At RPDP, we support educators through professional development. Professional development can occur in a variety of ways: Entire staff trainings, grade level meetings, one-on-one support, etc. We collaborate with administrators and teachers regarding the developing and strengthening math content knowledge, use of best practices in the classroom, we model lessons, and provide support for the use of quality instructional materials.

Providing educators with quality resources in regards to instructional materials is a continuous priority. We provide this support through math content overviews, the use of instructional materials, further practice/skill development materials, and through quality assessments/tasks. As we work to create these resources for educators, we may recommend other quality resources from time to time.

In recent years, some states have received funds to create quality instructional materials for ALL educators for ALL states to access. We have selected some of those materials that we believe support our vision of quality instructional materials that support teachers in providing a solid mathematical foundation for students. For more elementary math resources please visit Rpdp.net .

Adding and Subtracting Fractions

Mathematics, Grade 5

In this unit students extend what they have learned about fractions, including the meaning and uses of fractions and how to represent them. Students should know how to form equivalent fractions and how to compare fractions from earlier grades. They should have had experience with adding and subtracting fractions with like denominators.

Students will be asked to reason quantitatively, using benchmark fractions and estimation as a means of developing fraction number sense. Estimation can be useful in problem solving and checking to see if answers make sense.

The focus in grade five is to extend the conceptual understanding of adding and subtracting whole numbers to non-unit fractions with unlike denominators, developing fluency in adding and subtracting fractions. This is identified as Critical Area 1 in the 2011 Massachusetts Curriculum Frameworks for Mathematics. Students will be using improper fractions and mixed numbers in these applications, often switching between those representations of quantities. Students will need to regroup appropriately between whole numbers and fractions. Students will also become proficient in the use of the Mathematical Practices as outlined in the frameworks to demonstrate conceptual understanding.

These Model Curriculum Units are designed to exemplify the expectations outlined in the MA Curriculum Frameworks for English Language Arts/Literacy and Mathematics incorporating the Common Core State Standards, as well as all other MA Curriculum Frameworks. These units include lesson plans, Curriculum Embedded Performance Assessments, and resources. In using these units, it is important to consider the variability of learners in your class and make adaptations as necessary.

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Stage 1 Desired Results

ESTABLISHED GOALS

5.NF.1
Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with the denominators. *For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12.*

5.NF.2
Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g. by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result 2/5 + ½ = 3/7, by observing that 3/7 < ½.*

SMP.1 Make sense of problems and persevere in solving them.

SMP.2 Reason abstractly and quantitatively.

SMP.3 Construct viable arguments

Transfer

Students will be able to independently use their learning to…

Transfer Goals

1. Interpret and persevere in solving complex mathematical problems using strategic thinking and expressing answers with a degree of precision appropriate for the problem context. Is it okay to have 2 transfer goals?
2. Apply mathematical knowledge to analyze and model mathematical relationships in the context of a situation in order to make decisions, draw conclusions, and solve problems.

Meaning

UNDERSTANDINGS

Students will understand that…

1. Equivalent fractions are a powerful strategy for adding and subtracting fractions

2. Estimation provides a close answer that can be used to determine the reasonableness of answers or can be the solution to the problem

3. Addition/subtraction of fractions is similar to addition/subtraction of whole numbers

4. Equivalent calculations can be represented in a variety of models. (Ex. Visual fraction model, expressions, equations, number line)

ESSENTIAL QUESTIONS

1. How does equivalence help us solve problems?

2. When is it important to get a right answer and when is a close answer enough?

3. How can we use representations to perform and justify calculations?

4. How and why do we estimate?

5. How are adding and subtracting fractions similar and different than adding and subtracting whole numbers?

Acquisition

Students will know…

1. When to use equivalent fractions to find sums/differences

2. When benchmark fractions are used to make reasonable estimates of sums/differences

Students will be skilled at…

1. Finding common denominators to create equivalent fractions to find sums/differences

and critique the reasoning of others. SMP.4 Model with mathematics SMP.8 Look for an express regularity in repeated reasoning	3. Fractions can be represented in various ways in order to add/subtract 4. That, like positive whole numbers, when you add positive fractions you get a larger value, when you subtract positive fractions, you get a smaller value 5. How to use equivalent fractions and modeling to approach word problems involving addition and subtraction of fractions 6. Academic vocabulary: numerator, denominator, benchmark fraction, mixed number, equivalent fraction, sum, difference	2. Fluently adding fractions and mixed numbers 3. Fluently subtracting fractions and mixed numbers including regrouping 4. Relating different representations of addition and subtraction of fractions including language, visual, and equations 5. Estimating sums/differences using benchmark fractions 6. Communicating using appropriate notation and mathematical language (numerator, denominator, mixed number, equivalent fraction, benchmark fraction, sum, difference) 7. Solving word problems involving addition and subtraction of fractions

Stage 2 – Evidence

Evaluative Criteria	Assessment Evidence
See CEPA Rubric	Curriculum Embedded Performance Task: "Class Schedule" – The task is to assist the principal to figure out a new school schedule. The principal needs to get approval from the superintendent to make schedule changes such that that there is enough time in the 5 ½ hour day to do "everything" proposed in the schedule.
	OTHER EVIDENCE: Self Assessment: Journal Entry/ Teacher Discussion 1. How are fractions used in your everyday life? (Day 1) 2. When and why do you estimate? "When is estimation the appropriate strategy and when do you need precision?" (Day 2) 3. What do we know about subtraction of fractions? (Day 6) 4. How is addition/subtraction of whole numbers similar to addition/subtraction of fractions? (Day 4)

	5. Explain why you need common denominators and explain how you find common denominators? 6. YOU GET THE LAST WORD-one student shares their journal entry from the day before. (Day 2) (Day 7)
Anecdotal Records	Observation 1. Over or Under One (Day 2) 2. Word Problem of the Day/Week (Day 2) 3. What Common Denominator is Good To Use (Day 4) 4. What are some strategies we use to find common denominators? (Day 4) 5. Thank You For Making That Mistake (Day 5) (Day 6) 6. Find your "Equivalent Fraction", Find your "Group of 1" 7. Fraction Sums and Differences Card Game (Representation Matching) (Day 7)
Quiz & Test Results	Quiz/Test 1. Pre-assessment (Day 1) 2. Quiz on addition and subtraction of fractions, equivalent calculations, estimating fractions 3. Word Problems writing and solving 4. Exit Tickets (Day 2) (Day 3) (Day 4) Formative Assessment 1. Tell me all you know about the fraction 2/5 (Day 2) 2. Slates: Create a fact family (Day 5) 3. Thumbs up, Thumbs down (Day 5)

Stage 3 – Learning Plan

Summary of Key Learning Events and Instruction

Day One – Pre-assessment and Review

1. Pre-Assessment based on grade 4 vocabulary, mixed and improper fractions, models for fractions and for adding/subtracting with like denominators
2. Activator- The "Fraction" of Time Response Game.
3. Preview the need for substituting an equivalent fraction in order to find sums such as ½ + ¼
4. Exit Ticket- How are fractions used in your everyday life?

5. Homework- Find ways that you use fractions at home.

Day Two-Estimation to check reasonableness

6. "Word Problem of the Day" – Start with a problem where estimation is central
7. Students use mental math to estimate and add/subtract; show answers on white board that only teacher can see.
8. Journal Entry- "When and how do we estimate?"
9. Exit Slip- "If you have the fractions 5/11, what fraction(s) could you add so that the estimate is greater than 1?"

Day Three – Representing and solving addition problems in which one denominator needs to change

10. Present "Word Problem of the Day" involving addition requiring one denominator to change (link to estimation)
11. Review worked samples using Problem Solving Strategy as they think aloud (Problem solving strategy could be one already used by school/district or unit will provide an example such as **U**nderstand, **P**lan, **S**olve, **C**heck where U - draw a visual model, P - write as a solvable equation, S - calculate, and C – contextualize and estimate to assess reasonableness)
12. Optional: "Rule of Four" and show equations represented four ways (e.g., number sentence, words, number line, area model)
13. Exit Slip: Why do we need common denominators when adding fractions? What are some strategies for finding common denominators?
14. Video Clips

Day Four – Representing and solving addition problems in which both denominators need to change

15. "Word Problem of the Day" involving addition requiring both denominators to change (link to estimation)
16. Students work in small groups with addition problems and models in order to see patterns in their calculations so that they can generalize to formulate and present rule about finding common denominators - a/b+c/d = (ad+bc)/bd
17. Journal entry: *How is adding whole numbers similar to adding fractions?*
18. CEPA Part 1
19. Exit slip- What are some strategies for finding common denominators?

