Overview for 4th Graders

4. NBT.1.2.3.4.5.6.

Generalize Place Value Understanding for multi=digit whole numbers and apply that understanding to multi-digit arithmetic.

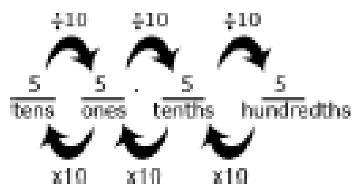
*Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000.

- Students generalize their understanding of place value to 1,000,000, understanding the relative sizes of numbers in each place.
- They apply their understanding of models for multiplication (equal-sized groups, arrays, area models), place value, and properties of operations, in particular the distributive property, as they develop, discuss, and use efficient, accurate, and generalizable methods to compute products of multi-digit whole numbers.
- Depending on the numbers and the context, they select and accurately apply appropriate methods to estimate or mentally calculate products.
- They develop fluency with efficient procedures for multiplying whole numbers; understand and explain why the procedures work based on place value and properties of operations; and use them to solve problems.
- Students apply their understanding of models for division, place value, properties of operations, and the relationship of division to multiplication as they develop, discuss, and use efficient, accurate, and generalizable procedures to find quotients involving multi-digit dividends.
- They select and accurately apply appropriate methods to estimate and mentally calculate quotients, and interpret remainders based upon the context.

Our Place Value System

In order to efficiently communicate ideas related to number, we must have a way of representing numbers symbolically. Writing math symbolically takes the place of describing mathematical thinking in words. The current symbolic system we use was developed around A.D. 800. Our Base Ten system, with place values, is a collection of properties and symbols that result in a systematic way to write all numbers. The two characteristics of our system are, place value, and grouping by ten. The 5 in the 58 has a place value representing the tens place; the face value of the 5 is five groups of ten, not five ones.

There is a "multiply by ten" relationship connecting any place with the place immediately to its left and a "divide by ten" relationship that connects it to the place immediately to its right:



Expanded Form to the Thousandths Place Value

1,000 + 500 + 60 + 8 = 1, 568

1 Thousand, 5 hundreds, 60 tens, and 8 ones can be written as 1,568.

5,467 $>$ 5, 367

4th graders need to read and write multi-digit whole numbers using base-ten numerals, number names, and expanded forms. Students need to compare two multi-digit numbers based on meanings of the digits in each place using, equivalent, not equivalent, greater than, or less than? ($=, \neq, >, <$)

1 0 0 0

5 0 0

6 0

8

Structure in Our Number System

In 4th grade students extend their understanding from 2nd and 3rd grade, where they learned their basic facts and computational strategies through properties of operations and place value. As the students learn the operations they are expected to apply them to solving word problems (Problem Situations).

Properties of Operations can also be built into students' explorations with true/false statements and open number sentences.

- 987 + 346 = 346 + 987 (Is this true for any two numbers?)
- 217 + □= 195 + 22
- I have more patches on my quilt than my friend. He has eight rows of four and I have five rows of seven. Do they have an equal number, not equivalent, greater than, or less than? ($=, \neq, >, <$)

Properties of Operations

The Properties of Operation have been used by the students in elementary school to learn their basic facts and computational strategies.

Properties of Operations for Addition

Name of Property	Symbolic Representation	How Students Might Describe the Pattern or Structure
Commutative	a + b = b+ c	"When you add two numbers in any order, you'll get the same answer."
Associative	(a + b)+ c = a + (b+ c)	"When you add three numbers, you can add the first two and then add the third or add the last two numbers and then add the first number. Either way, you will get the same answer."
Additive Identity (addition)	a + 0 =0+ a	"When you add zero to any number, you get the same number you started with."
Additive Identity (subtraction)	a - 0 = a	"When you subtract zero from any number, you get the number you started with."
Additive Inverse	a – a = 0 or a + (-a) = 0	"When you subtract a number from itself, you get zero."
Inverse Relationship of Addition and Subtraction	If a + b = c then c – b = a and c – a =b	"When you have a subtraction problem you can 'think addition' by using the inverse.

