Lesson 25: Finding Solutions to Make Equations True

Student Outcomes

- Students learn the definition of solution in the context of placing a value into a variable to see if that value makes the equation true.

Lesson Notes

In previous lessons, students used sentences and symbols to describe the values that, when substituted for the variable in an equation, resulted in a true number sentence. In this lesson, students will make the transition from their previous learning (e.g., substituting numbers into equations, writing complete sentences to describe when an equation results in a true number sentence, and using symbols to reduce the wordiness of a description) to today's lesson where they identify the value that makes an equation true as a *solution*.

As they did in previous lessons, students test for solutions by substituting numbers into equations and by checking whether the resulting number sentence is true. They have already seen how equations like $x = 3$ relate to the original equation and that it is valuable to find ways to simplify equations until they are in the form of $x =$ "a number." In the next lesson, students begin to learn the formal process of "solving an equation," that is, the process of transforming the original equation to an equation of the form $x =$ "a number," where it is easy to identify the solution.

Materials: Students will complete a matching game that needs to be cut out and prepared before the class period begins. Ideally, there should be 20 sets prepared, each in a separate bag, so that students may work in pairs. Specific directions for the game are below.

Classwork

Fluency Exercise (5 minutes): Division of Fractions

Sprint: Refer to the Sprints and Sprint Delivery Script sections in the Module Overview for directions on how to administer a Sprint.

Opening Exercise (5 minutes)

Opening Exercise

Identify a value for the variable that would make each equation or inequality into a true number sentence. Is this the only possible answer? State when the equation or inequality is true using equality and inequality symbols.

a. $3 + g = 15$

12 is the only value of g that will make the equation true. The equation is true when $g = 12$.

b. $30 > 2d$

Answers will vary. There is more than one value of d that will make the inequality true. The inequality is true when $d < 15$.

c. $\frac{15}{f} < 5$

Answers will vary. There is more than one value of f that will make the inequality true. The inequality is true when $f > 3$.

d. $42 \leq 50 - m$

Answers will vary. There is more than one value of m that will make the inequality true. The inequality is true when $m \leq 8$.

Example 1 (5 minutes)

Example 1

Each of the following numbers, if substituted for the variable, makes one of the equations below into a true number sentence. Match the number to that equation: $3, 6, 15, 16, 44$.

a. $n + 26 = 32$

6

b. $n - 12 = 32$

44

c. $17n = 51$

3

d. $4^2 = n$

16

e. $\frac{n}{3} = 5$

15

Discussion (2 minutes)

In most of the equations we have looked at so far, the numbers we used to substitute in for the variable have resulted in true number sentences. A number or value for the variable that results in a true number sentence is special and is called a "solution to the equation." In the example above, 6 is a solution to $n + 26 = 32$, 44 is a solution to $n - 12 = 32$, and so on.

Exercises (15 minutes)

Students work with a partner to match the equation with its solution. Please note that below are the answers for the activity. The actual game cut-out pieces are at the end of the lesson.

MP.3

$a + 14 = 36$	22
$\frac{c}{5} = 3$	15
$24 = e + 11$	13
$9 = \frac{45}{g}$	5
$1.5 + 0.5 = j$	2
$m = \frac{56}{8}$	7
$p + 13\frac{3}{4} = 32\frac{3}{4}$	19
$\frac{63}{r} = 7$	9

$3^3 = b$	27
$d - 10 = 32$	42
$32 = 4 \cdot f$	8
$43 = h - 17$	60
$9 \cdot \frac{1}{3} = k$	3
$n = 35.5 - 9.5$	26
$4 = \frac{1}{4}q$	16
$99 - u = 45$	54

Closing (8 minutes)