Day Five – Representing and solving subtraction of fractions and seeing connection to addition

20. Teacher led discussion about fact families and the relationship between addition and subtraction (inverse operations)
21. Journal Entry: *How are addition and subtraction of whole numbers similar to addition and subtraction of fractions?*
22. CEPA Part 2

Day Six – Representing and solving subtraction of fractions including regrouping

23. "Word Problem of the Day" - students create a fraction problem and write a story to match the problem
24. Journal Entry: Journal Entry: *What do we know about subtraction of fractions?*
25. CEPA Part 3

26. Exit slip- Create a subtraction problem with regrouping and describe how to solve it.
27. Video clips

Day Seven – Solidify models, reinforce concepts, and practice skills

28. Independent learning centers to provide extra support to students who need it, solidify for all, and extend for others - see resources such as National Library of Virtual Manipulatives website http://nlvm.usu.edu/en/nav/category_g_2_t_1.html and solve addition/equivalent fraction activities
    Technology Center Options: Adding fractions with unlike denominators video clip-
    http://www.mathplayground.com/howto_fractions_diffden.html
    http://www.khanacademy.org/math/arithmetic/fractions/v/adding-fractions-with-unlike-denominators?playlist

Day Eight – MP3 Project Poster

29. CEPA Final Performance Assessment

Day Nine Presentations/ Teacher & Peer Feedback/ Optional Review

Day 1	*Day 2*	*Day 3*	*Day 4*	*Day 5*
Pre Assessment & Review Activator: The "Faction" of Time Response Game.	**Using estimation to check reasonableness and solve problems** Word Problem of the Day Mental Math Activities: Two Walls and white board mental math Journal Entry- How and when do we estimate? Exit Slip- If you have the fractions 5/11, what fraction(s) could you add so that the estimate is greater than 1?	**Representing and solving addition with unlike denominators where one needs to be changed** (1/3+1/6) Present Word Problem of the Day (Represent the problem 1/3 + ½ using visual models) Review student samples using Problem Solving Strategy (**U**nderstand, **P**lan, **S**olve, **C**heck) as they think aloud Rule of 4- show an equation four ways Exit Slip on adding fractions	**Representing and solving addition with unlike denominators where two need to be changed** (1/2 + 1/7) Present Word Problem of the Day (unlike denominators) Turn & Talk- Work with addition problems and models in order to generalize how to find/use common denominators. Find a strategy or rule when one has to change both denominators in order to perform calculations Journal Entry- Why do we need common denominators to add fractions and how can we find common denominators? **CEPA Part 1 Begins**	**Representing and solving subtraction of fractions and seeing the connection to addition** Mental Math with slates to use as formative assessment related to fact families, inverse operations and subtracting fractions Small Groups- Students solve mental math fraction problems Journal Entry "How is addition/subtraction of whole numbers similar to addition/subtraction of fractions?" **CEPA Part 2**

Day 6	***Day 7***	***Day 8***	***Day 9***	
Representing and solving fraction subtraction problems with regrouping Word Problem of the Day Journal Entry Model Rule of Four **CEPA Part 3**	**Solidify models, reinforce concepts, and practice skills** Independent learning centers 1. Virtual manipulatives 2. Matching Memory Games 3. Persuasive Writing 4. Teacher Support 5. Enrichment	**CEPA Part 4** Give students time to create their poster/presentation	**CEPA Presentations** Peer/Teacher Feedback	

Lesson 1 –Fractions in the Real World

Brief Overview

In this lesson, students play the "Fraction" of Time! Response Game. This game requires students to reason abstractly and quantitatively about fractions using benchmark fractions as one of a reasonable strategy for comparing/estimating sums and differences of fractions as well as working on their "fraction sense". The set up of the game is designed so that students are constantly engaged in constructing viable arguments and critiquing the reasoning of others. (SMP.3) as they engage in the game. As you plan, consider the variability of learners in your class and make adaptations as necessary.

Estimated Time: 60 minutes

Resources for Lesson:

- Manipulatives as needed [Number line with 0, 1/2, 1 markings, Paper strips, Parallel number lines, Fraction models (circle and rectangles), Tape Diagrams, Graph paper, Geoboards]
- Computer Access
- Lesson 1 Unit Pre-Assessment
- Sample Problems for the "Fraction" of Time Response Game!
- Teacher resource for examples of models to represent fractions: *http://ime.math.arizona.edu/progressions*

Content Area/Course: Mathematics, Grade 5

Unit: Adding and Subtracting Fractions

Time (minutes): 60

Lesson 1: Fractions in the Real World

By the end of this lesson students will know and be able to:

See and use fractions in real life.

Essential Question(s) addressed in this lesson:

How can we use models to find and justify calculations?

Standard(s)/Unit Goal(s) to be addressed in this lesson (type each standard/goal exactly as written in the framework):

5.NF.1 Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with the denominators. For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12.
SMP.3 Construct viable arguments and critique the reasoning of others.

Instructional Resources/Tools (list all materials needed for this lesson):

Manipulatives as needed [Number line with 0, 1/2, 1 markings, Paper strips, Parallel number lines, Fraction models (circle and rectangles), Tape Diagrams, Graph paper, Geoboards]

Computer access

Lesson 1 Unit Pre Assessment handout

Sample Problems for "Fraction" of Time Response! Game handout (for teacher use only)

Anticipated Student Preconceptions/Misconceptions:

Denominators are subtracted from each other. Denominators do not have to be the same when adding or subtracting.

Instructional Tips/Strategies/Suggestions:

Differentiation Options:

Advanced: Ask students to create explain many ways fractions can be compared.

Remediation: (Type of remediation depends on student's needs)

- Use fraction sticks, number line, pattern blocks, etc. to model fractions.
- Compare fractions using benchmark fractions ¼, ½, and ¾.
- Fractional amounts can be represented in many ways, including the use of a tape diagram, parallel number lines, fractional pieces, number models, Cuisenaire rods, graph paper, etc.
- Use visuals /pictures with problems to accommodate ELLs as shown in Sample Problems for The "Fractions" of Time Response Game! (Lesson I Grade 5 MCU).

Pre-Assessment:

Fraction Lesson 1 Grade 5 MCU - Unit Pre-assessment handout).

What students need to know and are able to do coming into this lesson (including language needs):

Students are able to: Identify a fraction using a visual model. Identify benchmark fractions 1/3, ¼, ½, and 3/4. Add and subtract fractions with common denominators.

Lesson Sequence:

Pre-Assessment (15 to 30 minutes) See handout. *Note: Pre-assessment may be assigned as homework a few days earlier to help teacher gauge students' prior knowledge. Review pre-assessment outcomes to prepare Activator problems. Make a PowerPoint of the problems (one problem to a page).Please note that when students play the game it is through the discourse that students learn the content as well as engage in SMP.3 (Construct a viable argument and critique the reasoning of others.)*

Activator (depends on problems and time needed for game)

Teacher Talk: Today we are going to learn about fractions. We will be playing The "Fraction" of Time Response Game! Listen carefully to how the game is played.

Set up of game:

- Divide class into two teams: team A and team B.
- Team A and team B each pick a leader.
- Leader A and leader B sit by each other next to a buzzer (hand signals or call outs may be used instead of buzzer).
- Set timer (e.g. http://www.online-stopwatch.com/countdown-timer/) to 15 seconds.
- Present a fraction problem on the screen (use sample problems provided or select some from the pre-assessment depending on assessment outcomes. Make sure the problems selected lead to students' discussions /think alouds on the significance of the "whole").