Properties of Operation for Multiplication

Commutative	a x b = b x c	"When you multiply two numbers in any order, you will get the same answer."
Associative	(a x b) x c = a	"When you multiply three numbers, you can multiply the first two and then multiply the answer by the third or multiply the last two numbers and then multiply that answer by the first number. Either way, you will get the same answer."
Multiplicative Identity	a x 1 =1 x a = a	"When you multiply one by any number, you get the same number you started with."
Multiplicative Inverse	a x 1/a = 1/a = 1	When you multiply a number by its reciprocal, you will get one."
Inverse Relationship of Multiplication and Division	If a x b =c then c ÷ b = a and c ÷ a b	"When you have a division problem, you can 'think multiplication' by using the inverse."
Distributive (Multiplication over Addition)	a x (b + c) =a x b + a x c	"When you multiply two numbers, you can split one number into two parts (5 can be 2 + 3), multiply each part by the other number, and then add them together.

Fluency in Operations

In grades K-3 students learned understanding and application of the properties of operations and place value to achieve conceptual understanding of basic facts. Student's develop fluency with basic facts in addition, subtraction, **(+,-, facts by memory by 2nd Grade**), multiplication, and division (**x, ÷, facts by memory by 3rd Grade**). In third grade student learn to compute products and quotients within 100, using properties of operations and the base ten number system, and continue their development of alternative algorithms with multi-digit numbers. By the end of **4th** grades students are expected to apply and understand the **standard algorithm for addition and subtraction with multi-digit whole numbers**. In **5th** grade student are expected to apply and understand the **standard algorithm for multiplication with multi-digit whole numbers.** In **6th** grade student are expected to apply and understand the **standard algorithm for division with multi-digit whole numbers.**

Fluently add and Subtract Multi-digit Digit Whole Numbers Using the Standard Algorithm

Examples for addition: In fourth grade students make the connection between alternative algorithms and the standard US algorithm. It is important for students to experience algorithms where place value and properties of operations are apparent before the connection is made to the US standard algorithm.