- Let's look at the equation $8n = 72$. We know that 9 is a value that we can substitute for n that will result in a true number sentence. In previous lessons, we described this solution as "The equation is true when the value of n is 9" and noted that the equation is false when any number other than 9 is substituted for n, or when $n \neq 9$. Therefore, there is only one solution to $8n = 72$, and it is 9.
- We also saw that both statements (i.e., the numbers that make the equation true and the numbers that make it false) can be summarized with one sentence, "The equation is true when $n = 9$" because the values that make $n = 9$ true or false are the same as the values that make $8n = 72$ true or false. Thus, we can represent the solution as "The solution is 9," or $n = 9$.
- The next lesson shows the process for transforming an equation like $8n = 72$ until it is in the form $x = 9$. You have been doing this process for many years in tape diagrams and unknown angle problems, but now we will describe explicitly the steps you were following.

Note that the domain of the variable is just the set of numbers from which we are looking for solutions. For example, sometimes we only want to consider integers as solutions. In those cases, the domain of the variable would be the set of integer numbers.

Lesson Summary

VARIABLE: A *variable* is a symbol (such as a letter) that represents a number (i.e., it is a placeholder for a number).

A variable is a placeholder for "a number" that does not "vary."

EXPRESSION: An *expression* is a numerical expression or a result of replacing some (or all) of the numbers in a numerical expression with variables.

EQUATION: An *equation* is a statement of equality between two expressions.

If A and B are two expressions in the variable x, then $A = B$ is an equation in the variable x.

Teacher notes:

*A common description of a variable in the U.S. is "a quantity that varies." Ask yourselves, how can a quantity vary? A less vague description of a variable is "a place holder for a number"; this is better because it denotes a single, non-varying number.

The upside of the description of variable (and this is a point that must be made explicit to students) is that it is the user of the variable who controls what number to insert into the placeholder. Hence, it is the student who has the power to change or vary the number as they so desire. The power to vary rests with the student, not with the variable itself!

Exit Ticket (5 minutes)

Name _____ Date _____

Lesson 25: Finding Solutions to Make Equations True

Exit Ticket

Find the solution to each equation.

1. $7f = 49$

2. $1 = \frac{r}{12}$

3. $1.5 = d + 0.8$

4. $9^2 = h$

5. $q = 45 - 19$

6. $40 = \frac{1}{2}p$

Exit Ticket Sample Solutions

Find the solution to each equation.

1. $7f = 49$

   $f = 7$

2. $1 = \frac{r}{12}$

   $r = 12$

3. $1.5 = d + 0.8$

   $d = 0.7$

4. $9^2 = h$

   $h = 81$

5. $q = 45 - 19$

   $q = 26$

6. $40 = \frac{1}{2}p$

   $p = 80$

Problem Set Sample Solutions

Find the solution to each equation.

1. $4^3 = y$

   $y = 64$

2. $8a = 24$

   $a = 3$

3. $32 = g - 4$

   $g = 36$

4. $56 = j + 29$

   $j = 27$

5. $\frac{48}{r} = 12$

   $r = 4$

6. $k = 15 - 9$

   $k = 6$

7. $x \cdot \frac{1}{5} = 60$

   $x = 300$

8. $m + 3.45 = 12.8$

   $m = 9.35$

9. $a = 1^5$

   $a = 1$

$a + 14 = 36$	22	$3^3 = b$
27	$\frac{c}{5} = 3$	15
$d - 10 = 32$	42	$24 = e + 11$
13	$32 = 4 \cdot f$	8
$9 = \frac{45}{g}$	5	$43 = h - 17$

60	$1.5 + 0.5 = j$	2
$9 \cdot \frac{1}{3} = k$	3	$m = \frac{56}{8}$
7	$n = 35.5 - 9.5$	26
$p + 13\frac{3}{4} = 32\frac{3}{4}$	19	$4 = \frac{1}{4}q$
16	$\frac{63}{r} = 7$	9

$99 - u = 45$	54

Division of Fractions–Round 1

Number Correct: _____

Directions: Determine the quotients of the fractions.

