is the arrow that represents 5?

*5 units*

ii. What direction does it point?

*Right/up*

iii. How long is the arrow that represents  $-3$ ?

*3 units*

iv. What direction does it point?

*Left/down*

v. Which arrow is longer?

*The arrow that is five units long or the first arrow.*

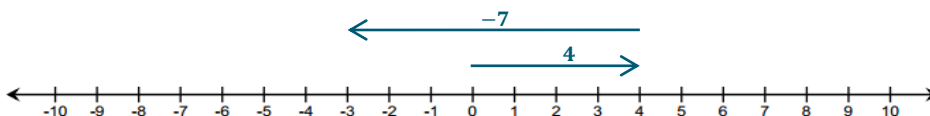
vi. What is the sum? If you were to represent the sum using an arrow, how long would the arrow be and what direction would it point?

$-3 + 5 = 2$

*The arrow would be 2 units long and point right/up.*

b. Represent the  $4 + (-7)$  using arrows on the number line.

$4 + (-7) = -3$



i. In the two examples above, what is the relationship between length of the arrow representing the sum and the lengths of the arrows representing the  $p$ -value and  $q$ -value?

*The length of the arrow representing the sum is equal to the difference of the absolute values of the lengths of both arrows representing the  $p$ -value and the  $q$ -value.*

ii. What is the relationship between the direction of the arrow representing the sum and the direction of arrows

representing the  $p$ -value and  $q$ -value?

*The direction of the arrow representing the sum has the same direction as the arrow of the addend with the greater absolute value.*

- iii. Write a rule that will give the length and direction of the arrow representing the sum of two values that have opposite signs.

*The length of the arrow of the sum is the difference of the absolute values of both the  $p$ -value and  $q$ -value, or the two addends. The direction of the arrow of the sum is the same as the direction of the longer arrow.*

The teacher writes the rule for adding integers with opposite signs.

**RULE:** Add rational numbers with opposite signs by subtracting the absolute values and using the sign of the integer with the greater absolute value.

### Exercise 3 (5 minutes)

Students work in pairs practicing addition with opposite signs. The teacher will monitor student work and provide support when necessary.

#### Exercise 3

- a. Circle the integer with the greater absolute value. Decide whether the sum will be positive or negative without actually calculating the sum.

i.  $-1 + 2$  \_\_\_\_\_ *Positive* \_\_\_\_\_

ii.  $5 + (-9)$  \_\_\_\_\_ *Negative* \_\_\_\_\_

iii.  $-6 + 3$  \_\_\_\_\_ *Negative* \_\_\_\_\_

iv.  $-11 + 1$  \_\_\_\_\_ *Negative* \_\_\_\_\_

- b. Find the sum.

i.  $-10 + 7$   
 $-3$

ii.  $8 + (-16)$   
 $-8$

iii.  $-12 + (65)$   
 $53$



iv.  $105 + (-126)$   
 $-21$

### Example 3 (5 minutes): Applying Integer Addition Rules to Rational Numbers

The teacher will pose the example to the whole class. Students will follow along in their student materials. The teacher will pose additional questions to the class.

- Which addend has the greatest absolute value (length of the arrow)? What direction does this arrow point?
  - $|6| = 6$  (The arrow length for 6 is 6 units long and to the right.)
  - $\left| -2\frac{1}{4} \right| = 2\frac{1}{4}$  (The arrow length for  $-2\frac{1}{4}$  is  $2\frac{1}{4}$  units long and to the left.)
  - The first addend has the greatest absolute value and the arrow will point to the right.
- What is the length of this arrow?
  - $|6| - \left| 2\frac{1}{4} \right| = 3\frac{3}{4}$
- What is the final sign? What is the direction of the resulting arrow?
  - Since 6 has the greater absolute value (arrow length), my answer will be positive, so  $6 + \left(-2\frac{1}{4}\right) = 3\frac{3}{4}$ .

#### Example 3: Applying Integer Addition Rules to Rational Numbers

Find the sum of  $6 + \left(-2\frac{1}{4}\right)$ . The addition of rational numbers follows the same rules of addition for integers.

