Lesson 11: Factoring Expressions

Student Outcomes

- Students model and write equivalent expressions using the distributive property. They move from expanded form to factored form of an expression.

Classwork

Fluency Exercise (5 minutes): GCF

Sprint: Refer to the Sprints and Sprint Delivery Script sections in the Module Overview for directions on how to administer a Sprint.

Example 1 (8 minutes)

Scaffolding:

For students struggling with variables, you can further solidify the concept by having them replace the variables with whole numbers to prove that the expressions are equivalent.

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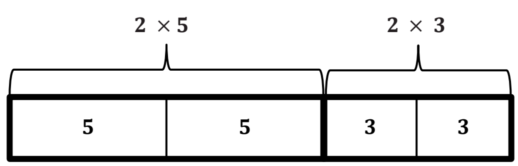
Example 1

a. **Use the model to answer the following questions.**



How many fives are in the model?

2

How many threes are in the model?

2

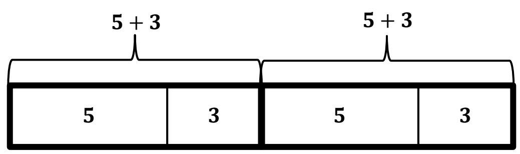
What does the expression represent in words?

The sum of two groups of five and two groups of three.

What expression could we write to represent the model?

$2 \times 5 + 2 \times 3$

b. Use the new model and the previous model to answer the next set of questions.



How many fives are in the model?

2

How many threes are in the model?

2

What does the expression represent in words?

Two groups of the sum of five and three.

What expression could we write to represent the model?

$(5+3)+(5+3)$ *or* $2(5+3)$

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c. Is the model in part (a) equivalent to the model in part (b)?

Yes, because both expressions have two 5*s and two* 3*s. Therefore,* $2\times 5+2\times 3=2(5+3)$.

d. What relationship do we see happening on either side of the equal sign?

On the left hand side, 2 *is being multiplied by* 5 *and then by* 3 *before adding the products together. On the right hand side, the* 5 *and* 3 *are added first and then multiplied by* 2.

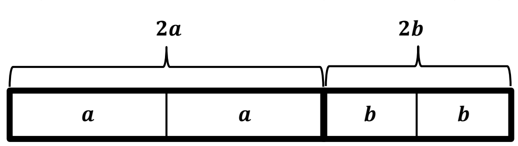
e. In Grade 5 and in Module 2 of this year, you have used similar reasoning to solve problems. What is the name of the property that is used to say that $2(5+3)$ is the same as $2\times 5+2\times 3$?

The name of the property is the distributive property.

Example 2 (5 minutes)

Example 2

Now, we will take a look at an example with variables. Discuss the questions with your partner.



What does the model represent in words?

a plus a plus b plus b, two a's plus two b's, two times a plus two times b

What does $2a$ mean?

$2a$ means that there are 2 a's or $2 \times a$.

How many a's are in the model?

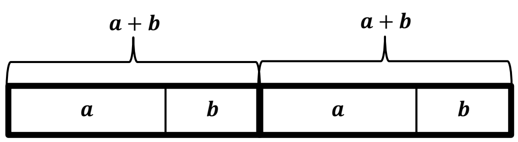
2

How many b's are in the model?

2

What expression could we write to represent the model?

$2a + 2b$



How many a's are in the expression?

2

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How many b's are in the expression?

2

What expression could we write to represent the model?

$(a + b) + (a + b) = 2(a + b)$

Are the two expressions equivalent?

Yes, both models include 2 a's and 2 b's. Therefore, $2a + 2b = 2(a + b)$

Example 3 (8 minutes)

Example 3

Use GCF and the distributive property to write equivalent expressions.

1. $3f + 3g =$ ______ $3(f + g)$ ______

What is the question asking us to do?

We need to rewrite the expression as an equivalent expression in factored form, which means the expression is written as the product of factors. The number outside of the parentheses is the GCF.

How would Problem 1 look if we expanded each term?