- When the problem is presented leader A and leader B get a "Fraction" of Time (up to 15 seconds) to either accept (solve the problem) or not accept (pass the problem) for his/her team by hitting the buzzer (or raising hand or calling out). If neither leader responds teacher makes the call.
- The team that selects the problem, say team A gets another "Fraction" of Time (three minutes) to think aloud, discuss and agree upon the solution. The conversations have to be public and loud for the teacher and opposing team B to listen in. Team B members cannot talk amongst themselves but can take notes during this time.
- At the end of the three minutes, the timer signals time's up and the solution has to be presented by leader A. The team has to arrive at the solution by consensus.
- If the solution is correct, teacher presents a second problem and the process is repeated. If the solution is incorrect the opposing team (B) gets a "Fraction of Time" (up to two minutes since they have the advantage of listening in the first time) to discuss aloud and agree upon the correct solution. The now opposing team A listens in/takes notes but no talk is allowed. At the end of the time leader B has to present the agreed upon solution. The winning team scores points.

Turn and Talk –(depends on time remaining) Teacher: Now that you have played the game I want you to reflect on what you know about factions and think about how you use them everyday life. Turn and talk to your partner /group.

Teacher listens while circulating.

Summarizer - (10 min) Exit Ticket or Journal entry:

How are fractions used in your everyday life?

Homework - Look for fractions at home. Be prepared to share your list in class tomorrow.

OPTIONAL: Create a Questions/Concerns Poster- Allow students an opportunity to add something on a sticky note and post to the Questions/Concerns chart fractions are used in the real world and what would you like to learn about fractions. I want you to Turn & Talk to your table mates and tell them how your answered that question. (Allow time for discussion.)

SAMPLE PROBLEMS FOR "FRACTION" OF TIME RESPONSE! GAME (Lesson I Grade 5 MCU)

1. ¾ of a candy bar or 3/8 of the candy bar. Which one if you want more?

_____

2. 3/4 of 

3/8 of 

If you want more which one would you choose? Why? Give mathematical reasoning for your response.

---

3. Sarah has 3/8 of a bag of chips left. Her mom said she could put it on the shopping list only if the chips were down to ½ of a bag. Should Sarah put the bag of chips on the shopping list?

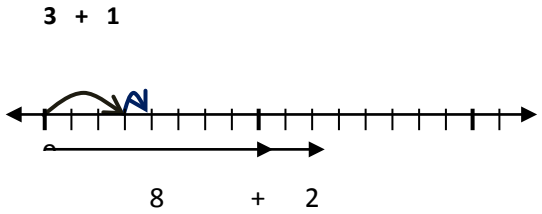
4. A recipe for a mystery punch calls for 1/5 of a cup of pineapple juice and 2/3 of a cup of mango juice. How many cups of punch will the recipe make?

5. David and Stan are brothers. During snack time their mom gave David 2/5 of the cake and Stan 6/15 of the same cake. David and Stan began arguing. Stan said he got the smaller piece. David said he got the smaller piece. Who is right?

6. Dave is having a friend over for pizza. He wants to make sure he orders enough food. Jeff said he can eat 1/3 of the large pizza that is planning to order. Dave usually eats about ¼ of the same large pizza. Will one pizza be enough for them to share?

7. The school band practiced 2 ¾ hours on Saturday and 3 2/3 hours on Sunday. Was the band's total more or less than 6 hours.

8. 3/8 + ½ = 4/10 or 2/5. Is this correct? If no give the correct answer and model!



Fraction Lesson 1 Grade 5 MCU - Unit Pre-Assessment

(May be assigned as homework the previous day)

Name: _____ Date: _____

1. What is a fraction?

_____

_____

_____

2. Explain why fractions are used.

_____

_____

_____

3. Write an equivalent fraction for ¼? _____

Use pictures, models or words to explain how you found the equivalent fraction in problem 3.

_____

_____

_____

4. Compare the following fraction. Use <, >, =.

¼ _____3/4 2/3 _____ 2/6 1/6 _____2/4

½ _____ 2/4

5. Is 2/5 greater than or less than ½? Explain your reasoning. _____

6. What is a common denominator for ¼ and 2/3? _____

7. Using words, pictures or models explain how you found the common denominator in problem 6.

_____

_____

_____

Problem: **Visual Representation:**

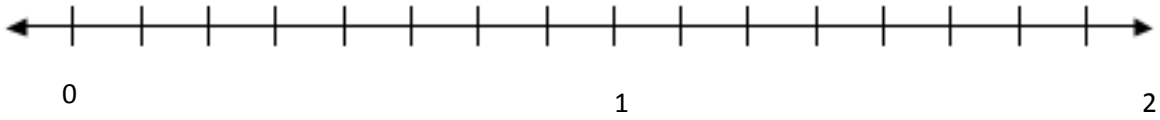
8. 5/8 + 1/8 = _____

9. 1- 2/5= _____

10. ¾ + 1 ½ = _____

11. 3 ¼ - 1 ¾ = _____

12. Plot the following fractions on the number line: 2/3, ¾, 7/8, ½, 1/8



13. Explain how to compare fractions. Use numbers, words, and visual representations in your explanation.

_____

_____

_____

_____

Lesson 2 –Using Benchmark Fractions as a Strategy for Estimating

Brief Overview

In this lesson, students will use visual fraction models to represent problems and learn to use benchmark fractions as a reasonable strategy for estimating sums and differences of fractions. As you plan, consider the variability of learners in your class and make adaptations as necessary.

Estimated Time: 60 minutes

Resources for Lesson:

- Number lines labeled with endpoints 0 and 1, and a midpoint of 1/2
- Manipulatives as needed [Number line with 0, 1/2, 1 markings, Paper strips, Parallel number lines, Fraction models (circle and rectangles), Tape Diagrams, Graph paper, Geoboards]
- Access to the Internet
- *http://ime.math.arizona.edu/progressions (Teacher resource for visual representations of fractions* - Draft 3–5 progression on Number and Operations—Fractions)

Content Area/Course: Mathematics, Grade 5

Unit: Adding and Subtracting Fractions

Time (minutes): 60

Lesson #2: Using Benchmark Fractions as a Strategy for Estimating

By the end of this lesson students will know and be able to:

Use stated benchmarks such as (0, 1/2, 1) to make reasonable estimates of sums and differences of fractions.

Use estimation as a means of determining the reasonableness of sums and differences.

Describe strategies for estimating the sums and differences of fractions.

Essential Question(s) addressed in this lesson:

How and why do we estimate?

Standard(s)/Unit Goal(s) to be addressed in this lesson (type each standard/goal exactly as written in the framework):

5.NF.2 Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g. by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. For example, recognize an incorrect result 2/5 + ½ = 3/7, by observing that 3/7 < ½.

SMP.2 Reason abstractly and quantitatively.

SMP.3 Construct viable arguments and critique the reasoning of others.

Instructional Resources/Tools (list all materials needed for this lesson):

Use of number lines labeled with endpoints 0 and 1, and a midpoint of 1/2

Fractional amounts can be represented in many ways, including the use of a tape diagram, parallel number lines, fractional pieces, number models, Cuisenaire rods, graph paper, etc.

Anticipated Student Preconceptions/Misconceptions:

- Inability to mentally relate the size of fraction to 0, 1/2, or 1 whole or more.
- Converting fractions to equivalents with common denominators rather than reasoning mentally with benchmarks.
- Inexperience with visualizing fractions.

Instructional Tips/Strategies/Suggestions:

Fractional amounts can be represented in many ways, including the use of a tape diagram, parallel number lines, fractional pieces, number models, Cuisenaire rods, graph paper, etc.

Pre-Assessment:

Tell me all you know about the fraction 2/5.

Students may need to use number line as reference.

Stacked number lines could be used to illustrate generalizations. Stack lines that show 0-1 whole; 0 1/2, 2/2;

0, 1/4, 2/4, 3/4, 4/4; 0, 1/6, 2/6, 3/6, 4/6, 5/6, 6/6; etc.

By drawing /highlighting a vertical line along the one half markings and one whole students may find it easier to discern the patterns suggested in the Wrap Up.

What students need to know and are able to do coming into this lesson (including language needs):

Students have created equivalent fractions using models, manipulatives, and calculations in earlier grades. Students know the benchmark fractions and are able to use them in comparing the size of fractions. Students can add and subtract fractions with like denominators mentally and this helps in developing reasonable estimates before using standard algorithm.

Teacher Notes:

VOCABULARY:

Estimation, Denominator, Numerator, Equivalent fractions

Benchmark fractions

SUGGESTED MANIPULATIVES & MODELS:

Number line with 0, 1/2, 1 markings

Paper strips

Parallel number lines

Fraction models (circle and rectangles)

Tape Diagrams

Graph paper

Geoboards

ESTIMATION:

Strengthening estimation skills can help develop a student's understanding of computation with fractions.

