Stations for Solving Systems Using Substitution

Instruction

Goal: To provide opportunities for students to develop concepts and skills related to solving systems of linear equations using substitution

Common Core Standards

Expressions and Equations

Analyze and solve linear equations and pairs of simultaneous linear equations.

8.EE.8. Analyze and solve pairs of simultaneous linear equations.

- Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.
- Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection.
- Solve real-world and mathematical problems leading to two linear equations in two variables.

Student Activities Overview and Answer Key

Station 1

Students will be given 10 blue algebra tiles to represent the coefficient of x, 10 yellow algebra tiles to represent the coefficient of y, and 40 red algebra tiles to represent the constant. They will also be given index cards with x, y, +, –, and = written on them.

Students use the algebra tiles and index cards to model a system of linear equations. They rearrange the tiles and index cards to solve for x. Then they solve for y to find the solution to the system of linear equations.

Answers

1. x and y
2. $x = -y + 10$
3. $2(-y + 10) + y = 15$; $y = 5$
4. Substitute y into the first equation to find $x = 5$.
5. $x + y = 10$ and $5 + 5 = 10$; $2x + y = 15$ and $2(5) + 5 = 15$

Instruction

Station 2

Students will be given systems of linear equations. They will work together to solve each system of linear equations using the substitution method. They will show their work and explain how to double-check their answers.

Answers

1. $x = y + 3$

   $5(y + 3) - y = -5$

   $5y + 15 - y = -5$

   $y = -5$

   and

   $5x - (-5) = -5$

   $x = -2$

2. For $5x = 4y + 1$, $x = \frac{4}{5}y + \frac{1}{5}$, so $7\left(\frac{4}{5}y + \frac{1}{5}\right) + 4y = 11$, $y = 1$ and for $5x - 4(1) = 1$, $x = 1$.

3. Solve for one variable in the first equation. Substitute this quantity in for that variable in the second equation. Solve for the variable still in the equation. Then substitute your answer for that variable into the original equation to find the value of the other variable.
4. Substitute x and y into both equations to make sure they satisfy both equations.
5. You substitute one variable in terms of the other variable to find x or y.

Station 3

Students will be given a system of linear equations and nine index cards with the following equations written on them:

$10x = 30$	$y = 1$	$2x - 12x + 30 = 0$	$y = 2x - 5$	$-10x + 30 = 0$
$6 - y = 5$	$x = 3$	$2x - 6(2x - 5) = 0$	$2(3) - y = 5$	

Each index card represents a step in solving a system of linear equations by substitution. Students work together to arrange the steps in the correct order. They give the strategy they used to arrange the index cards.

Answers

1. $y = 2x - 5$
2. $2x - 6(2x - 5) = 0$
3. $2x - 12x + 30 = 0$
4. $-10x + 30 = 0$
5. $10x = 30$
6. $x = 3$
7. $2(3) - y = 5$
8. $6 - y = 5$
9. $y = 1$
10. Answers will vary. Possible answer: I looked for one variable written in terms of the other variable. Then I solved by substitution in a step-by-step manner.

Station 4

Students will be given systems of linear equations that have either infinite solutions or no solutions. They will try to find the solution of the system. Then they will realize that lines with the same slope and *y*-intercept have infinite solutions. They will also find that lines that have the same slope but different *y*-intercepts have no solutions.

Answers

1. $y = 4 - x$
   $2x + 2(4 - x) = 8$
   $8 = 8$
2. Answers will vary.
3. –1
4. –1
5. yes; yes
6. When lines are parallel and have the same *y*-intercept, they are the same line. This means the system of linear equations has an infinite number of solutions.
7. $4 \neq 10$

8. no solutions
9. 2
10. 2
11. yes
12. no
13. The lines have the same slope, but different *y*-intercepts. There are no solutions to the system of linear equations.

Materials List/Setup

Station 1 10 blue algebra tiles to represent the coefficient of *x*, 10 yellow algebra tiles to represent the coefficient of *y*, and 40 red algebra tiles to represent the constant; index cards with *x*, *y*, +, –, and = written on them

Station 2 none

Station 3 nine index cards with the following written on them:

$10x = 30$	$y = 1$	$2x - 12x + 30 = 0$	$y = 2x - 5$	$-10x + 30 = 0$
$6 - y = 5$	$x = 3$	$2x - 6(2x - 5) = 0$	$2(3) - y = 5$	

Station 4 none

Instruction

Discussion Guide

To support students in reflecting on the activities and to gather some formative information about student learning, use the following prompts to facilitate a class discussion to "debrief" the station activities.

Prompts/Questions

1. How do you solve systems of linear equations using substitution?
2. How many solutions does a system of linear equations have if the equations have the same slope and *y*-intercept?
3. How many solutions does a system of linear equations have if the equations have the same slope, but different *y*-intercepts?
4. Why does the "substitution method" have this name?

