At RPDP, we support educators through professional development. Professional development can occur in a variety of ways: Entire staff trainings, grade level meetings, one-on-one support, etc. We collaborate with administrators and teachers regarding the developing and strengthening math content knowledge, use of best practices in the classroom, we model lessons, and provide support for the use of quality instructional materials.

Providing educators with quality resources in regards to instructional materials is a continuous priority. We provide this support through math content overviews, the use of instructional materials, further practice/skill development materials, and through quality assessments/tasks. As we work to create these resources for educators, we may recommend other quality resources from time to time.

In recent years, some states have received funds to create quality instructional materials for ALL educators for ALL states to access. We have selected some of those materials that we believe support our vision of quality instructional materials that support teachers in providing a solid mathematical foundation for students. For more elementary math resources please visit Rpdp.net .

Fractions: Size Matters

Mathematics, Grade 4

The emphasis of this unit is to develop proficiency with fraction equivalency and comparison for 4th grade students. This unit addresses components of Critical Area (2) in the Massachusetts Mathematics Curriculum Frameworks. Students will be able to recognize and generate equivalent fractions and use various models to guide their thinking (number lines, fraction bars, area models, etc.). Students will be able to compare and order fractions with different numerators and different denominators using mathematical notation, precise language, reasoning, benchmark fractions, and visual models. This unit will integrate mathematical practices by asking students to *reason abstractly and quantitatively* (SMP.2) about fractions, and to *construct viable arguments* (SMP.3) about the comparisons and equivalence of fractions in the solution of problems as they *look for and make use of structure*. (SMP.7)

This unit, "Size Matters", should be placed sometime after units teaching multiplication and division and factors and multiples. Students should have experience writing word problems about addition and subtraction situations.

These Model Curriculum Units are designed to exemplify the expectations outlined in the MA Curriculum Frameworks for English Language Arts/Literacy and Mathematics incorporating the Common Core State Standards, as well as all other MA Curriculum Frameworks. These units include lesson plans, Curriculum Embedded Performance Assessments, and resources. In using these units, it is important to consider the variability of learners in your class and make adaptations as necessary.

Table of Contents

Stage 1 Desired Results

STABLISHED GOALS

Massachusetts Curriculum Framework for Mathematics, 2011:

4.NF.1. Explain why a fraction a/b is equivalent to a fraction (n x a)/(n x b) by using visual fraction models, with attention to how the numbers and sizes of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

4.NF2. Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model. (Denominators are limited to 2,3,4,5,6,8,10,12 and 100.)

SMP.2 Reason abstractly and quantitatively.
SMP.3 Construct viable arguments and critique the reasoning of others.

Transfer

Students will be able to independently use their learning to…

Apply mathematical knowledge to analyze and model mathematical relationships in the context of a situation in order to make decisions, draw conclusions and solve problems.

Meaning

UNDERSTANDINGS

Students will understand that…

1. Equal quantities of the same whole can be expressed as different equivalent fractions; at various times one fraction might be more useful than another.
2. Fractions can be compared using a variety of visual models
3. Equivalent fractions can be represented in a variety of visual models (e.g. number line, area model, fraction bars/circles, pattern blocks).
4. The values of the numerator and denominator help you compare and reason about fractions.

ESSENTIAL QUESTIONS

1. How do you know what the size of a fraction is?
2. How do I know my answer makes sense?
3. How can I use models to compare fractions?
4. How does equivalence help us solve problems?
5. How can I use language to explain my thinking about the size of fractions?

SMP.7 Look for and make use of structure. ***Massachusetts Curriculum Framework for English Language Arts and Literacy, 2011:*** 4.W.2.D Use precise language and domain-specific vocabulary to inform about or explain the topic 4.RI.7 Interpret information presented visually, orally or quantitatively and explain how the information contributes to an understanding of the text in which it appears	***Acquisition***	
	Students will know…	***Students will be skilled at…***
	1. The meaning of benchmark fractions. 2. How to model, find and explain the relationship between the pairs of numerators and pairs of denominators of equivalent fractions. 3. How to use models and number lines to compare fractions.	1. Recognizing and generating equivalent fractions. 2. Comparing and ordering fractions using benchmarks fractions; reasoning about the relationships of the numerator and denominator. 3. Drawing and locating fractions on a number line. 4. Creating a visual model to justify the equivalence or comparison of two fractions. 5. Communicating using appropriate mathematical language and notation including numerator, denominator, equivalent fraction, benchmark fraction, >, < and =.

Stage 2 – Evidence	
Evaluative Criteria	**Assessment Evidence**
See Rubric.	**CURRICULUM EMBEDED PERFORMANCE ASSESSMENT (PERFORMANCE TASKS):** **Equal Shares** Students will determine if the way cakes are shared at a class party is fair. Students compare numbers of pieces of cake to determine if each child gets an equal share. This CEPA can be given to students at the end of day 1 in Lesson 4. (Alternatively, the teacher may want to wait until the end of this lesson.) **Picture Day at Insect School** Students order insects by length from least to greatest and display their results on a number line. Students then place an unfamiliar fraction using benchmark fractions. This CEPA is intended to assess work included in all the lessons, and should be given at the end of the unit.
	OTHER EVIDENCE: Quizzes, in-class projects and exercises to show that… • Given a set of fractions students can place them in order from smallest to greatest accurately • Given a fraction students can generate equivalents • Given a group of fractions students can identify equivalents • Given a set of fractions students can use multiple representations to compare them • Students can justify their reasoning about fractions using visual models, benchmark fractions and written and oral arguments Quiz: Comparing Fractions - explain your reasoning. Self-Evaluation.