Estimation is most useful in situations when the solution cannot be computed quickly or easily.

Real life situations frequently require estimation, not precise calculations.

BENCHMARK:

Standard values, like 1/2 and 1, can be used to compare fractions and estimate sums and differences.

Lesson Sequence:

The overall goal of this lesson is to model the use of benchmarks as a reasonable strategy for estimating sums and differences as well as work on "fraction sense". Students often learn the procedures for adding and subtracting fractions, but are unable to reason about or make sense of fractions.

Journal Entry- When and why do you estimate?

Introduction – 15 MINUTES

Teacher Talk:

Students may have had some experiences using benchmarks to compare fractions.

Beginning with pre-assessment 2/5, have students share what they know about the example. Students' responses are likely to include a picture representation showing 2/5 of an area or set. Some may describe how they pictured it in their mind. Others may describe the fraction in relation to 1/2. Record responses so all students can "see" the possibilities.

Note: The pre-assessment should lead into discussion of how benchmarks like 1/2 can help in estimation.

Guided Practice – 30 MINUTES

Present this problem to students:

Several large, same-size pizzas were ordered for a party. Three children each ate a different kind of pizza. Jane had 1/3 of a pepperoni pizza, Jim ate 4/8 of a cheese pizza, and John finished 3/5 of the veggie pizza. Who had the most pizza? Explain how you know.

Teacher Talk:

In addressing the latter example, probe students to find out how they arrived at their solutions. Common responses will likely include visual representations and using common denominators. If no one offers the use of benchmark strategies, the teacher should model the thinking out aloud. All strategies should be listed as a reference for students.

Pay close attention to the abstract and quantitative reasoning in students' thinking. Are they relating the problem to a physical or mental model? Do they consider the relative size of the fraction in their solutions? For example, do they say that 2/5 is close to, but less than, one-half? Do they solve the pizza problem by describing the fraction of pizza consumed by comparing the amount to 1/2? (SMP.2: Reason abstractly and quantitatively.) Call on some students to explain their thinking. Call on others to analyze this thinking. (SMP.3: Construct viable arguments and critique the reasoning.)

Provide number lines for students with markings but without labels. Lead students to create a number line for reference by labeling 0, 1/2, 1, to begin. Lines should be posted as well for demonstration purposes. Additional number lines may be needed later in the lesson.

Have students turn and talk to their shoulder partner about the fraction 1/2 and how they might compare a fraction like 3/7 to that benchmark WITHOUT finding a common denominator. What are some characteristics of 3/7 that would make it possible to determine if it was larger or smaller than 1/2.

Provide some real life examples of situations that indicate "about half", such as 12/25 students ride the bus to and from school, or about half the class, 8/15 boys are wearing sneakers, etc.

Pose the question 3/8 + 4/9 and have students estimate if it is more or less than one.

Again, ask for an explanation, reinforcing the solutions that use the benchmark 1/2 to mentally solve.

Additional examples: 1/2 + 1/3

Contrast with an estimate of 12/13 + 7/8. Have students model how this is different from previous examples, yet uses another benchmark of one whole. In this case students should realize that both fractions are close to a whole, so the sum has to be greater than 1.

Next student should use estimates to explain how one would know the answer is reasonable.

OPTIONAL – SIMILAR EXAMPLES

A running track is one kilometer long. If a runner jogs for 1/6 of a kilometer and runs 2/3 of a km will she/he complete the distance around the track?

Estimate: Strategy:

The ribbon is 15/12 of a yard long. If 6/7 of the ribbon is used for a package, about how much will be left?

Estimate: Strategy:

Over or Under One *(*adapted from *Elementary and Middle School Mathematics: Teaching Developmentally, 8th Edition).*

Explain to students they are to estimate sums and difference of fractions. They are to decide if the answers are **over** or **under** one for each problem as it is displayed. Students can write on slates and display or use "thumbs up'' to indicate over and "thumbs down" for under.

On electronic Whiteboard or overhead display examples of addition and subtraction problems for about 10 seconds, doing several in a row. Then return to each problem and discuss the estimates

Possible estimation examples: 11/12 – 3/4 2/3 + 1/7 4/3 – 3/5 3/2 + 1/4

5/9 + 1/8 + 1/7 5/3 – 1/4

During discussion have students refer to number line, fraction circles or other similar area models, or give real life examples in justifying their answer.

Wrap Up: 15 minutes

Guide students to generalize about the fractions they think are closer to 0, those closer to 1/2, and those closer to 1. They should be making statements that describe the relationship between the numerator and denominator. For example, fractions closer to 1/2 have numerators that are about one half of the denominator. Fractions closer to 1 have numerators and denominators that are closer together.

* Adapted from NCTM Focus in Grade 5: *Teaching with Curriculum Focal Points*. 2009

Reys, Barbara J., Kim, Ok-Kyeong, and Bay, Jennifer. 1999. "Establishing Fraction Benchmarks." *Mathematics Teaching in Middle School,* 4 (May) 530-532.

Van de Walle, John A., Karen S. Karp; Jennifer M. Bay-Williams; with contributions by Jonathan Wray. *Elementary and Middle School Mathematics: Teaching Developmentally*. Eighth edition. 2013 pp 316-319.

Van de Walle, John A. and LouAnn H. Lovin. *Teaching Student-Centered Mathematics.* Grades 5-8. 2006 pp 74– 80.

Summative Assessment:

Exit Slip: If you have the fraction 5/11, what fraction(s) could you use so that the estimate was greater than 1?

Lesson 3 – Representing and Adding Fractions with Unlike Denominators, Convert One Denominator

Brief Overview: Students have had experience with adding fractions with like denominators in the previous grades. In this lesson, students will practice adding fractions with like denominators and learn how to add fractions with unlike denominators by first making them into equivalent fractions. In this lesson one denominator is manipulated. As you plan, consider the variability of learners in your class and make adaptations as necessary.

Estimated Time: 60 minutes

Resources for Lesson: Manipulatives as needed (Number line with 0, 1/2, 1 markings, Paper strips, Parallel number lines, Fraction models (circle and rectangles), Tape Diagrams, Graph paper, Geoboards)

- Internet access
- Rule of Four Template

Content Area/Course: Mathematics, Grade 5

Unit: Adding and Subtracting Fractions

Time (minutes): 60

Lesson #3: Representing and Adding Fractions with Unlike Denominators, Convert One Denominator

By the end of this lesson students will know and be able to:

Add fractions with like and unlike denominators accurately.

Represent fractions with equivalent fractions to aid in solving addition problems.

Explain when, why, and how to create equivalent fractions to successfully add fractions

Essential Question(s) addressed in this lesson:

How does equivalence help us solve problems?

Standard(s)/Unit Goal(s) to be addressed in this lesson (type each standard/goal exactly as written in the framework):

5.NF.1 Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with the denominators. For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12.

SMP.2: Reason abstractly and quantitatively.

SMP.4 Model with mathematics

Instructional Resources/Tools (list all materials needed for this lesson):

Manipulatives (Number line with 0, 1/2, 1 markings, Paper strips, Parallel number lines, Fraction models (circle and rectangles), Tape Diagrams, Graph paper, Geoboards)

Internet access.

Anticipated Student Preconceptions/Misconceptions:

Ignoring the need for common denominators in using the standard algorithm for calculations

Inability to identify a reasonable common denominator, given two (or more) denominators.

Ability to find an equivalent denominator, but neglecting to change the numerator by the same amount.

Inaccurate representations or calculations.

Inability to create a model that shows a common whole that can be divided to create equivalent fractions.

Instructional Tips/Strategies/Suggestions:

Fractional amounts can be represented in many ways, including the use of a tape diagram, parallel number lines, fractional pieces, number models, Cuisenaire rods, graph paper, etc.

What students need to know and are able to do coming into this lesson (including language needs):

Students have created equivalent fractions using models, manipulatives, and calculations in earlier grades and current unit. Students are able to use and understand how benchmark fractions can assist in developing reasonable estimates before using standard algorithm.

Information for Teacher:

The representative whole must be the same size, no matter what denominator is used in the calculations.

