NVACS: Number and Operations in Base Ten (5.NBT)

Overview for 5th Graders

5. NBT

Understand the Place Value System. (5.NBT.1, 2, 3, 4)

Perform Operations with multi-digit whole numbers and with decimals to hundredths. (5.NBT.5, 6, 7)

Students develop understanding of why division procedures work based on the meaning of base-ten numerals and properties of operations. They finalize fluency in these computations, and make reasonable estimates of their results. Students use the relationship between decimals and fractions, as well as the relationship between finite decimals and whole numbers (i.e., a finite decimal multiplied by an appropriate power of 10 is a whole number), to understand and explain why the procedures for multiplying and dividing finite decimals make sense. They compute products and quotients of decimals to hundredths efficiently and accurately.

Our Place Value System

In order to efficiently communicate ideas related to number, we must have a way of representing numbers symbolically. Writing math symbolically takes the place of describing mathematical thinking in words. The current symbolic system we use was developed around A.D. 800. Our Base Ten system, with place values, is a collection of properties and symbols that result in a systematic way to write all numbers. The two characteristics of our system are, place value, and grouping by ten. The 5 in the 58 has a place value representing the tens place; the face value of the 5 is five groups of ten, not five ones.

Structure in Our Number System

In grades 3-5, students apply the properties of addition and multiplication as they learn their basic facts and computational strategies. As the students learn them they are also expected to apply them to solving problems. Properties of Operations can be built into students' explorations with true/false statements and open number sentences.

4567 + 346 = 346 + 4567 (Is this true for any two numbers?)

217 + □= 195 + 22

I have more patches on my quilt than my friend.

He has eight rows of four and I have five rows of seven.

Do they have an equal number, not equivalent, greater than, or less than? (=, ≠, >, <)

Properties of Operations

The Properties of Operation have been used by the students in elementary school to learn their basic facts and computational strategies.

Properties of Operations for Addition

Name of Property	Symbolic Representation	How Students Might Describe the Pattern or Structure
Commutative	a + b = b+ c	"When you add two numbers in any order, you'll get the same answer."
Associative	(a + b)+ c = a + (b+ c)	"When you add three numbers, you can add the first two and then add the third or add the last two numbers and then add the first number. Either way, you will get the same answer."
Additive Identity (addition)	a + 0 =0+ a	"When you add zero to any number, you get the same number you started with."
Additive Identity (subtraction)	a - 0 = a	"When you subtract zero from any number, you get the number you started with."
Additive Inverse	a – a = 0 or a + (-a) = 0	"When you subtract a number from itself, you get zero."
Inverse Relationship of Addition and Subtraction	If a + b = c then c – b = a and c – a =b	"When you have a subtraction problem you can 'think addition' by using the inverse.

Properties of Operation for Multiplication

Commutative	a x b = b x c	"When you multiply two numbers in any order, you will get the same answer."
Associative	(a x b) x c = a	"When you multiply three numbers, you can multiply the first two and then multiply the answer by the third or multiply the last two numbers and then multiply that answer by the first number. Either way, you will get the same answer."
Multiplicative Identity	a x 1 =1 x a = a	"When you multiply one by any number, you get the same number you started with."
Multiplicative Inverse	a x 1/a = 1/a = 1	When you multiply a number by its reciprocal, you will get one."
Inverse Relationship of Multiplication and Division	If a x b =c then c ÷ b = a and c ÷ a b	"When you have a division problem, you can 'think multiplication' by using the inverse."
Distributive (Multiplication over Addition)	a x (b + c) =a x b + a x c	"When you multiply two numbers, you can split one number into two parts (5 can be 2 + 3), multiply each part by the other number, and then add them together.

