Lesson 3: Equivalent Ratios

Student Outcomes

- Students develop an intuitive understanding of equivalent ratios by using tape diagrams to explore possible quantities of each part when given the part-to-part ratio. Students use tape diagrams to solve problems when the part-to-part ratio is given and the value of one of the quantities is given.

- Students formalize a definition of equivalent ratios: Two ratios, $A : B$ and $C : D$, are equivalent ratios if there is a positive number, c, such that $C = cA$ and $D = cB$.

Classwork

Exercise 1 (5 minutes)

This exercise continues to reinforce the students' ability to relate ratios to the real world, as practiced in Lessons 1 and 2. Provide students with time to think of a one-sentence story problem about a ratio.

Exercise 1

Write a one-sentence story problem about a ratio.

Answers will vary. The ratio of the number of sunny days to the number of cloudy days in this town is $3 : 1$**.**

Write the ratio in two different forms.

$3 : 1$ *3 to* 1

Have students share their sentences with each other in pairs or trios. Ask a few students to share with the whole class.

Exercise 2 (15 minutes)

Ask students to read the problem, and then describe in detail what the problem is about without looking back at the description, if possible. This strategy encourages students to really internalize the information given as opposed to jumping right into the problem without knowing the pertinent information.

- Let's represent this ratio in a table.

The Length of Shanni's Ribbon (in inches)	The Length of Mel's Ribbon (in inches)
7	3
14	6
21	9

- We can use a tape diagram to represent the ratio of the lengths of ribbon. Let's create one together.

Walk through the construction of the tape diagram with students as they record with you.

- How many units should we draw for Shanni's portion of the ratio?
  - *Seven*

- How many units should we draw for Mel's portion of the ratio?
  - *Three*

Exercise 2

Shanni and Mel are using ribbon to decorate a project in their art class. The ratio of the length of Shanni's ribbon to the length of Mel's ribbon is $7: 3$.

Draw a tape diagram to represent this ratio.

Shanni ⬚⬚⬚⬚⬚⬚⬚

Mel ⬚⬚⬚

- What does each unit on the tape diagram represent?
  - *Allow students to discuss; they should conclude that they do not really know yet, but each unit represents some unit that is a length.*

- What if each unit on the tape diagrams represents 1 inch? What are the lengths of the ribbons?
  - *Shanni's ribbon is 7 inches; Mel's ribbon is 3 inches.*

- What is the ratio of the lengths of the ribbons?
  - $7: 3$ *(Make sure that the students feel comfortable expressing the ratio itself as simply the pair of numbers $7: 3$ without having to add units.)*

- What if each unit on the tape diagrams represents 2 meters? What are the lengths of the ribbons?
  - *Shanni's ribbon is 14 meters; Mel's ribbon is 6 meters.*

- How did you find that?
  - *Allow students to verbalize and record using a tape diagram.*

- What is the ratio of the length of Shanni's ribbon to the length of Mel's ribbon now? Students may disagree; some may say it is $14: 6$, and others may say it is still $7: 3$.
  - *Allow them to debate and justify their answers. If there is no debate, initiate one:* A friend of mine told me the ratio would be (provide the one that no one said, either $7: 3$ or $14: 6$). Is she right?

- What if each unit represents 3 inches? What are the lengths of the ribbons? Record. Shanni's ribbon is 21 inches; Mel's ribbon is 9 inches. Why?
  - 7 *times* 3 *equals* 21; 3 *times* 3 *equals* 9.

- If each of the units represents 3 inches, what is the ratio of the length of Shanni's ribbon to the length of Mel's ribbon?
  - *Allow for discussion as needed.*

> **Scaffolding:**
>
> If students do not see that each unit represents a given length, write the length of each unit within the tape diagram units, and have students add them to find the total.


- We just explored three different possibilities for the length of the ribbon; did the number of units in our tape diagrams ever change?
  - *No.*
- What did these three ratios, $7:3$, $14:6$, $21:9$, all have in common?
  - *Write the ratios on the board. Allow students to verbalize their thoughts without interjecting a definition. Encourage all to participate by asking questions of the class with respect to what each student says, such as, "Does that sound right to you?"*
- Mathematicians call these ratios *equivalent*. What ratios can we say are equivalent to $7:3$?
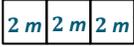

Shanni and Mel are using ribbon to decorate a project in their art class. The ratio of the length of Shanni's ribbon to the length of Mel's ribbon is $7:3$.
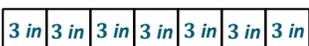
Draw a tape diagram to represent this ratio.