Both the numerator and denominator must be increased or decreased by the same amount when equivalent fractions are generated or identified.

VOCABULARY: Least common multiple, Denominator, Numerator, Equivalent fractions, Benchmark fractions

Lesson Sequence[+]:

Activator:

Teacher Talk - Let's discuss how we can solve word problems. (Guide students in discovering the problem solving strategy of *Understand, Plan, Solve, (Try), & Check* . For an explanation of the model visit: http://library.thinkquest.org/25459/learning/problem/

Note: The modeling of this strategy makes explicit the decontextualization and contextualization of the problem to students. SMP.2 (Reason abstractly and quantitatively).

Teacher Models: "On Saturday, 3/4 of a fifth grade class went to see a new movie. If ½ of the class went to the afternoon show, what fraction of the class went to the evening show?"

Teacher Talk: First, let's read the problem and understand what it is asking. During this part of the problem solving strategy, you should underline key words. I would also like you to circle what the question is asking you to do. (Allow time for students to reread text, and then call on students to share what words should be underlined and what question should be circled.)

Next, we need to PLAN how to solve the problem. We have already worked on adding fractions with like denominators. Raise your hand if you can explain how we do this. (See optional video clips to use during discussion.)

Video Clip for adding fractions with like denominators:

http://www.khanacademy.org/math/arithmetic/fractions/v/adding-fractions-with-like-denominators

How can we add fractions that have unlike denominators? (Allow Discussion)

Video Clip for adding fractions with unlike denominators:

http://www.mathplayground.com/howto_fractions_diffden.html

http://www.khanacademy.org/math/arithmetic/fractions/v/adding-fractions-with-unlike-denominators

Let's look at how we should solve this problem. (Discuss how to use multiples to find common denominators. Students should discover that only one denominator needs to be changed in this problem. Allow time for students to SOLVE the problem and then CHECK for accuracy.)

Groups solve with support from teacher: "At a class party 4/8 of a vegetarian pizza and ¼ of a cheese pizza were eaten. How much pizza was eaten all together?

Groups should be able to describe patterns/ strategies for finding common denominators for each group. They should also be able to offer other examples that would fit each category.

Optional: Model the Rule of 4 Poster*- Use area models, number sentence, pictures, words, paper strips, number lines, etc. to prove answer through representation. Create a poster that includes four ways to represent the problem.

[+]Adapted from NCTM Focus in Grade 5: Teaching with Curriculum Focal Points 2009

Review outcomes of the lesson:

Students should be able to describe and use different methods or strategies to find common denominators necessary to calculate sums of fractions.

Preview outcomes for the next lesson:

Extending the use of common denominators in subtraction of fractions and with mixed numbers.

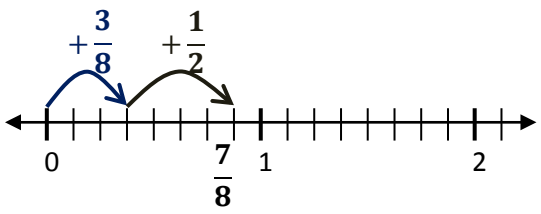
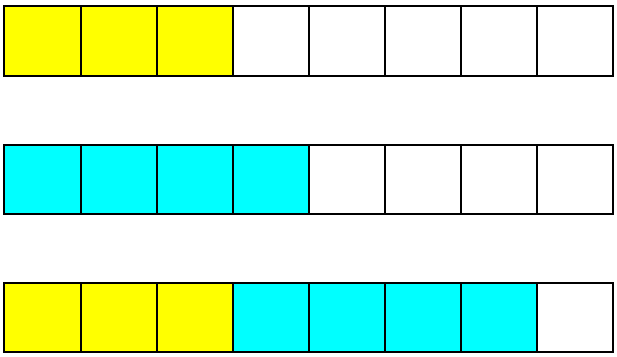
Summative Assessment:

EXIT SLIP: Why do we need common denominators to add fractions?

Lesson 3 Attachment 1: ***Representing and solving addition problems with unlike denominators where one fraction needs to be changed.***

Here students examine a real- world context and use diagrams or models to map the relationships between the fractions to draw conclusions. (SMP.4: Model with mathematics).

P**aul is making a dog house for his cub scouting project. His group plans to donate dog houses to a local animal shelter. He takes a piece of wood that is 3/8 of an inch thick and glues it to a piece of wood that is 1/2 of an inch thick. Together, what is the thickness of the two pieces of wood?**

3/8 + ½ =  	3/8 + ½ = 
Quinn demonstrates his ability to correctly show the problem using a number line.	Ryan attempts to draw a picture that represents the fractions being added to each other.
3/8 + ½ = 3/8 + 4/8= 7/8 Sabastine correctly writes the number model.	3/8 + ½ = The first thing I needed to do was to find equivalent fractions so that I could have both fractions have the same denominator. I know the multiples of 2 are 2, 4, 6, and 8. I then made an equivalent fraction for ½ which would be 4/8. Now that both fractions had the same denominator, I added the fractions together, 3/8+4/8=7/8. So the answer to this question is 7/8.

Lesson 3 Attachment 2: Representing and solving addition problems with unlike denominators where one fraction needs to be changed using the **Rule of Four Poster**.

Problem:

_____

_____

Directions: Show your solutions in multiple ways.

Lesson 4 – Representing and Adding Fractions with Unlike Denominators, Convert Both Denominators

Brief Overview: Students will practice adding fractions with like denominators and learn how to add fractions with unlike denominators by first making them into equivalent fractions. In this lesson both denominators are manipulated. As you plan, consider the variability of learners in your class and make adaptations as necessary.

Estimated Time: 60 minutes

Resources for Lesson:

- Manipulatives as needed (Number line with 0, 1/2, 1 markings, Paper strips, Parallel number lines, Fraction models (circle and rectangles), Tape Diagrams, Graph paper, Garboards)
- CEPA Part 1

Content Area/Course: Mathematics, Grade 5

Unit: Adding and Subtracting Fractions

Time (minutes): 60

Lesson #4: Representing and Adding Fractions with Unlike Denominators, Convert Both Denominators

By the end of this lesson students will know and be able to:

Add fractions with like and unlike denominators accurately.

Represent fractions with equivalent fractions to aid in solving addition problems.

Explain when, why, and how to create equivalent fractions to successfully add fractions.

Essential Question(s) addressed in this lesson:

How does equivalence help us solve problems?

Standard(s)/Unit Goal(s) to be addressed in this lesson (type each standard/goal exactly as written in the framework):

5.NF.1 Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with the denominators. For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12.

SMP.4 Model with mathematics

SMP.8 Look for an express regularity in repeated reasoning

Instructional Resources/Tools (list all materials needed for this lesson):

Manipulatives Number line with 0, 1/2, 1 markings, Paper strips, Parallel number lines, Fraction models (circle and rectangles), Tape Diagrams, Graph paper, Geoboards)

CEPA Part 1

Anticipated Student Preconceptions/Misconceptions:

Ignoring the need for common denominators in using the standard algorithm for calculations

Inability to identify a reasonable common denominator, given two (or more) denominators.

Ability to find an equivalent denominator, but neglecting to change the numerator by the same amount.

Inaccurate representations or calculations.

Inability to create a model that shows a common whole that can be divided to create equivalent fractions.

Instructional Tips/Strategies/Suggestions:

Fractional amounts can be represented in many ways, including the use of a tape diagram, parallel number lines, fractional pieces, number models, Cuisenaire rods, etc. ***The representative whole must be the same size, no matter what denominator is used in the calculations.*** Both the numerator and denominator must be increased or decreased by the same amount when equivalent fractions are generated or identified.

Pre-Assessment:

Problem of the Day, 4/8 + 1/4

Changing only one denominator in developing equivalent fractions for calculations

Journal Entry- How is adding whole numbers similar to adding fractions.

What students need to know and are able to do coming into this lesson (including language needs):

Students have created equivalent fractions using models, manipulatives, and calculations in earlier grades and current unit. Students are able to use

and understand how benchmark fractions can assist in developing reasonable estimates before using standard algorithm.