1.	$\frac{1}{2} \div \frac{3}{5}$	
2.	$\frac{5}{6} \div \frac{1}{5}$	
3.	$\frac{3}{7} \div \frac{6}{11}$	
4.	$\frac{2}{5} \div \frac{8}{9}$	
5.	$\frac{1}{6} \div \frac{9}{10}$	
6.	$\frac{11}{12} \div \frac{8}{9}$	
7.	$\frac{5}{6} \div \frac{10}{13}$	
8.	$\frac{7}{8} \div \frac{13}{15}$	
9.	$\frac{3}{5} \div \frac{7}{9}$	
10.	$\frac{14}{17} \div \frac{13}{20}$	
11.	$3\frac{1}{2} \div 4\frac{4}{5}$	
12.	$6\frac{1}{5} \div 6\frac{3}{4}$	
13.	$2\frac{1}{4} \div 3\frac{1}{8}$	
14.	$1\frac{3}{5} \div \frac{7}{8}$	
15.	$\frac{1}{5} \div 4\frac{1}{2}$	

16.	$6\frac{7}{8} \div 11\frac{2}{3}$	
17.	$5\frac{5}{6} \div 3\frac{1}{2}$	
18.	$10\frac{5}{8} \div 12\frac{3}{7}$	
19.	$9\frac{1}{3} \div 8\frac{2}{5}$	
20.	$\frac{3}{4} \div 6\frac{7}{10}$	
21.	$2\frac{1}{3} \div 3\frac{5}{6}$	
22.	$2\frac{4}{5} \div 7\frac{9}{10}$	
23.	$5\frac{8}{9} \div 3\frac{3}{5}$	
24.	$12\frac{5}{9} \div 5$	
25.	$1\frac{5}{6} \div 2\frac{6}{7}$	
26.	$10 \div 5\frac{8}{9}$	
27.	$14\frac{3}{5} \div 10$	
28.	$7\frac{9}{11} \div 1\frac{9}{10}$	
29.	$15\frac{2}{3} \div 24$	
30.	$32 \div 12\frac{6}{7}$	

Division of Fractions– Round 1 [KEY]

Directions: Determine the quotients of the fractions.

1.	$\frac{1}{2} \div \frac{3}{5}$	$\frac{5}{6}$
2.	$\frac{5}{6} \div \frac{1}{5}$	$\frac{25}{6} = 4\frac{1}{6}$
3.	$\frac{3}{7} \div \frac{6}{11}$	$\frac{33}{42} = \frac{11}{14}$
4.	$\frac{2}{5} \div \frac{8}{9}$	$\frac{18}{40} = \frac{9}{20}$
5.	$\frac{1}{6} \div \frac{9}{10}$	$\frac{10}{54} = \frac{5}{27}$
6.	$\frac{11}{12} \div \frac{8}{9}$	$\frac{99}{96} = 1\frac{1}{32}$
7.	$\frac{5}{6} \div \frac{10}{13}$	$\frac{65}{60} = 1\frac{1}{12}$
8.	$\frac{7}{8} \div \frac{13}{15}$	$\frac{105}{104} = 1\frac{1}{104}$
9.	$\frac{3}{5} \div \frac{7}{9}$	$\frac{27}{35}$
10.	$\frac{14}{17} \div \frac{13}{20}$	$\frac{280}{221} = 1\frac{59}{221}$
11.	$3\frac{1}{2} \div 4\frac{4}{5}$	$\frac{35}{48}$
12.	$6\frac{1}{5} \div 6\frac{3}{4}$	$\frac{124}{135}$
13.	$2\frac{1}{4} \div 3\frac{1}{8}$	$\frac{72}{100} = \frac{18}{25}$
14.	$1\frac{3}{5} \div \frac{7}{8}$	$\frac{64}{35} = 1\frac{29}{35}$
15.	$\frac{1}{5} \div 4\frac{1}{2}$	$\frac{2}{45}$