Think, Pair, Share

Have students jot down their own responses to questions, then discuss with a partner (who was not in their station group), and then discuss as a whole class.

Suggested Appropriate Responses

1. Solve for one variable in the first equation. Substitute this quantity in for that variable in the second equation. Solve for the variable still in the equation. Then substitute your answer for that variable into the original equation to find the value of the other variable.
2. infinite number of solutions
3. no solutions
4. You substitute one variable in terms of the other variable to find x or y.

Possible Misunderstandings/Mistakes

- Incorrectly solving for one variable when rewriting an equation in step 1 of the process
- Incorrectly substituting x in the first equation for y in the second equation
- Not double checking that the values of x and y satisfy both equations
- Not realizing that lines that have the same slope and *y*-intercept are actually the same line. These have an infinite number of solutions to the system of linear equations.
- Not realizing that parallel lines that have different *y*-intercepts will have no solutions for the system of linear equations

NAME:

Expressions and Equations

Set 5: Solving 2-by-2 Systems by Substitution

Station 1

At this station, you will find 10 blue algebra tiles to represent the coefficient of x, 10 yellow algebra tiles to represent the coefficient of y, and 40 red algebra tiles to represent the constant. You will also be given index cards with x, y, +, −, and = written on them.

As a group, arrange the algebra tiles and index cards to model the system of linear equations below:

$$\begin{cases} x + y = 10 \\ 2x + y = 15 \end{cases}$$

1. What values do you need to find in order to solve this system of linear equations?

2. Rearrange $x + y = 10$ to solve for x. Write your answer on the line below.

   _____

3. Substitute what you found in problem 2 into the equation $2x + y = 15$. Show your work in the space below.

   Now solve for y. What is the value of y?

   _____

4. How can you use this value of y to find x?

   What is the value of x?

   _____

5. Verify that x and y satisfy both equations. Show your work in the space below.

NAME:

Expressions and Equations

Set 5: Solving 2-by-2 Systems by Substitution

Station 2

Use substitution to find the solutions for each system of linear equations. Show your work.

1. $\begin{cases} 5x - y = -5 \\ x - y = 3 \end{cases}$

2. $\begin{cases} 5x - 4y = 1 \\ 7x + 4y = 11 \end{cases}$

3. What strategies did you use to find x and y?

4. How can you double-check your solutions for x and y?

5. Why is this method called the "substitution" method?

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Expressions and Equations

Set 5: Solving 2-by-2 Systems by Substitution

Station 3

At this station, you will find nine index cards with the following written on them:

$10x = 30$ $\quad y = 1$ $\quad 2x - 12x + 30 = 0$ $\quad y = 2x - 5$ $\quad -10x + 30 = 0$

$6 - y = 5$ $\quad x = 3$ $\quad 2x - 6(2x - 5) = 0$ $\quad 2(3) - y = 5$

Given: $\begin{cases} 2x - y = 5 \\ 2x - 6y = 0 \end{cases}$

Each index card represents one step in solving the linear system of equations by substitution. Shuffle the index cards. Work together to arrange the index cards in the correct order of solving the system of linear equations by substitution.

List the steps below.

1. _____
2. _____
3. _____
4. _____
5. _____
6. _____
7. _____
8. _____
9. _____
10. What strategy did you use to find the order of the steps?

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Expressions and Equations

Set 5: Solving 2-by-2 Systems by Substitution

Station 4

Work as a group to solve this system of linear equations by substitution.

$$\begin{cases} x + y = 4 \\ 2x + 2y = 8 \end{cases}$$

1. What happens when you solve this equation by substitution?

2. Based on problem 1, how many solutions do you think this system of linear equations has?

   _____

3. What is the slope of $x + y = 4$? _____

4. What is the slope of $2x + 2y = 8$? _____

5. Are these two lines parallel? _____

   Do these lines have the same y-intercept? _____

6. How can you relate the slopes and y-intercepts of each equation to the number of solutions for the linear system of equations?

continued

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Expressions and Equations

Set 5: Solving 2-by-2 Systems by Substitution

Work as a group to solve this system of linear equations by substitution.

$$\begin{cases} y = 2x + 4 \\ y = 2x + 10 \end{cases}$$

7. What happens when you solve this equation by substitution?

8. Based on problem 7, how many solutions do you think this system of linear equations has?

   _____

9. What is the slope of $y = 2x + 4$? _____

10. What is the slope of $y = 2x + 10$? _____

11. Are these two lines parallel? _____

12. Do these lines have the same y-intercept? _____

13. How can you relate the slopes and y-intercepts of each equation to the number of solutions for the linear system of equations?