Shanni

7 inches
3 inches
$7:3$

Mel

Shanni 2 m 2 m 2 m 2 m 2 m 2 m 2 m
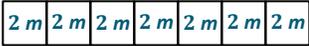
14 meters
6 meters
$14:6$

Mel 2 m 2 m 2 m

Shanni 3 in 3 in 3 in 3 in 3 in 3 in 3 in

21 inches
9 inches
$21:9$

Mel 3 in 3 in 3 in

Exercise 3 (8 minutes)

Work as a class or allow students to work independently first and then go through as a class.

Exercise 3

Mason and Laney ran laps to train for the long-distance running team. The ratio of the number of laps Mason ran to the number of laps Laney ran was 2 to 3.
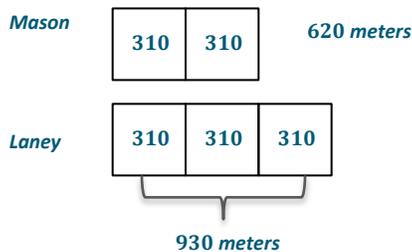
  a. **If Mason ran 4 miles, how far did Laney run? Draw a tape diagram to demonstrate how you found the answer.**

4 miles

Mason 2 mi | 2 mi

Laney 2 mi | 2 mi | 2 mi   6 miles

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b. **If Laney ran 930 meters, how far did Mason run? Draw a tape diagram to determine how you found the answer.**

Mason

| 310 | 310 |

620 *meters*

Laney

| 310 | 310 | 310 |

930 *meters*

c. **What ratios can we say are equivalent to** $2:3$**?**

$4:6$ *and* $620:930$

Exercise 4 (7 minutes)

Allow students to work the exercise independently and then compare their answers with a neighbor's answer.

Exercise 4

Josie took a long multiple-choice, end-of-year vocabulary test. The ratio of the number of problems Josie got incorrect to the number of problems she got correct is $2:9$.

a. **If Josie missed 8 questions, how many did she get correct? Draw a tape diagram to demonstrate how you found the answer.**

8

Incorrect

| 4 | 4 |

Correct

| | | | | | | | | |

36 *correct*

4×9
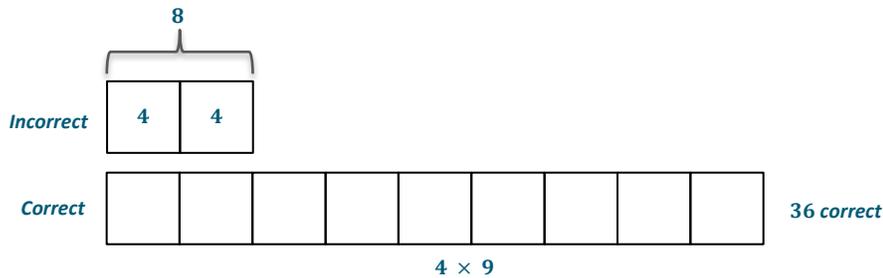
b. **If Josie missed 20 questions, how many did she get correct? Draw a tape diagram to demonstrate how you found the answer.**

20

Incorrect

| 10 | 10 |

Correct

| | | | | | | | | |

90 *correct*

10×9
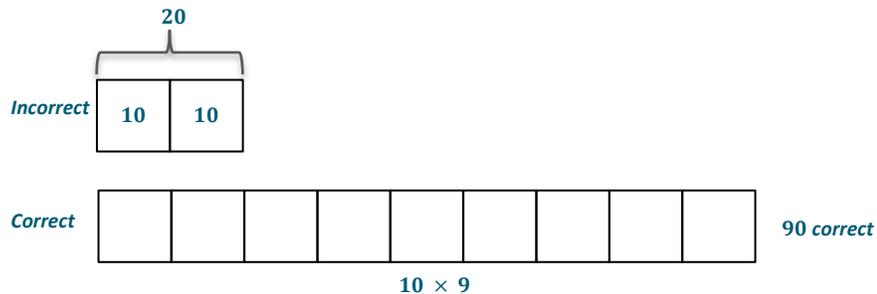
c. **What ratios can we say are equivalent to** $2:9$**?**

$8:36$ *and* $20:90$

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d. Come up with another possible ratio of the number Josie got incorrect to the number she got correct.

5	5

10:45

$5 \times 9 = 45$

e. How did you find the numbers?

Multiplied 5×2 *and* 5×9

f. Describe how to create equivalent ratios.

Multiply both numbers of the ratio by the same number (any number you choose).