Relational Thinking with Base Ten Application

If the numbers are like the ones in the example below the students could use both relational thinking and base ten understanding to agree or disagree with the following equation.

$$6000 \div 1000 = 600 \div 100$$

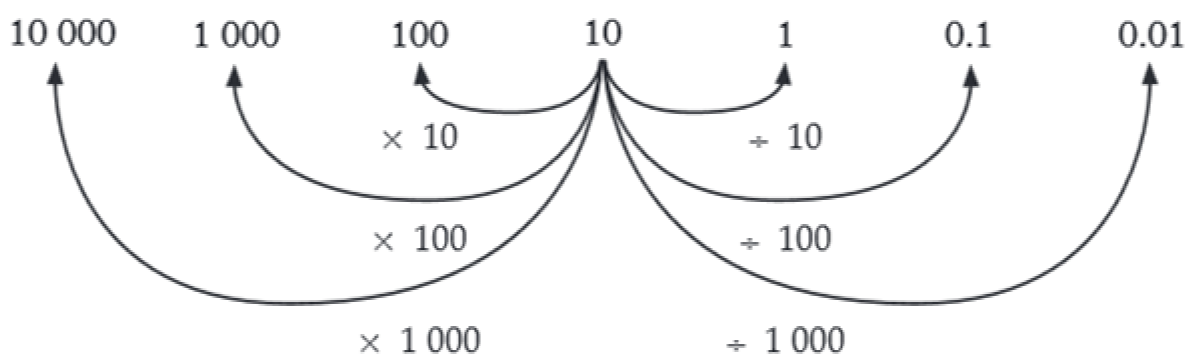
Fluency in Operations

In fifth grade student learn to compute products fluently using the standard algorithm. In third and fourth grade the students used the properties of operations and the base ten number system and used alternative algorithms to compute products efficiently.

Powers of 10

Place value is a powerful structure that makes complex problems relatively simple.

There is a "multiply by ten" relationship connecting any place with the place immediately to its left and a "divide by ten" relationship that connects it to the place immediately to its right:



10000 x 10000= 100,000,000 ($10^4 \times 10^4 = 10^8$)

Expanded Form to the Thousandths Place Value

1 0 0 0
5 0 0
6 0
8

1 Thousand, 5 hundreds, 60 tens, and 8 ones can be written as 1,568.

Division Procedures for Whole Numbers

In this model students are taking away groups that are multiples of ten until there are no more groups of multiples of 10 to take away. The students can now take away other groups of multiples. In this case the student tool away 4 multiples of four and subtracted 16. One is left which is 3 less than another group of 4, so there is a remainder of 1. 234 **R**. 1

4)937	
– 400	100
537	
– 400	100
137	
– 80	20
57	
– 40	10
17	
– 16	4
1	234 r1

Note: In this example there is a lower case r and then the remainder. It is suggested that you write remainders with a capital R because students may confuse it with a 'variables 'in 5th grade. 'Variables' are typically written in lower case. Formulas are written in Upper Case.

*Students are not expected to use the standard division algorithm until 6th grade. The above division algorithm is one that is based on place value. Repeated groups are subtracted to find the quotient and any remainder.

Multiplication Procedures for Whole Numbers

$$\begin{aligned} &= (5 \times 60) + (5 \times 8) \\ &= \quad 300 \quad + \quad 40 \\ &= 340 \end{aligned}$$

x	60	8		
5	300 +	40	=	340

$$\begin{array}{ll} 5 \times 60 & 300 \\ 5 \times 8 & \underline{+\,40} \\ & 340 \end{array} \quad \text{to} \quad \begin{array}{rl} 68 & \\ \underline{\times 5} & \\ 40 & 5 \times 8 \\ \underline{300} & 5 \times 60 \\ 340 & \end{array} \quad \text{to} \quad \begin{array}{r} {}^{4}68 \\ \underline{\times 5} \\ 340 \end{array}$$

Students are introduced to a short method of multiplication that involves the recording of the regrouped "ones" above the tens value (See the model on the right.) Students need to make connections between the partial product model and the standard algorithm model.

Computation with Decimals

Students should have a firm understanding of place value and a connection between decimals and fractions. In the NVACS it states that 5th graders should "apply their understanding of models of decimals, decimal notation, properties of operations to add and subtract decimals to hundredths. They develop fluency in these computations, and make reasonable estimates of their results. Students use the relationship between decimals and fractions, as well as the relationship between finite decimals and whole numbers (i.e., a finite decimal multiplies by an appropriate power of 1 is a whole number), to understand and explain why the procedures for multiplying and dividing finite decimals make sense" CCSSO, 21, p.33

Add, Subtract, Multiply, and Divide with Decimals

Because of the uniformity of the base-ten system, students use the same place value understanding for adding and subtracting decimals that they used for adding and subtracting whole numbers. Students align the place values and compute. It can help students to put the zeros in the problem to line up the place values in the number. (e.g., a whole number is not written with a decimal. If adding or subtracting with other decimals and decimal point and zeros can be added. Whole number of 52 can be written as 52.00 if you are subtracting 1.2.)