$3 \cdot f + 3 \cdot g$

What is the GCF in Problem 1?

3

How can we use the GCF to rewrite this?

3 goes on the outside and $f + g$ will go inside the parentheses. $3(f + g)$

- Let's use the same ideas for Problem 2. Start by expanding the expression and naming the GCF.

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2. $6x + 9y =$ _____ $3(2x + 3y)$ _____

What is the question asking us to do?

We need to rewrite the expression as an equivalent expression in factored form, which means the expression is written as the product of factors. The number outside of the parentheses is the GCF.

How would Problem 2 look if we expanded each term?

$2 \cdot 3 \cdot x + 3 \cdot 3 \cdot y$

What is the GCF in Problem 2?

The GCF is 3.

How can we use the GCF to rewrite this?

I will factor out the 3 from both terms and place it in front of the parentheses. I will place what is left in the terms inside the parentheses: $3(2x + 3y)$.

3. $3c + 11c =$ _____ $c(3 + 11)$ _____

Is there a greatest common factor in Problem 3?

Yes, when I expand I can see that each term has a common factor c.

$3 \cdot c + 11 \cdot c$

Rewrite the expression using the distributive property.

$c(3 + 11)$

4. $24b + 8 =$ _____ $8(3b + 1)$

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Explain how you used GCF and the distributive property to rewrite the expression in Problem 4.

I first expanded each term. I know that 8 goes into 24, so used it in the expansion.

$2 \cdot 2 \cdot 2 \cdot 3 \cdot b + 2 \cdot 2 \cdot 2$

I determined that $2 \cdot 2 \cdot 2$, or 8, is the common factor. So, on the outside of the parentheses I wrote 8, and on the inside I wrote the leftover factor, $3b + 1$. $8(3b + 1)$.

Why is there a 1 in the parentheses?

When I factor out a number, I am leaving behind the other factor that multiplies to make the original number. In this case, when I factor out an 8 from 8, I am left with a 1 because $8 \times 1 = 8$.

How is this related to the first two examples?

In the first two examples, we saw that we could rewrite the expressions by thinking about groups.

We can either think of $24b + 8$ as 8 groups of $3b$ and 8 groups of 1 or as 8 groups of the sum of $3b + 1$. This shows that $8(3b) + 8(1) = 8(3b + 1)$ is the same as $24b + 8$.

Exercises (12 minutes)

If times allows, you could have students practice these questions on white boards or small personal boards.

Exercises

1. **Apply the distributive property to write equivalent expressions.**

   a. $7x + 7y$

      $7(x + y)$

   b. $15g + 20h$

      $5(3g + 4h)$

   c. $18m + 42n$

      $6(3m + 7n)$

   d. $30a + 39b$

      $3(10a + 13b)$

   e. $11f + 15f$

      $f(11 + 15)$

   f. $18h + 13h$

      $h(18 + 13)$

   g. $55m + 11$

      $11(5m + 1)$

h. $7 + 56y$

$7(1 + 8y)$

2. Evaluate each of the expressions below.

a. $6x + 21y$ and $3(2x + 7y)$ $x = 3$ and $y = 4$

$6(3) + 21(4)$	$3(2 \cdot 3 + 7 \cdot 4)$
$18 + 84$	$3(6 + 28)$
102	$3(34)$
102	102

b. $5g + 7g$ and $g(5 + 7)$ $g = 6$

$5(6) + 7(6)$	$6(5 + 7)$
$30 + 42$	$6(12)$
72	72

c. $14x + 2$ and $2(7x + 1)$ $x = 10$

$14(10) + 2$	$2(7 \cdot 10 + 1)$
$140 + 2$	$2(70 + 1)$
142	$2(71)$
142	142

d. Explain any patterns that you notice in the results to parts (a)–(c).

Both expressions in parts (a)–(c) evaluated to the same number when the indicated value was substituted for the variable. This shows that the two expressions are equivalent for the given values.

e. What would happen if other values were given for the variables?