- a. Find the absolute values of the numbers.

$$|6| = 6$$

$$\left| -2\frac{1}{4} \right| = 2\frac{1}{4}$$

- b. Subtract the absolute values.

$$\textcircled{6} - 2\frac{1}{4} = 6 - \frac{9}{4} = \frac{24}{4} - \frac{9}{4} = \frac{15}{4} = 3\frac{3}{4}$$

- c. The answer will take the sign of the number that has the greater absolute value.

Since 6 has the greater absolute value (arrow length), my answer will be positive  $3\frac{3}{4}$ .

### Exercise 4 (5 minutes)

Students work independently while solving practice problems.

Exercise 4

Solve the following problems. Show your work.

- a. Find the sum of  $-18 + 7$ .

$$|-18| = 18$$

$$|7| = 7$$

$$18 - 7 = 11$$

$$-11$$

- b. If the temperature outside was 73 degrees at 5:00 p.m., but it fell 19 degrees by 10:00 p.m., what is the temperature at 10:00 p.m.? Write an equation and solve.

$$73 + (-19)$$

$$73 - 19 = 54$$

$$54$$

$$|73| = 73$$

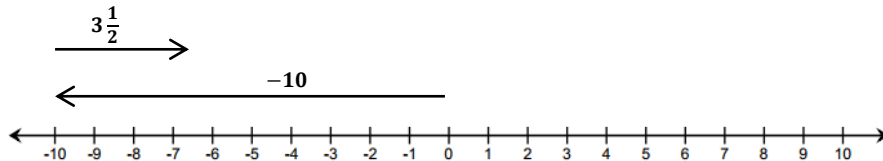
$$|-19| = 19$$

- c. Write an addition sentence, and find the sum using the diagram below.

$$-10 + 3\frac{1}{2} = -10 + \frac{7}{2} = -\frac{20}{2} + \frac{7}{2} = -\frac{13}{2} = -6\frac{1}{2}$$

$$\left| \frac{-20}{2} \right| = \frac{20}{2}$$

$$\left| \frac{7}{2} \right| = \frac{7}{2}$$



Closing (3 minutes)

The teacher calls on students at random to summarize the lesson.

- What are the rules of adding numbers with opposite signs?
  - *The rule for adding numbers with opposite signs is to subtract the absolute values and then the answer will take the sign of the number with the largest absolute value.*
- What is the sum of  $-3 + (-8)$ ?
  - *The sum of  $-3 + (-8)$  is  $-11$ .*
- What do you think the rules would be for subtracting numbers with same sign? (Do not spend too much time on this question. Allow students to verbally experiment with their responses.)
  - *Answers will vary.*

Scaffolding:

- To help build confidence, allow students time to “turn and talk” with partners before posing questions.

Lesson Summary

- Add integers with the same sign by adding the absolute values and using the common sign.
- Steps to adding integers with opposite signs:
  1. Find the absolute values of the integers.
  2. Subtract the absolute values.
  3. The answer will take the sign of the integer that has the greater absolute value.
- To add rational numbers, follow the same rules used to add integers.



**Exit Ticket (5 minutes)**

Name \_\_\_\_\_

Date \_\_\_\_\_

## Lesson 4: Efficiently Adding Integers and Other Rational Numbers

**Exit Ticket**

- Write an addition problem that has a sum of  $-4\frac{3}{5}$  and
  - Both addends ( $p$ -value and  $q$ -value) have the same sign.
  
  
  
  
  
  
  
  
  
  
  - The two addends ( $p$ -value and  $q$ -value) have different signs.
  
  
  
  
  
  
  
  
  
  
- In the Integer Game, what card would you need to draw to get a score of 0 if you have a  $-16$ ,  $-35$ , and 18 in your hand?



### Exit Ticket Sample Solutions

1. Write an addition problem that has a sum of  $-4\frac{4}{5}$  and

a. Both addends ( $p$ -value and  $q$ -value) have the same sign.

*Answers will vary.*  $-1\frac{4}{5} + (-3) = -4\frac{4}{5}$ .

b. The two addends ( $p$ -value and  $q$ -value) have different signs.

*Answers will vary.*  $1.8 + (-6.6) = -4.8$ .

2. In the Integer Game, what card would you need to draw to get a score of 0 if you have a  $-16$ ,  $-35$ , and  $18$  in your hand?