Look at the **alternative addition algorithm** example on the right. When students are developing their understanding of addition with multi-digit numbers they can use math talk to discuss the process with place value understanding. Students can describe how in the ones place value, five and six equals 11 ones (1 ten and 1 one), and when that one ten is regrouped from the ones place value is added to the 3 and 8 in the tens column it equals 12 tens (1 hundred and 2 tens), and then that one 1 hundred from the tens column is regrouped from the tens place value is added to the 2 and the 3 in the hundreds column it equals 6 hundred. Answer is 621.

```
                                      10
  235  →  200 + 30 + 5  →  200 + 30 + 5
+ 386  →  300 + 80 + 6  →  300 + 80 + 6
 ____     ____________     ____________

           100  10
       →  200 + 30 + 5
       →  300 + 80 + 6  →  621
          600 + 20 + 1
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Now look at the example to your right which is the **standard addition algorithm**. Students can use this shortcut with the underlying understanding they have developed in provious grades. They should know that the 1's above the tens and hundreds place value are really a 10 and a 100.

$$\begin{array}{r} \scriptstyle 11 \\ 235 \\ +\ 487 \\ \hline 722 \end{array}$$

Examples for Subtraction:

$$\begin{array}{r} \\ 856 \\ -\ 138 \\ \hline 718 \end{array} \qquad \begin{array}{l} \ 40\ \ 16 \\ 800 + \not{50} + \not{6} \\ \underline{100 + 30 + 8} \\ 700 + 10 + 8 \ = 718 \end{array}$$

The example above shows how a student made an equivalent equation of 50 = 40 +10. The student then regrouped the 10 to add it to the 6 in the ones place value. The student then subtracted the problem to equal 718. This is just one example of an alternative algorithm students may use and find efficient that develops their place value understanding before making a connection to the standard algorithm.

Multiplication Procedures for Whole Numbers

$$\begin{aligned} &= (5 \times 60) + (5 \times 8) \\ &= \ \ 300 \ \ + \ \ 40 \\ &= 340 \end{aligned}$$

x	60	8	
5	300 +	40	= 340

$$\begin{array}{ll} 5 \times 60 & \ \ 300 \\ 5 \times 8 & \underline{+\ 40} \\ & \ \ 340 \end{array} \quad \text{to} \quad \begin{array}{rl} 68 & \\ \underline{\times 5} & \\ 40 & 5 \times 8 \\ \underline{300} & 5 \times 60 \\ 340 & \end{array} \quad \text{to} \quad \begin{array}{r} {}^{4}68 \\ \underline{\times 5} \\ 340 \end{array}$$

Example: Students are introduced to a short method of multiplication that involves the recording of the regrouped "ones" above the tens value (See the model on the right.) Students need to make connections between the partial product model and the standard algorithm model.

Division Procedures for Whole Numbers

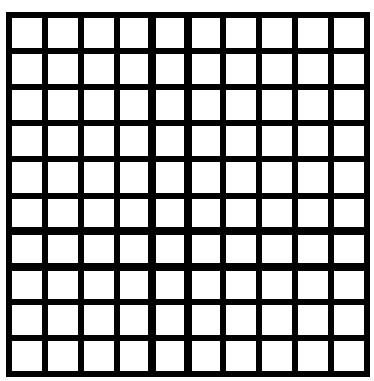
In this model students are taking away groups that are multiples of ten until there are no more groups of multiples of 10 to take away. The students can now take away other groups of multiples. In this case the student tool away 4 multiples of four and subtracted 16. One is left which is 3 less than another group of 4, so there is a remainder of 1. 234 **R**. 1

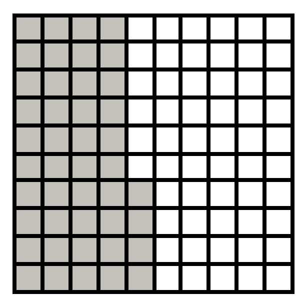
4) 937	
– 400	100
537	
– 400	100
137	
– 80	20
57	
– 40	10
17	
– 16	4
1	234 r1

Note: In this example there is a lower case r and then the remainder. It is suggested that you write remainders with a capital R because students may confuse it with a 'variables 'when they are introduced to them in 5th grade. 'Variables' are typically written in lower case. Formulas are written in Upper Case.

*Students are not expected to use the standard division algorithm until 6th grade. The above division algorithm is one that is based on place value. Repeated groups are subtracted to find the quotient and any remainder.

In 4th grade Students should have a firm understanding of place value and a connection between decimals and fractions.

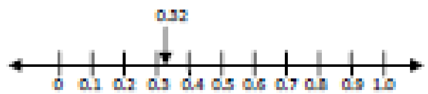




The model on the left can represent a whole with a 100 parts and be used as a model for decimals.

The model above is an example of how 0.44 = 4 tenths and 4 can be modeled on the 100 grid with shading.

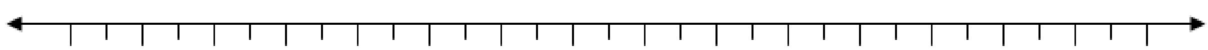
4th graders use decimal notation and compare two decimals to hundredths based on meanings of the digits in each place using, equivalent, not equivalent, greater than, or less than? (=, ≠, >, <)



Place Value to Understand Rounding

Students should go beyond simply applying an algorithm or procedure for rounding. The expectation is that students have a deep understanding of place value and number sense and can explain and reason about the answers they get when they round.

Number Line Example: Students can use differently segmented number lines, depending on the numbers they need to round, to give them a visual model of the number system.



The open number line to the right shows a model of How this tool can be used to prove their mathematical reasoning. Is 368 closer to 350 or 400? Or is 368 closer to 300 or 400? Students have to know what 'range of numbers' have to be considered when rounding numbers.