16.	$6\frac{7}{8} \div 11\frac{2}{3}$	$\frac{165}{280} = \frac{33}{56}$
17.	$5\frac{5}{6} \div 3\frac{1}{2}$	$\frac{70}{42} = 1\frac{2}{3}$
18.	$10\frac{5}{8} \div 12\frac{3}{7}$	$\frac{595}{696}$
19.	$9\frac{1}{3} \div 8\frac{2}{5}$	$\frac{140}{126} = 1\frac{1}{9}$
20.	$\frac{3}{4} \div 6\frac{7}{10}$	$\frac{30}{268} = \frac{15}{134}$
21.	$2\frac{1}{3} \div 3\frac{5}{6}$	$\frac{42}{69} = \frac{14}{23}$
22.	$2\frac{4}{5} \div 7\frac{9}{10}$	$\frac{140}{395} = \frac{28}{79}$
23.	$5\frac{8}{9} \div 3\frac{3}{5}$	$\frac{265}{162} = 1\frac{103}{162}$
24.	$12\frac{5}{9} \div 5$	$\frac{113}{45} = 2\frac{23}{45}$
25.	$1\frac{5}{6} \div 2\frac{6}{7}$	$\frac{77}{120}$
26.	$10 \div 5\frac{8}{9}$	$\frac{90}{53} = 1\frac{37}{53}$
27.	$14\frac{3}{5} \div 10$	$\frac{73}{50} = 1\frac{23}{50}$
28.	$7\frac{9}{11} \div 1\frac{9}{10}$	$\frac{860}{209} = 4\frac{24}{209}$
29.	$15\frac{2}{3} \div 24$	$\frac{47}{72}$
30.	$32 \div 12\frac{6}{7}$	$\frac{224}{90} = 2\frac{22}{45}$

Division of Fractions–Round 2

Number Correct: _____

Directions: Determine the quotients of the fractions.

Improvement: _____

1.	$\frac{1}{2} \div \frac{3}{4}$	
2.	$\frac{3}{5} \div \frac{1}{7}$	
3.	$\frac{5}{6} \div \frac{1}{8}$	
4.	$\frac{3}{8} \div \frac{5}{9}$	
5.	$\frac{1}{4} \div \frac{2}{5}$	
6.	$\frac{7}{8} \div \frac{9}{10}$	
7.	$\frac{8}{11} \div \frac{4}{5}$	
8.	$\frac{5}{6} \div \frac{7}{12}$	
9.	$\frac{3}{4} \div \frac{8}{9}$	
10.	$\frac{7}{11} \div \frac{4}{7}$	
11.	$1\frac{3}{4} \div \frac{1}{2}$	
12.	$\frac{1}{10} \div 2\frac{3}{4}$	
13.	$5\frac{2}{3} \div \frac{7}{9}$	
14.	$\frac{5}{6} \div 6\frac{1}{2}$	
15.	$\frac{6}{7} \div 2\frac{1}{3}$	

16.	$\frac{7}{8} \div 2\frac{1}{4}$	
17.	$\frac{3}{4} \div 2\frac{3}{5}$	
18.	$4\frac{1}{5} \div 2\frac{1}{3}$	
19.	$4\frac{3}{8} \div \frac{2}{7}$	
20.	$\frac{4}{5} \div 2\frac{1}{8}$	
21.	$1\frac{1}{2} \div 3\frac{5}{6}$	
22.	$3\frac{2}{3} \div 2\frac{1}{4}$	
23.	$4\frac{3}{5} \div 1\frac{3}{4}$	
24.	$7\frac{1}{2} \div 6\frac{1}{3}$	
25.	$3\frac{4}{5} \div 2\frac{9}{10}$	
26.	$3\frac{5}{6} \div 2\frac{1}{2}$	
27.	$3\frac{3}{4} \div 4\frac{1}{8}$	
28.	$5 \div 4\frac{5}{6}$	
29.	$3\frac{1}{4} \div 2$	
30.	$8 \div 5\frac{1}{3}$	

Division of Fractions–Round 2 [KEY]

Directions: Determine the quotients of the fractions.