Information for Teacher:

VOCABULARY:

Least common multiple

Denominator

Numerator

Equivalent fractions

Benchmark fractions

SUGGESTED MANIPULATIVES & MODELS:

Number line with 0, 1/2, 1 markings

Paper strips

Parallel number lines

Fraction models (circle and rectangles)

Tape Diagrams

Graph paper

Geoboards

SUPPORTING FRACTION OPERATION SENSE:

Using visual models, like fraction strips, and number lines should support the student's ability to visualize fractional amounts. (SMP.4: Model with mathematics.) However, it is also important for students to routinely think about fractions as quantities and how the fractions are related to one another and to 1 whole. Benchmark fractions such as 0, 1/2, or 1 should be used to estimate reasonableness of sums and difference.

ESTIMATION:

Using benchmark fractions of 1/2, 1 whole, or more than 1 whole may provide insight into student thinking. For example, given the algorithm 1/2 + 1/3, students may estimate answer closer to 1 whole. When the student calculates the sum given is 1/2+1/3 = 2/5 – a clear mismatch from estimate that indicates misunderstanding related to the denominator.

Lesson Sequence:

This lesson continues to build on the use of estimation and benchmark fractions in completing addition of fractions. Picking up from Lesson 3, the lesson should extends the use of common denominators and the process of changing both denominators in completing addition of fractions with unlike denominators

Introduction: 10 MINUTES

Teacher Talk: Yesterday we were working on adding fractions. In our pizza example, we added 4/8 + ¼. Who can explain how to add these two fractions together? (Try to guide discussion so that estimation is discussed.)

Review of Day 3 problem: 4/8 + 1/4 – students describe steps necessary to successfully complete calculation with standard algorithm, INCLUDING the use of estimation using benchmark fraction to assess reasonableness of answer

Have students suggest similar examples, (i.e., 3/4 + 1/12; 2/8 + 1/4)

Guided Practice: 20 MINUTES

Teacher Talk: Let's try another problem. Introduce new scenario: Amelia and Asante are working on making patchwork quilts. Amelia finished making ½ of a quilt and Asanti finished making 2/3 of a quilt. How many quilts got made?

Estimate answer using benchmark fractions

students consider how to solve – talk with shoulder partner

Use area models, paper strips, parallel number lines, etc. to prove answer through representation.

Students should also record a number sentence that matches the representation.

Teacher Talk: How can these two fractions be added? (Discuss using multiples to find a common denominator.)

Questions to consider:

- Were estimates reasonable?
- Do the equations accurately match the representations?

Provide each small group with fraction addition problems that have common patterns. See Lesson 4: What Common Denominator is good to use?

Groups should be able to describe patterns/ strategies for finding common denominators for each group. In one set of fractions one denominator is a multiple of the other so it is necessary to only change one denominator, for example 3/16 +3/8. In the other set, a common denominator can be determined by multiply both denominators together, like 6/4 + 1/3. Students should also be able to offer other examples that would fit each category. This exercise is linked to SMP.8 (Look for and express regularity in repeated reasoning.).

Wrap-Up: 10 MINUTES

What are some strategies we use to find common denominators?

Introduce Curriculum Embedded Performance Assessment (CEPA): 20 minutes

This task is going to be addressed over the course of lessons 4, 5, and 6, with a final product being developed at the end of the unit. (See CEPA for full sequence).

Teacher Talk: Your task is to assist the principal to figure out a new school schedule. The principal needs to get approval from the superintendent to make these changes and to prove that there is enough time in the 5 ½ hour day to do everything.

CEPA Part 1:

The principal has to allow 2/3 hour for lunch (which includes recess). She also wants to have 1/6 an hour for morning meeting.

- Estimate if the time for lunch, recess and morning meeting is more than, less than, or equal to one hour. Explain why you think your answer is reasonable.
- Share your answer and estimate with a partner. (SMP.3: Construct viable arguments and critique the reasoning of others.) Calculate the actual amount of time for these two activities. Was your estimate correct? Why or Why not?

Review outcomes of the lesson:

Students should be able to describe and use different methods or strategies to find common denominators necessary to calculate sums of fractions.

Preview outcomes for the next lesson:

Extending the use of common denominators in subtraction of fractions and with mixed numbers

Summative Assessment:

EXIT SLIP: What are some strategies for finding common denominators?

Lesson 4: What Common Denominator Is Good to Use?

Solve the problems. How did you decide on what common denominator to use?

3/16 + 3/8 = **4/10 + 1/5 =** **2/3 + 3/12 =**

6/4 + 1/3 = **3/7 + 1/9 =** **2/4 + 3/5 =**

Write your "rule" or strategy for finding common denominators before solving the equations.

Rule of Four* Template (Used in Lesson 3, 4, 5)

Problem:

_____

Directions: Show your solutions in multiple ways.

Lesson 5 – Representing and Solving Fraction Subtraction Problems

Brief Overview: In this lesson, students will practice subtracting fractions with like denominators and learn how to subtract fractions with unlike denominators by first making them into equivalent fractions.

As you plan, consider the variability of learners in your class and make adaptations as necessary.

Estimated Time: 60 minutes

Resources for Lesson:

- Manipulatives as needed [Number line with 0, 1/2, 1 markings, Paper strips, Parallel number lines, Fraction models (circle and rectangles), Tape Diagrams, Graph paper, Geoboards]
- Slates or white boards
- Index cards
- Internet Access
- CEPA Part 2
- Rule of Four Template

Content Area/Course: Mathematics, Grade 5

Unit: Adding and Subtracting Fractions

Time (minutes): 60

Lesson #5: Representing and Solving Fraction Subtraction Problems

By the end of this lesson students will know and be able to:

Represent and solve fraction subtraction problems and understand the connection between the subtraction and addition of fractions

Essential Question(s) addressed in this lesson:

How can we use models to find and justify calculations?

Standard(s)/Unit Goal(s) to be addressed in this lesson (type each standard/goal exactly as written in the framework):

5.NF.1 Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with the denominators. For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12.

SMP.4 Model with mathematics

SMP.8 Look for an express regularity in repeated reasoning

Instructional Resources/Tools (list all materials needed for this lesson):

Manipulatives [Number line with 0, 1/2, 1 markings, Paper strips, Parallel number lines, Fraction models (circle and rectangles), Tape Diagrams, Graph paper, Geoboards]

Slates or white boards

Index cards

Internet access

Anticipated Student Preconceptions/Misconceptions:

Denominators are subtracted from each other.

Denominators do not have to be the same when adding or subtracting

Instructional Tips/Strategies/Suggestions:

Differentiation: Ask students to create subtraction problems. Give them the difference and ask them to come up with multiple ways of getting that particular answer. Challenge them to use mixed numbers and different denominators for their number sentences

For instance: Find 5 ways to get 5/6 as the difference of a subtraction problem.

Remediation: (Type of remediation depends on student's needs)

Pre-Assessment:

Ask students to create and addition and subtraction fact family of their own. Circulate the room and check student work

What students need to know and are able to do coming into this lesson (including language needs):

Students are able to:

Find common denominators

Create equivalent fractions

Solve fraction addition problems with both like and unlike denominators.

Students may treat numerators and denominators as different "numbers" in calculating. For example, given the problem 2/3 + 3/4 the student might

add both numerators and add both denominators and provide the solution 2/3 + 3/4 = 5/7.

Information for Teacher:

SUGGESTED MANIPULATIVES & MODELS:

Number line with 0, 1/2, 1 markings

Paper strips

Parallel number lines

Fraction models (circle and rectangles)

Tape Diagrams

Graph paper

Geoboards

Lesson Sequence:

CEPA Part 2 (15 min.)

The principal wants to extend morning meeting to include current events and exercise in a period that is 3/2 hour. Lunch and recess still uses 2/3 of an hour.

Estimate if the time used is more than, less than, or equal to two hours.

Explain why you think your answer is reasonable. (SMP.3: Construct viable arguments and critique the reasoning of others.)

Share your answer and estimate with a partner. Calculate the actual amount of time used by these two periods. Was your estimate correct? Why or Why not?

When students have the solution ask students to make a fact family with the number sentence. As students come up with the related number sentences, write them on the board. You may want to make addition examples if necessary.

Introduction: (15 minutes)

Formative Assessment (Slates)

Ask students to create an addition and subtraction fact family of their own. Circulate the room to check student work.

At the beginning of the lesson ask students:

What is the opposite operation to addition?

What is different between addition and subtraction?

Ask them if they know how to create a fact family.