$-16 + (-35) + 18 = -33$ , so I would need to draw a 33 because 33 is the additive inverse of  $-33$ .  
 $-33 + 33 = 0$ .

### Problem Set Sample Solutions

Students must understand the rules for addition of rational numbers with same and opposite signs. The Problem Set presents multiple representations of these rules including diagrams, equations, and story problems. Students are expected to show their work or provide an explanation where necessary to justify their answers. Answers can be represented in fraction or decimal form.

1. Find the sum. Show your work to justify your answer.

a.  $4 + 17$

$4 + 17 = 21$

b.  $-6 + (-12)$

$-6 + (-12) = -18$

c.  $2.2 + (-3.7)$

$2.2 + (-3.7) = -1.5$

d.  $-3 + (-5) + 8$

$-3 + (-5) + 8 = -8 + 8 = 0$

e.  $\frac{1}{3} + \left(-2\frac{1}{4}\right)$

$\frac{1}{3} + \left(-2\frac{1}{4}\right) = \frac{1}{3} + \left(-\frac{9}{4}\right) = \frac{4}{12} + \left(-\frac{27}{12}\right) = -\frac{23}{12} = -1\frac{11}{12}$

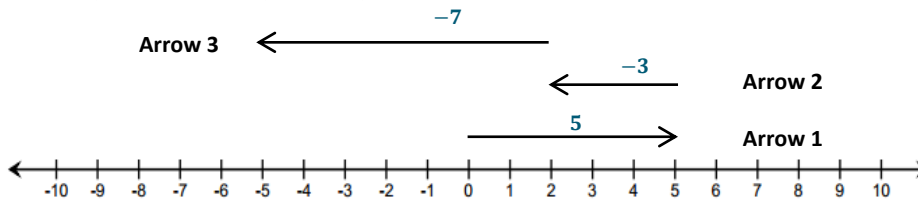
2. Which of these story problems describes the sum  $19 + (-12)$ ? Check all that apply. Show your work to justify your answer.

X Jared’s dad paid him \$19 for raking the leaves from the yard on Wednesday. Jared spent \$12 at the movie theater on Friday. How much money does Jared have left?

\_\_\_\_\_ Jared owed his brother \$19 for raking the leaves while Jared was sick. Jared’s dad gave him \$12 for doing his chores for the week. How much money does Jared have now?

X Jared’s grandmother gave him \$19 for his birthday. He bought \$8 worth of candy and spent another \$4 on a new comic book. How much money does Jared have left over?

3. Use the diagram below to complete each part.



a. Label each arrow with the number the arrow represents.

b. How long is each arrow? What direction does each arrow point?

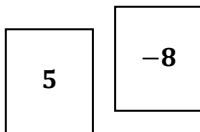
Arrow	Length	Direction
1	5	right
2	3	left
3	7	left

c. Write an equation that represents the sum of the numbers. Find the sum.

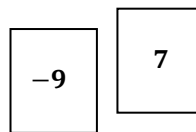
$5 + (-3) + (-7) = -5$

4. Jennifer and Katie were playing the Integer Game in class. Their hands are represented below.

Jennifer’s Hand



Katie’s Hand



a. What is the value of each of their hands? Show your work to support your answer.



*Jennifer's hand has a value of  $-3$  because  $5 + (-8) = -3$ . Katie's hand has a value of  $-2$  because  $-9 + 7 = -2$ .*

- b. If Jennifer drew two more cards, is it possible for the value of her hand not to change? Explain why or why not.

*It is possible for her hand not to change. Jennifer could get any two cards that are the exact opposites such as a  $-2$  and  $2$ . Numbers that are exact opposites are called additive inverses, and they sum to  $0$ . Adding the number  $0$  to anything will not change the value.*

- c. If Katie wanted to win the game by getting a score of  $0$ , what card would she need? Explain.

*Katie would need to draw a  $2$  because the additive inverse of  $-2$  is  $2$ .  $-2 + 2 = 0$ .*

- d. If Jennifer drew a  $-1$  and a  $-2$ , what would be her new score? Show your work to support your answer.

*Jennifer's new score would be  $-6$  because  $-3 + (-1) + (-2) = -6$ .*