Stage 3 – Learning Plan

Summary of Key Learning Events and Instruction

The Learning plan will consist of 5 topics:

1. Using different wholes
2. Comparison of Fractions
3. Benchmark fractions
4. Generating Equivalent Fractions
5. Comparison of Fractions with either common numerators or denominators

Topic 1. Using Different Wholes Lesson 1

- Pre-assessment: Use manipulatives to show fractions
- Activities:
  - Video from Teacher's Domain: *Equal Amount of Gold* with activity (manipulatives available) and discussion
  - Problems with Different Wholes (square pizzas and miles run) – Group work includes manipulatives and number lines, followed by whole class discussion
  - Pattern Block Activity
- Assessment/Homework problem (Brownies or similar)

Topic 2. Comparison of Fractions Lesson 2

- Warm-up: Sharing candy bars
- Activities
  - Formative assessment question
  - Fraction Flash Game: use fraction cards, compare size of fraction, winner is one with larger fraction (4.NF2)
  - Comparison of fractions using Kits/manipulatives
  - Optional Computer activity Comparing fractions www.visualfractions.com/CompareL/comparel.html (4.NF2)
- Formative assessment/independent practice: worksheet listing pairs of fractions, students determine correct symbol, < = or > and provide a visual representation for each
- Homework: Write fraction stories
- Activities
  - Put fractions on clothespin number line
  - Comparisons using fraction cards
- Assessment: Melissa's Answer

Topic 3. Benchmark Fractions Lesson 3

- Warm-up: Stickers questions and discussion
- Activities
  - Ordering Benchmarks
  - Benchmark Measuring Cup
  - Sorting Fractions
  - Benchmark Number Line
- Formative Assessments
  - Fraction Boxes
  - Mr. Sullivan's Class

Topic 4. Generating Equivalent Fractions Lesson 4

- Pre-assessment
  - Fractions equal to ½
- Activities
  - Teacher's Domain video, Orange Crystals followed by questions and discussion
  - Illuminations activity – Equivalent Fractions http://illuminations.nctm.org/ActivityDetail.aspx?ID=80 , can be done on individual computers or using interactive white board, students use Recording sheets.
  - Find the pattern: show various fractions in numerical form as discovery activity to derive multiplicative relationship of equivalent fractions.
  - Whole class discussion on multiplying and dividing numerator and denominator by the same number to generate equivalent fractions
- Formative assessment/independent practice
  - Worksheet to generate equivalent fractions and give a visual model for each
- Activities
  - Coins as fractions of a dollar guiding questions
  - Fractions to coins card game
  - Quilt designs
  - Candy Store Problem
- CEPA – Party Day

Topic 5. Comparing Fractions with Like Denominators or Numerators Lesson 5

- Warm-up: Robbie's Recipe

- Activities
  - Guided Discussion on fraction comparisons
  - Pizza problems
  - Running races
- Assessment
  - Choose 3 fractions
- CEPA – Picture Day at Insect Street School

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Lesson 1 - Using Different Wholes

Brief Overview: This lesson looks closely at what a "whole" means, and how using different wholes changes the size of the fractions. As you plan, consider the variability of learners in your class and make adaptations as necessary.

Estimated Time: 60 minutes

Resources for Lesson:

- Internet Access http://www.teachersdomain.org/asset/vtl07_vid_equalamtgo/. This video provides a situation/context where the equivalence of fractional parts allows a solution to a problem. Students need to be able focus on the story told.
- A range of story problems that require comparisons of fractions where the wholes may be the same or the wholes may be different. Students need to be able to represent these fractions using area models and number lines so comparisons can be made as well as using the context of the story problem to consider what makes sense and why.
- Manipulative materials that provide flexibility as to what represents the whole, such as pattern blocks, graph paper, fraction circles, etc. may also be helpful.

Content Area/Course: Mathematics, Grade 4

Unit: Fractions: Size Matters

Time (minutes): 60-90 minutes

Lesson 1: Using Different Wholes

By the end of this lesson students will know and be able to:

Explain why comparisons are valid only when the two fractions refer to the same whole.

Essential Question (s) addressed in this lesson:

How do you know what the size of a fraction is? Why is the size of the whole important?

Standard(s)/Unit Goal(s) to be addressed in this lesson (type each standard/goal exactly as written in the framework):

4.NF2. Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model. (Denominators are limited to 2,3,4,5,6,8,10,12 and 100.)

SMP.2: Reason abstractly and quantitatively.

SMP.3 Construct viable arguments and critique the reasoning of others.