Closing (5 minutes)

Ask students to share their answers to part (f); then summarize by presenting the following definition:

Two ratios $A:B$ and $C:D$ are __equivalent__ if there is a positive number, c, such that $C = cA$ and $D = cB$. Ratios are equivalent if there is a positive number that can be multiplied by both quantities in one ratio to find the corresponding quantities in the second ratio.

Note that if students do not have a sufficient grasp of algebra, they should not use the algebraic definition. It is acceptable to use only the second definition.

Lesson Summary

Two ratios $A:B$ and $C:D$ are equivalent ratios if there is a positive number, c, such that $C = cA$ and $D = cB$.

Ratios are equivalent if there is a positive number that can be multiplied by both quantities in one ratio to equal the corresponding quantities in the second ratio.

Exit Ticket (5 minutes)

Name _____   Date_____

Lesson 3: Equivalent Ratios

Exit Ticket

Pam and her brother both open savings accounts. Each begin with a balance of zero dollars. For every two dollars that Pam saves in her account, her brother saves five dollars in his account.

1. Determine a ratio to describe the money in Pam's account to the money in her brother's account.

2. If Pam has 40 dollars in her account, how much money does her brother have in his account? Use a tape diagram to support your answer.

3. Record the equivalent ratio.

4. Create another possible ratio that describes the relationship between the amount of money in Pam's account and the amount of money in her brother's account.

Exit Ticket Sample Solutions

Pam and her brother both open savings accounts. Each begin with a balance of zero dollars. For every two dollars that Pam saves in her account, her brother saves five dollars in his account.

1. Determine a ratio to describe the money in Pam's account to the money in her brother's account.

   $2: 5$

2. If Pam has 40 dollars in her account, how much money does her brother have in his account? Use a tape diagram to support your answer.

   40

   Pam | 20 | 20 |
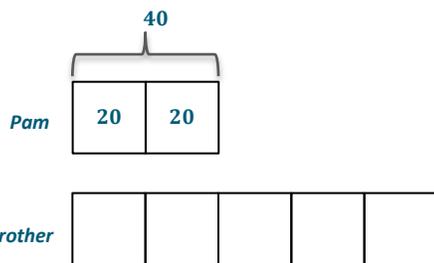
   Brother | | | | | |

3. Record the equivalent ratio.

   $40: 100$

4. Create another possible ratio that describes the relationship between the amount of money in Pam's account and the amount of money in her brother's account.

   Answers will vary. $4: 10, 8: 20,$ *etc.*
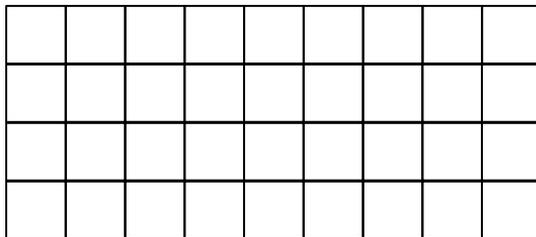
Problem Set Sample Solutions

1. Write two ratios that are equivalent to $1: 1$.

   Answers will vary. $2: 2, 50: 50,$ *etc.*

2. Write two ratios that are equivalent to $3: 11$.

   Answers will vary. $6: 22, 9: 33,$ *etc.*

3.

   a. The ratio of the width of the rectangle to the height of the rectangle is ___9___ to ___4___.

b.   If each square in the grid has a side length of 8 mm, what is the width and height of the rectangle?

72 mm wide and 32 mm high.

4.   For a project in their health class, Jasmine and Brenda recorded the amount of milk they drank every day. Jasmine drank 2 pints of milk each day, and Brenda drank 3 pints of milk each day.

a.   Write a ratio of the number of pints of milk Jasmine drank to the number of pints of milk Brenda drank each day.

2: 3

b.   Represent this scenario with tape diagrams.

Jasmine

Brenda

c.   If one pint of milk is equivalent to 2 cups of milk, how many cups of milk did Jasmine and Brenda each drink? How do you know?

Jasmine drank 4 cups of milk, and Brenda drank 6 cups of milk. Since each pint represents 2 cups, I multiplied Jasmine's 2 pints by 2 and multiplied Brenda's 3 pints by 2.

d.   Write a ratio of the number of cups of milk Jasmine drank to the number of cups of milk Brenda drank.

4: 6

e.   Are the two ratios you determined equivalent? Explain why or why not.

2: 3 and 4: 6 are equivalent because they represent the same value. The diagrams never changed, only the value of each unit in the diagram.