



Lesson 3: The Relationship of Multiplication and Addition

Student Outcomes

- Students build and clarify the relationship of multiplication and addition by evaluating identities such as $3 \cdot g = g + g + g$.

Lesson Notes


Students will continue to use the squares from Lessons 1 and 2 to create tape diagrams. Each pair of students will need 30 squares to complete the activities.


Classwork


Opening Exercise (5 minutes)

Opening Exercise

Write two different expressions that can be depicted by the tape diagram shown. One expression should include addition, while the other should include multiplication.

a. 
Possible answers: $3 + 3 + 3$ or 3×3

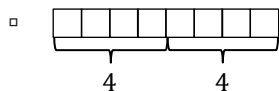
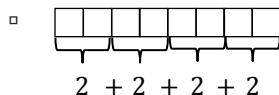
b. 
Possible answers: $8 + 8$ or 2×8

c. 
Possible answers: $5 + 5 + 5$ or 3×5

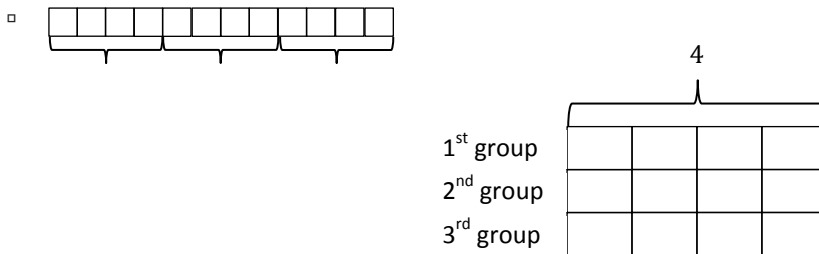
Discussion (17 minutes)

Provide each pair of students with a collection of 30 squares, which they will use to create tape diagrams throughout the lesson.

- One partner builds a tape diagram to represent the expression $2 + 2 + 2 + 2$, while the other partner builds a tape diagram to represent 4×2 .



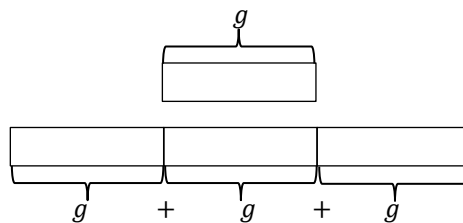
- What do you notice about the two tape diagrams you created?
 - *Possible answer: Although the tape diagrams represent two different expressions, they each have the same number of squares.*
- Why are the two tape diagrams the same? What does it say about the value of the expressions?
 - *The two tape diagrams are the same because the values of the expressions are equivalent.*
- If both expressions yield the same value, is there an advantage to using one over the other?
 - *Answers will vary.*
- Since each tape diagram has the same number of squares, can we say the two expressions are equivalent? Why or why not?
 - *Possible answer: The two expressions are equivalent because they represent the same value. When evaluated, both expressions will equal 8.*
- Therefore, $2 + 2 + 2 + 2 = 4 \times 2$. Let's build a new set of tape diagrams. One partner builds a tape diagram to represent the expression 3×4 , while the other partner builds a tape diagram to represent the expression $4 + 4 + 4$.



- Is 3×4 equivalent to $4 + 4 + 4$? Why or why not?
 - *Possible answer: The two expressions are equivalent because when each of them is evaluated, they equal 12, as we can see with our tape diagrams.*
- Using variables, write an equation to show the relationship of multiplication and addition.

Provide students with time to create an equation.

- *Possible answer: $3g = g + g + g$. Emphasize that each g represents the same number.*



MP.2

- $3g$ is the same as writing $3 \times g$, but we no longer use the \times for multiplication because it looks like a variable and can become confusing. When a number is next to a variable with no sign, multiplication is implied.
- In the two previous lessons, we talked about identities. Is the equation $3g = g + g + g$ also an identity? Why or why not?
 - *Possible answer: The equation $3g = g + g + g$ is an identity because we can replace g with any number, and the equation will always be true.*

Exercises (15 minutes)

Students can continue to work with the given squares and with their partner to answer the following questions.