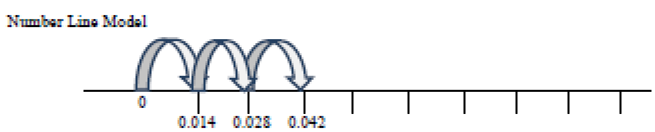
Before students are asked to multiply and divide with decimals students they should have experiences estimating. Students can build on their experiences they have had with the whole numbers in grade 1-4 and apply these understandings.

3.6 + 1.7 (A students might estimate the sum to be larger than 5 because 3.6 is more than 3 ½ and 1.7 is more than 1 ½.)

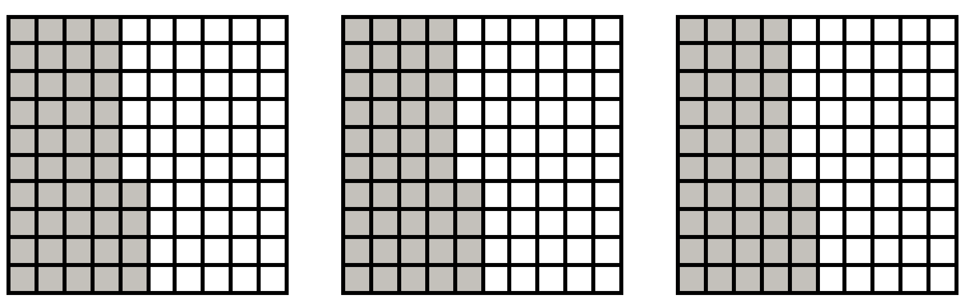
6 x 2.4 (A student might estimate and answer between 12 and 18 since 6 x 2 is 12 and 6 x 3 is 18. Another student might give an estimate of a little less than 15 because s/he figures that answer to be very close , but smaller than 6 x 2 ½ and think of 2 ½ groups of 6 as 12 (2 groups of 6) + 3(1/2 of a group of 6

Examples of Multiplication

Example 1: A paperclip costs $0.014 to make. How much would 3 paperclips cost to make?



Example 2: A candy bar costs $ 0.44. How much 3 candies would bars cost? Grid Paper model can be used to represent 4 tenths and 4 hundredths. Use the model to solve.



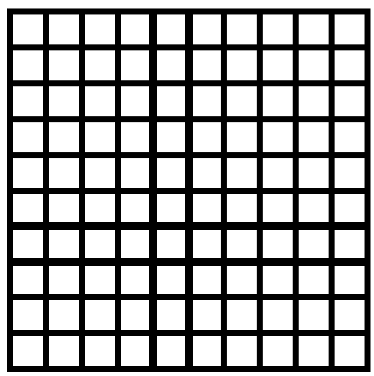
Answer: $1.32

Examples of Division

Example 1:

A relay race lasts 4.65 miles. The relay team has 3 runners. If each runner goes the same distance, how far does each team member run? Students can make an estimate first, find their actual answer by computation, and then compare them.

100 grids can be used to show a model of the above word problem. First represent 4.65 hundredths and then divided into 3 sections with labeling or coloring the three parts to show the solution of 1.55 miles

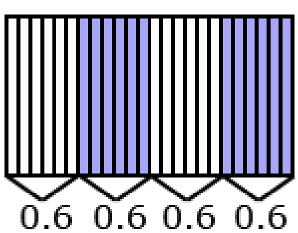


Example 2:

Each individual section represents 1 tenth (0.1) of a whole (1.0).

The model below represents 24 = 24 X (0.1)

2.4 ÷ 4 = 0.6



Place Value to Understand Rounding

Students should go beyond simply applying an algorithm or procedure for rounding. The expectation is that students have a deep understanding of place value and number sense and can explain and reason about the answers they get when they round.

Number Line Example: Students can use the number line to give them a visual model of the number system and round to the nearest whole number or tenth.