What makes four number sentences a fact family?

Teacher Talk: All of the preceding questions serve to activate students' prior understanding of the relationship between addition and subtraction. If students have difficulty remembering what a fact family is, put some simple examples on the board.

Ex: 3+2=5

2+3=5

5-2=3

5-3=2

The Lesson – Ask students (10 minutes)

How do you subtract whole numbers?

How do you subtract fractions? (*Keep in mind, fractions ARE numbers.)*

Do some mental math fraction subtraction problems

Provide students with materials to represent and solve the problem:

Rule of Four Template, Number Line, Area Model, Equation, and Fact Family

Sample fact families to put on index cards:

5/6 – 1/6 = 4/6 ⅝–⅛= 4/8 2/4 – ¼ = ¼

¾–¼=2/4

Formative assessment: Check for understanding (Thumbs up, Thumbs down)

Teacher Talk:

What else can we say about these fraction problems? Look for students to realize all of the problems had common denominators and noticing that fractions can be arranged in fact families. The relationship between addition and subtraction holds true with fractions. Students should also notice the similarity to whole number addition and subtraction. (SMP.8:Look for and express regularity in repeated reasoning.) Other misconceptions may arise that may need to be clarified.

How do you feel right now with subtracting fractions? Teacher looks for students that give thumbs down. These students may need additional small group instruction

Turn and Talk – (15 min)

Put a subtraction problem on the board with unlike denominators (½–⅓=). *(You may want to point out the unlike denominators to students.)*

Ask students to talk to a peer and come up with a plan on how to solve the subtraction problem. As students discuss circulate among students and ask focusing/scaffolding questions if needed.

Deliberate Practice (15 min)

Have a whole class discussion on how to subtract fractions with unlike denominators. Also, solve some subtraction problems as a whole group; be sure to include mixed numbers. (Discover that what you do with adding fractions is the same with subtracting fractions.)

Optional: *See Rule of 4 handout. Teacher will omit one of the sections and have the students construct the missing piece.*

3/5–½= 3 ¾– 2 1/12 = 9/6–⅔=

Deliberate practice closure: Thank you for making that mistake!

During practice problems make note of common mistakes made by students or difficult problems. Review them with the class.

Review strategies used to find common denominators

Review the meaning of subtraction

Provide manipulatives for visual students

Break down the process of solving fractions

Have students write or illustrate a "cheat sheet" on how to solve a fraction subtraction problem.

Homework:

Have students choose one of the four fraction problems used in class and create a word problem that would correctly represent the number model.

Summative Assessment:

Exit Ticket or Journal entry: How is addition and subtraction of whole numbers similar to addition and subtraction of fractions? How are they different

Lesson 6 – Representing and Solving Fraction Subtraction Problems with Regrouping

Brief Overview: In this lesson, students will continue to learn how to subtract fractions (including mixed numbers) with unlike denominators by regrouping and converting into equivalent fractions. As you plan, consider the variability of learners in your class and make adaptations as necessary.

Estimated Time: 60 minutes

Resources for Lesson:

- Manipulatives as needed [Number line with 0, 1/2, 1 markings, Paper strips, Parallel number lines, Fraction models (circle and rectangles), Tape Diagrams, Graph paper, Geoboards]
- Internet access & SMART Board (optional)
- CEPA Part 3

Content Area/Course: Mathematics, Grade 5

Unit: Adding and Subtracting Fractions

Time (minutes): 60

Lesson #6: Representing and Solving Fraction Subtraction Problems with Regrouping

By the end of this lesson students will know and be able to:

Represent and solve fraction subtraction problems with regrouping

Essential Question(s) addressed in this lesson:

How can we use models to find and justify calculations?

Standard(s)/Unit Goal(s) to be addressed in this lesson (type each standard/goal exactly as written in the framework):

5.NF.1 Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with the denominators. For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12.

SMP.3 Construct viable arguments and critique the reasoning of others.

Instructional Resources/Tools (list all materials needed for this lesson):

Manipulatives as needed.

SMART Board Lessons
http://exchange.smarttech.com/search.html?q=%22fractions%22

Birmingham Grid for Learning
http://www.bgfl.org/bgfl/15.cfm?s=15&p=250,index

PBS Teacher
http://www.pbs.org/teachers/search/resources/?q=fractions&x=0&y=0

Teacher Domain
http://www.teachersdomain.org/search/?q=fractions&fq_grade=PK&fq_grade=PS

Anticipated Student Preconceptions/Misconceptions:

Denominators are subtracted from each other.

Denominators do not have to be the same when adding or subtracting

Instructional Tips/Strategies/Suggestions:

Advanced Students: Ask students to create subtraction problems with regrouping. Give them the difference and ask them to come up with multiple ways of getting that particular answer.

For instance: Find 5 ways to get 5/6 as the difference of a subtraction problem.

Remediation: (Type of remediation depends on student's needs)

1. Review how to find common denominators
2. Provide manipulatives for visual students
3. Break down the process of solving fractions

Have students write or illustrate a "cheat sheet" on how to solve a fraction subtraction problem

Pre-Assessment:

Journal: What do we know about subtraction of fractions with like and unlike denominators?

Provide students with deliberate practice problems and provide immediate feedback to students by monitoring their practice. (See attached handout) Students may use manipulatives with their practice problems.

What students need to know and are able to do coming into this lesson (including language needs):

Students are able to: Find common denominators. Create equivalent fractions. Turn mixed numbers into fractions. Solve fraction addition problems with both like and unlike denominators. Solve subtraction problems without regrouping.

Lesson Sequence:

Activator - Journal entry: (10 minutes)

What do we know about subtraction of fractions with unlike and like denominators?

After Journal, have students share their observations about fraction subtraction, record observations on the white board

Formative Assessment

By the student responses to the journal prompt, decide if students need a refresher about subtraction or if students are prepared for new information

The Lesson – (10 Minutes) You may choose to provide students with fraction manipulatives prior to this lesson or during the new instruction. Present problem: Ami had 5 ½ of a pizza. Sandy had 3 6/7 of the same pizza. How much pizza was left over?

Turn and Talk (5 minutes): *This strategy supports SMP.3 (Construct viable arguments and critique the reasoning of others.)* Ask students to talk to a peer and come up with a plan on how to solve the subtraction problem. As students discuss, circulate among students and ask focusing/scaffolding questions. Show http://www.youtube.com/watch?v=7N5tpnRyV5o as one possible method.

After, have a whole group discussion about how the problems can be solved when regrouping is necessary. Have students who have successfully developed a procedure for solving the regrouping problem to "teach" the class what they did at the board or with their manipulatives.

Sample problems:

6 5/10 - 3 8/10 =

11- 6 ¼ =

9 2/5 – 4 7/10 =

7 1/3 – 4 ¾ =

Solve the problems as a class. After, list the steps for solving in the board

Formative assessment: Check for understanding (Thumbs up, Thumbs down)

Teacher Talk: Show the following videos on subtracting mixed numbers with regrouping. After the videos, ask probing questions concerning understanding.

Video subtracting mixed numbers with regrouping - http://www.youtube.com/watch?v=4LYtLMmDuzw&feature=relmfu -

Video subtracting mixed numbers with regrouping word problem - http://www.youtube.com/watch?v=tj9oLZYA-AQ&feature=relmfu

Formative Assessment –Deliberate practice (20 min)

Provide students with deliberate practice problems and provide immediate feedback to students by monitoring their practice. Students may use manipulatives with their practice problems.

Sample: 8 2/5 – 3 4/5 =

Method 1: Regroup 1 as 5/5

7 7/5 – 3 4/5 = 4 3/5

Method 2: Subtract by adding on to find unknown addend

3 4/5 + 1/5 to equal 4 + 4 to equal 8, plus 2/5 to equal 8 2/5

3 4/5 + 1/5 to equal 4 + 4 to equal 8, plus 2/5 to equal 8 2/5

**Note: Method 2 can be thought of in terms of jumps on a number line.

Deliberate practice closure: Thank you for making that mistake! (10 min)

During practice problems make note of common mistakes made by students or difficult problems. Review them with the class, while thanking students for making mistakes, so learning can happen.

CEPA Part 3

After checking her calculations, 2 1/6 hours are to be used for the lunch/recess period and the extended morning meeting. With a 5 ½ hour day, how much time is left for Math, ELA, Science and Social Studies?