Exercises

1. Write the addition sentence that describes the model and the multiplication sentence that describes the model.



$$5 + 5 + 5 \text{ and } 3 \times 5$$

2. Write an equivalent expression to demonstrate the relationship of multiplication and addition.

a. $6 + 6$

$$2 \times 6$$

b. $3 + 3 + 3 + 3 + 3 + 3$

$$6 \times 3$$

c. $4 + 4 + 4 + 4 + 4$

$$5 \times 4$$

d. 6×2

$$2 + 2 + 2 + 2 + 2 + 2$$

e. 4×6

$$6 + 6 + 6 + 6$$

f. 3×9

$$9 + 9 + 9$$

g. $h + h + h + h + h$

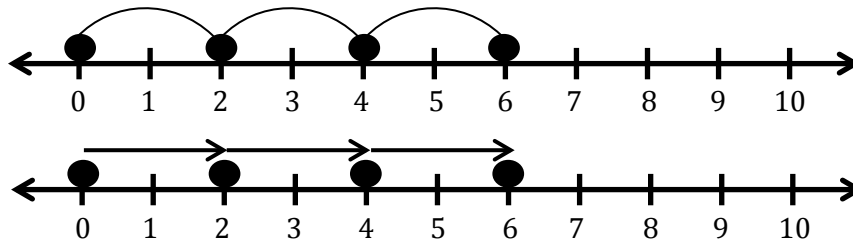
$$5h$$

h. $6y$

$$y + y + y + y + y + y$$

3. Roberto is not familiar with tape diagrams and believes that he can show the relationship of multiplication and addition on a number line. Help Roberto demonstrate that the expression 3×2 is equivalent to $2 + 2 + 2$ on a number line.

Possible answer: The first number line shows that there are 3 groups of 2, resulting in 6. The second number line shows the sum of $2 + 2 + 2$, resulting in 6.



Since both number lines start at 0 and end at 6, the expressions are equivalent.

4. Tell whether the following equations are true or false. Then, explain your reasoning.

a. $x + 6g - 6g = x$

The equation is true because it demonstrates the addition identity.

b. $2f - 4e + 4e = 2f$

The equation is true because it demonstrates the subtraction identity.

5. Write an equivalent expression to demonstrate the relationship between addition and multiplication.

a. $6 + 6 + 6 + 6 + 4 + 4 + 4$

$4 \times 6 + 3 \times 4$

b. $d + d + d + w + w + w + w + w$

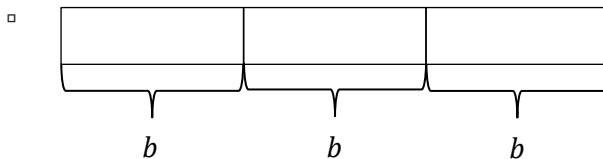
$3d + 5w$

c. $a + a + b + b + b + c + c + c + c$

$2a + 3b + 4c$

Closing (4 minutes)

- Create a diagram that models 3 groups of size b .



- Write two equivalent expressions that represent this model.

□ *Possible answers: $3b$, $b + b + b$*

- Peter says that since the addition expression yields the same value as the multiplication expression, he will always choose to use the addition expression when solving these types of problems. Convince Peter that he may want to reconsider his position.
 - *Answers will vary, but should include the idea that when the group size is large, it is more advantageous to multiply instead of add.*

Exit Ticket (4 minutes)

Name _____

Date _____

Lesson 3: The Relationship of Multiplication and Addition

Exit Ticket

Write an equivalent expression to show the relationship of multiplication and addition.

1. $8 + 8 + 8 + 8 + 8 + 8 + 8 + 8$

2. 4×9

3. $6 + 6 + 6$

4. $7h$

5. $j + j + j + j + j$

6. $u + u + u + u + u + u + u + u + u + u$



Exit Ticket Sample Solutions

Write an equivalent expression to show the relationship of multiplication and addition.

1. $8 + 8 + 8 + 8 + 8 + 8 + 8 + 8 + 8$

9×8

2. 4×9

$9 + 9 + 9 + 9$

3. $6 + 6 + 6$

3×6

4. $7h$

$h + h + h + h + h + h + h$

5. $j + j + j + j + j$

$5j$

6. $u + u + u + u + u + u + u + u + u + u$

$10u$

Problem Set Sample Solutions

Write an equivalent expression to show the relationship of multiplication and addition.

1. $10 + 10 + 10$

3×10

2. $4 + 4 + 4 + 4 + 4 + 4 + 4$

7×4

3. 8×2

$2 + 2 + 2 + 2 + 2 + 2 + 2 + 2$

4. 3×9

$9 + 9 + 9$

5. $6m$

$m + m + m + m + m + m$

6. $d + d + d + d + d$

$5d$