Because the two expressions in each part are equivalent, they evaluate to the same number, no matter what value is chosen for the variable.

Closing (3 minutes)

How can use you use your knowledge of GCF and the distributive property to write equivalent expressions?

We can use our knowledge of GCF and the distributive property to change expressions from standard form to factored form.

Find the missing value that makes the two expressions equivalent.

$4x + 12y$ ____4____ $(x + 3y)$

$35x + 50y$ ____5____ $(7x + 10y)$

$18x + 9y$ ____9____ $(2x + y)$

$32x + 8y$ ___8___ $(4x + y)$

$100x + 700y$ ___100___ $(x + 7y)$

Explain how you determine the missing number.

I would expand each term and determine the greatest common factor. The greatest common factor is the number that is placed on the blank line.

> Lesson Summary
>
> **AN EXPRESSION IN FACTORED FORM: An expression that is a product of two or more expressions is said to be in *factored form*.**

Exit Ticket (4 minutes)

Name ______________________________ Date ______________

Lesson 11: Factoring Expressions

Exit Ticket

Use greatest common factor and the distributive property to write equivalent expressions in factored form.

1. $2x + 8y$

2. $13ab + 15ab$

3. $20g + 24h$

Exit Ticket Sample Solutions

Use greatest common factor and the distributive property to write equivalent expressions in factored form.

1. $2x + 8y$

   $2(x + 4y)$

2. $13ab + 15ab$

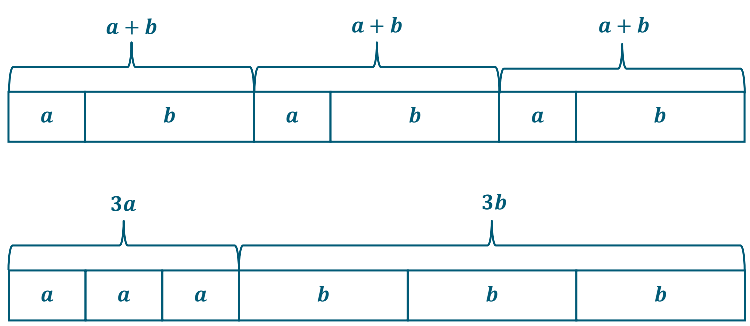
   $ab(13 + 15)$

3. $20g + 24h$

   $4(5g + 6h)$

Problem Set Sample Solutions

1. Use models to prove that $3(a + b)$ is equivalent to $3a + 3b$.



2. Use greatest common factor and the distributive property to write equivalent expressions in factored form for the following expressions.

   a. $4d + 12e$

      $4(d + 3e)$ *or* $4(1d + 3e)$

   b. $18x + 30y$

      $6(3x + 5y)$

   c. $21a + 28y$

      $7(3a + 4y)$

   d. $24f + 56g$

      $8(3f + 7g)$

Number Correct: ______

Greatest Common Factor–Round 1

Directions: Determine the greatest common factor of each pair of numbers.

1.	GCF of 10 and 50		16.	GCF of 45 and 72	
2.	GCF of 5 and 35		17.	GCF of 28 and 48	
3.	GCF of 3 and 12		18.	GCF of 44 and 77	
4.	GCF of 8 and 20		19.	GCF of 39 and 66	
5.	GCF of 15 and 35		20.	GCF of 64 and 88	
6.	GCF of 10 and 75		21.	GCF of 42 and 56	
7.	GCF of 9 and 30		22.	GCF of 28 and 42	
8.	GCF of 15 and 33		23.	GCF of 13 and 91	
9.	GCF of 12 and 28		24.	GCF of 16 and 84	
10.	GCF of 16 and 40		25.	GCF of 36 and 99	
11.	GCF of 24 and 32		26.	GCF of 39 and 65	
12.	GCF of 35 and 49		27.	GCF of 27 and 87	
13.	GCF of 45 and 60		28.	GCF of 28 and 70	
14.	GCF of 48 and 72		29.	GCF of 26 and 91	
15.	GCF of 50 and 42		30.	GCF of 34 and 51	

Greatest Common Factor–Round 1 [KEY]

Directions: Determine the greatest common factor of each pair of numbers.