1.	$\frac{1}{2} \div \frac{3}{4}$	$\frac{4}{6} = \frac{2}{3}$
2.	$\frac{3}{5} \div \frac{1}{7}$	$\frac{21}{5} = 4\frac{1}{5}$
3.	$\frac{5}{6} \div \frac{1}{8}$	$\frac{40}{6} = 6\frac{2}{3}$
4.	$\frac{3}{8} \div \frac{5}{9}$	$\frac{27}{40}$
5.	$\frac{1}{4} \div \frac{2}{5}$	$\frac{5}{8}$
6.	$\frac{7}{8} \div \frac{9}{10}$	$\frac{70}{72} = \frac{35}{36}$
7.	$\frac{8}{11} \div \frac{4}{5}$	$\frac{40}{44} = \frac{10}{11}$
8.	$\frac{5}{6} \div \frac{7}{12}$	$\frac{60}{42} = 1\frac{3}{7}$
9.	$\frac{3}{4} \div \frac{8}{9}$	$\frac{27}{32}$
10.	$\frac{7}{11} \div \frac{4}{7}$	$\frac{49}{44} = 1\frac{5}{44}$
11.	$1\frac{3}{4} \div \frac{1}{2}$	$\frac{14}{4} = 3\frac{1}{2}$
12.	$\frac{1}{10} \div 2\frac{3}{4}$	$\frac{4}{110} = \frac{2}{55}$
13.	$5\frac{2}{3} \div \frac{7}{9}$	$\frac{153}{21} = 7\frac{2}{7}$
14.	$\frac{5}{6} \div 6\frac{1}{2}$	$\frac{10}{78} = \frac{5}{39}$
15.	$\frac{6}{7} \div 2\frac{1}{3}$	$\frac{18}{49}$

16.	$\frac{7}{8} \div 2\frac{1}{4}$	$\frac{28}{72} = \frac{7}{18}$
17.	$\frac{3}{4} \div 2\frac{3}{5}$	$\frac{15}{52}$
18.	$4\frac{1}{5} \div 2\frac{1}{3}$	$\frac{63}{35} = 1\frac{4}{5}$
19.	$4\frac{3}{8} \div \frac{2}{7}$	$\frac{245}{16} = 15\frac{5}{16}$
20.	$\frac{4}{5} \div 2\frac{1}{8}$	$\frac{32}{85}$
21.	$1\frac{1}{2} \div 3\frac{5}{6}$	$\frac{18}{46} = \frac{9}{23}$
22.	$3\frac{2}{3} \div 2\frac{1}{4}$	$\frac{44}{27} = 1\frac{17}{27}$
23.	$4\frac{3}{5} \div 1\frac{3}{4}$	$\frac{92}{35} = 2\frac{22}{35}$
24.	$7\frac{1}{2} \div 6\frac{1}{3}$	$\frac{45}{38} = 1\frac{7}{38}$
25.	$3\frac{4}{5} \div 2\frac{9}{10}$	$\frac{190}{145} = 1\frac{9}{29}$
26.	$3\frac{5}{6} \div 2\frac{1}{2}$	$\frac{46}{30} = 1\frac{8}{15}$
27.	$3\frac{3}{4} \div 4\frac{1}{8}$	$\frac{120}{132} = \frac{10}{11}$
28.	$5 \div 4\frac{5}{6}$	$\frac{30}{29} = 1\frac{1}{29}$
29.	$3\frac{1}{4} \div 2$	$\frac{13}{8} = 1\frac{5}{8}$
30.	$8 \div 5\frac{1}{3}$	$\frac{24}{16} = 1\frac{1}{2}$