- Estimate if the time left is more than, less than, or equal to three hours? Explain why you think your answer is reasonable.
- Share your answer and estimate with a partner.
- Calculate the actual amount of time left. Was your estimate correct? Why or Why not?

Summative Assessment: Exit Ticket or Journal entry: Create a subtraction problem with regrouping and describe how to solve it.

Lesson 7 –Fraction Math Centers

Brief Overview: In this lesson students will use and practice what they learned about fractions in lessons 1-6 in math centers. The centers format and the activities in this lesson are designed so that students engage in productive struggle that is supported by their peers (SMP.1: Make sense of problems and persevere in solving them.) and public discourse of the mathematical concepts of fraction that they have learned. (SMP.3: Construct viable arguments and critique the reasoning of others.) As you plan, consider the variability of learners in your class and make adaptations as necessary.

Estimated Time: 60 minutes

Resources for Lesson:

- Manipulatives as needed [Number line with 0, 1/2, 1 markings, Paper strips, Parallel number lines, Fraction models (circle and rectangles), Tape Diagrams, Graph paper, Geoboards]
- Access to Internet

Closest to One

Directions:

1. Player one chooses two fraction cards and adds the two fractions together by finding common denominators.

2. Player two chooses two fraction cards and adds the two fractions together by finding common denominators.

3. The player that is closest to 1 wins the round.

4. Continue play for 10 rounds.

5. The winner is the player that wins the most rounds.

½ Fraction Match Memory ½

Directions:

1. Place all cards face down.

2. Player one turns two cards over and tries to create a match of equivalent fractions.
    a. If the cards match, the player takes both cards and turns two more cards over.
    b. If the cards do not match, the cards are turned back over and the turn is over.

3. Player two takes a turn.

4. Play continues until all the cards are matched.

5. The winner is the player with the most matches at the end of the game.

$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$
$\frac{1}{3}$	$\frac{3}{4}$	$\frac{2}{8}$

$\frac{2}{6}$	$\frac{1}{6}$	$\frac{6}{8}$
$\frac{1}{12}$	$\frac{3}{12}$	$\frac{6}{12}$
$\frac{3}{}$	$\frac{4}{}$	$\frac{5}{}$

6	6	6
$\frac{1}{7}$	$\frac{2}{7}$	$\frac{3}{7}$
$\frac{4}{7}$	$\frac{5}{7}$	$\frac{6}{7}$

Grade 5 Unit Lesson 7–Fraction Math Centers

Additional Activities to use for Centers

Modeling Fractions with Cuisenaire Rods

http://www.teachersdomain.org/resource/rttt12.math.cuisenaire/

Locating and ordering fractions on a number line

http://www.teachersdomain.org/resource/vtl07.math.number.nums.lpnumbline/

Using Recipes to Add Fractions and Convert Improper Fractions to Proper Fractions or Mixed Numbers

http://www.teachersdomain.org/resource/vtl07.math.number.fra.lpmixednum/

Illustrativemathematics.org website for background information

http://illustrativemathematics.org/standards/k8

Curriculum Embedded Performance Task (CEPA) Summary

New School Schedule

This task requires students to apply concepts and skills related to adding and subtracting fractions with unlike denominators. To be successful students must be able find equivalent fractions and regroup as necessary between whole numbers and fractions. It is necessary to be able to represent fractions and whole numbers as improper fractions or mixed numbers.

Portions of the task will be part of lessons 4, 5, and 6. The final product builds off these sessions and is completed at the end of the unit. There is also a persuasive essay component that is included as a center activity on day 7.

Today students will start to work on the final portion of the Curriculum Embedded Performance Assessment (CEPA).

Teachers should circulate and assist students to ensure that they are demonstrating an understanding of how to add and subtract fractions. (See handout Grade 5 CEPA).

GRADE 5 CEPA

Your principal has asked for your help. Propose a daily schedule for the 5 ½ hours of school. Remember to include the following:

Subject/Activity	Time Required
Morning meeting	1/6 of an hour
Lunch	½ hour
Recess	1/3 of an hour
Math	5/4 of an hour
ELA	3/2 hours

Using the time requirements above, create a schedule that also incorporates two more periods for science/social studies and specials (Art/music/PE).

1. Prepare a detailed schedule identifying the fraction of the school day that is spent on each subject.
2. Write a persuasive letter to your principal explaining why he/she should choose your schedule. Be sure to justify your schedule.
3. Create a visual model that shows fractional part of each day that is used for each subject.
4. Show your calculations so that each fractional piece added together would equal the 5 ½ hours of the day. (Remember to use equivalent fractions and other strategies to help you add fractions with unlike denominators.)

Your task is to assist the principal to figure out a new school schedule. The principal needs to get approval from the superintendent to make these changes and to prove that there is enough time in the 5 ½ hour day to do everything.

Part 1 (on day 4)

The principal has to allow 2/3 hour for lunch (which includes recess). She also wants to have 1/6 an hour for morning meeting.

- Estimate if the time for lunch, recess and morning meeting is more than, less than, or equal to one hour. Explain why you think your answer is reasonable.
- Share your answer and estimate with a partner.
- Calculate the actual amount of time for these two activities. Was your estimate correct? Why or Why not?

Part 2 (on day 5)

The principal wants to extend morning meeting to include current events and exercise in a period that is 3/2 hour. Lunch and recess still uses 2/3 of an hour.

- Estimate if the time used is more than, less than, or equal to two hours. Explain why you think your answer is reasonable.
- Share your answer and estimate with a partner.
- Calculate the actual amount of time used by these two periods. Was your estimate correct? Why or Why not?

Part 3 (on Day 6)

After checking her calculations, 2 1/6 hours are to be used for the lunch/recess period and the extended morning meeting. With a 5 ½ hour day, how much time is left for Math, ELA, Science and Social Studies?

- Estimate if the time left is more than, less than, or equal to three hours? Explain why you think your answer is reasonable.
- Share your answer and estimate with a partner.
- Calculate the actual amount of time left. Was your estimate correct? Why or Why not?

Part 4 (on day 7)

In order to justify the new schedule, the principal needs to show that they can fill all of their allotted time (5 ½ hours) with educational activities.

Keeping in mind the time already proposed for. Is it reasonable for the principal to suggest ½ hour for science, 5/4 hours for math and 1/6 for social studies?

If you think the principal is correct in proposing the times above prepare a poster with four different displays representing the amount of time used. Be sure to use evidence to support your answer.

Grade 5 CEPA (Performance Assessment) Rubric

Calculations	Visual Representation	Explanation * (Persuasive Letter)	Quality of Work (Neat, thorough, proper conventions)
4 Accurately found common denominators in order to add/subtract fractions	4 Visual representations accurately matches the numerical calculations and labeled correctly	4 Takes a clear position and supports it consistently with well-chosen reasons and/or examples; uses persuasive strategy to convey an argument.	4 Project shows strong evidence e that a lot of effort and time were put forth.
3 Calculations were mostly correct. Error was due to carelessness but did not impact the overall schedule.	3 Visual representations mostly matched the numerical calculations	3 Takes a clear position and supports it with some relevant reasons and/or examples; there is some development of the essay.	3 Project shows that some effort and time were put forth.
2 Calculations were mostly inaccurate and impacted the overall schedule.	2 Visual representations were mostly inaccurate and did not match the numerical calculations. Some or inaccurate labeling.	2 Takes a position and provides uneven support; may lack development in parts or be repetitive OR essay is no more than a well-written beginning.	2 Project shows that little effort and time put forth was inconsistent.
1 Calculations were attempted but not accurate.	1 Visual representations were attempted, but inaccurate and did not match the numerical calculations. Missing or incorrect labeling.	1 Attempts to take a position (addresses topic), but position is very unclear OR takes a position, but provides minimal or no support; may only paraphrase the prompt.	1 Projects show that hardly any effort and time was put forth.
0 Calculations were not attempted.	0 Visual representations were not attempted. No labels.	0 No attempt made to take a position.	0 Project was not completed.

** Persuasive writing component should be consistent with the school's writing program*

Sample self-assessment rubric:
http://www.pvusd.k12.ca.us/departments/GATE/dweck/documents/effective_feedback_mindsets_packet_effort_rubrics.pdf