1.	GCF of 10 and 50	10	16.	GCF of 45 and 72	9
2.	GCF of 5 and 35	5	17.	GCF of 28 and 48	4
3.	GCF of 3 and 12	3	18.	GCF of 44 and 77	11
4.	GCF of 8 and 20	4	19.	GCF of 39 and 66	3
5.	GCF of 15 and 35	5	20.	GCF of 64 and 88	8
6.	GCF of 10 and 75	5	21.	GCF of 42 and 56	14
7.	GCF of 9 and 30	3	22.	GCF of 28 and 42	14
8.	GCF of 15 and 33	3	23.	GCF of 13 and 91	13
9.	GCF of 12 and 28	4	24.	GCF of 16 and 84	4
10.	GCF of 16 and 40	8	25.	GCF of 36 and 99	9
11.	GCF of 24 and 32	8	26.	GCF of 39 and 65	13
12.	GCF of 35 and 49	7	27.	GCF of 27 and 87	3
13.	GCF of 45 and 60	15	28.	GCF of 28 and 70	14
14.	GCF of 48 and 72	24	29.	GCF of 26 and 91	13
15.	GCF of 50 and 42	2	30.	GCF of 34 and 51	17

Number Correct: ______

Improvement: ______

Greatest Common Factor–Round 2

Directions: Determine the greatest common factor of each pair of numbers.

1.	GCF of 20 and 80		16.	GCF of 33 and 99	
2.	GCF of 10 and 70		17.	GCF of 38 and 76	
3.	GCF of 9 and 36		18.	GCF of 26 and 65	
4.	GCF of 12 and 24		19.	GCF of 39 and 48	
5.	GCF of 15 and 45		20.	GCF of 72 and 88	
6.	GCF of 10 and 95		21.	GCF of 21 and 56	
7.	GCF of 9 and 45		22.	GCF of 28 and 52	
8.	GCF of 18 and 33		23.	GCF of 51 and 68	
9.	GCF of 12 and 32		24.	GCF of 48 and 84	
10.	GCF of 16 and 56		25.	GCF of 21 and 63	
11.	GCF of 40 and 72		26.	GCF of 64 and 80	
12.	GCF of 35 and 63		27.	GCF of 36 and 90	
13.	GCF of 30 and 75		28.	GCF of 28 and 98	
14.	GCF of 42 and 72		29.	GCF of 39 and 91	
15.	GCF of 30 and 28		30.	GCF of 38 and 95	

Greatest Common Factor–Round 2 [KEY]

Directions: Determine the greatest common factor of each pair of numbers.

1.	GCF of 20 and 80	20	16.	GCF of 33 and 99	33
2.	GCF of 10 and 70	10	17.	GCF of 38 and 76	38
3.	GCF of 9 and 36	9	18.	GCF of 26 and 65	13
4.	GCF of 12 and 24	12	19.	GCF of 39 and 48	3
5.	GCF of 15 and 45	15	20.	GCF of 72 and 88	8
6.	GCF of 10 and 95	5	21.	GCF of 21 and 56	7
7.	GCF of 9 and 45	9	22.	GCF of 28 and 52	4
8.	GCF of 18 and 33	3	23.	GCF of 51 and 68	17
9.	GCF of 12 and 32	4	24.	GCF of 48 and 84	12
10.	GCF of 16 and 56	8	25.	GCF of 21 and 63	21
11.	GCF of 40 and 72	8	26.	GCF of 64 and 80	16
12.	GCF of 35 and 63	7	27.	GCF of 36 and 90	18
13.	GCF of 30 and 75	15	28.	GCF of 28 and 98	14
14.	GCF of 42 and 72	6	29.	GCF of 39 and 91	13
15.	GCF of 30 and 28	2	30.	GCF of 38 and 95	19