SMP.7 Look for and use of structure

Instructional Resources/Tools (list all materials needed for this lesson)

1. Internet Access http://www.teachersdomain.org/asset/vtl07_vid_equalamtgo/. This video provides a situation/context where the equivalence of fractional parts allows a solution to a problem. Students need to be able focus on the story told.

2. A range of story problems that require comparisons of fractions where the wholes may be the same or the wholes may be different. Some are included within the lesson. Teachers may supplement these with similar problems. The problems may be provided to students in whatever format is most suitable to the specific class. Students need to be able to represent these fractions using area models and number lines so comparisons can be made as well as using the context of the story problem to consider what makes sense and why. Note: Using multiple representations of fractions help students communicate their own reasoning as well as critique the reasoning of others. (SMP.3: *Construct a viable argument and critique the reasoning of others*).

3. Manipulative materials that provide flexibility as to what represents the whole, such as pattern blocks, graph paper, fraction circles, etc. may also be helpful.

Anticipated Student Preconceptions/Misconceptions

It doesn't matter if the fractions are related to the same whole;

When students construct visual representations of two fractions for the purpose of making comparisons, the whole may differ in size, rather than being the same.

This also comes up when students use visual representations to add two fractions and need to determine their whole in order to identify the solution, e.g., 3/4 + 3/4 may look like 6/8 because they are thinking of the wholes associated with each fraction making up one new whole associated with the sum.

This also comes up when students construct visual representations of improper fractions.

Instructional Tips/Strategies/Suggestions:

Provide manipulatives (tiles or unit blocks) and/or graph paper for students to represent the pizzas.

Number lines may be a good representation for the distances.

By looking at the student work on these two problems the teacher can assess formatively what the student understands. (the 4 quarters within a pizza are the same size, the relative sizes of 12 x 12 and 8 x 8 pizzas, attempts to make ¼ the same size for both the large and small pizza, etc.)

Extension: students use number lines to represent the pizza fractions and area models to represent the fractions in the running problem. These are useful representations and are consistent with SMP.2: *Reason abstractly and quantitatively*, which has to do with de-contextualizing and re-contextualizing the mathematics of the problem.

While students may feel that the answers to these questions are obvious, the important work is providing a strong justification for their solutions.

The needs of ELL and special education students would be addressed by placing them in small groups where they have opportunities to participate at their levels of expertise with many informal opportunities to represent and talk about their thinking.

Pre-Assessment

Build a representation of 3/4. Explain how your representation shows this fraction.

Swap papers with a partner.

Do you agree or disagree that the representation shows 3/4? Why or why not?

Have different student show their representation of 3/4. Ask students: "Do all the representations look alike? Are they all the same size? Why do we call them all ¾?

This assessment is designed to provide information about how students build representations of fractions and the extent to which their representations clearly indicate the whole. It is also designed to begin to raise questions about how the relationship between the size of the whole and the size of the parts. These understandings lay important groundwork for how students compare fractions and the importance of keeping the whole the same. Note: When "students swap papers with a partner" teachers are presented with opportunities to listen in /observe students as they construct arguments, communicate them to others, and critique the arguments of others (SMP.3) while reasoning abstractly and quantitatively (SMP.2).

What students need to know and are able to do coming into this lesson (including language needs):

Students need to understand how to determine fractional parts of a whole and connect these fractional parts to the numerator and denominator of a fraction using area models and number lines. They also need to be able to represent fractions using area models and number lines with some degree of fluency and accuracy so visual comparisons (as well as numerical comparisons) can be made.

Lesson Sequence

Intro: Introduce this unit about comparisons of fractions with a video on equivalent fractions. Show students the following video… (5 mins)

Teachers' Domain – Equal Amount of Gold

http://www.teachersdomain.org/asset/vtl07_vid_equalamtgo/

Focus: As you watch, think about the strategy the CyberSquad uses to solve the problem. How does knowing about the relationship between different fractions help them? What fractions do they encounter here? How are these different fractions related? The focus questions encourage students to reason abstractly and quantitatively (SMP.2).

Guided Practice: (15 mins)

Students use a prepared template to respond to the following questions: Template (Handout 1) is found at: http://www.teachersdomain.org/resource/vtl07.math.number.nums.lpnumbline/

- What did the CyberSquad know about these fractions that helped them share the gold equally between Vin and Pan?
- How did they decide which gold bars represented which fractions? For example, how did they know the bar they called one-fourth, was really one-fourth?
- Draw a diagram showing the equivalencies between eighths, fourths, halves, and a whole.

The rest of this lesson is designed to consist of a set of tasks that students work on in small groups. The teacher is responsible for posing the task, making sure students are clear on the task itself, circulating while students work on the task in small groups in order to attend to student thinking, and then facilitating a whole group discussion about the mathematics that emerged in the small groups.

The teacher, while circulating, should attend to the thinking going on in each group and how that thinking is getting represented on paper, asking questions that provide more information about the student thinking and also help move that thinking forward. The teacher should also be thinking about which solutions might be presented to the class and in what order, as well as the main mathematical points to make during this discussion.

Conclude the discussion with the question, could you still share the bars equally, if you said the second to the largest bar is one whole? Then ask what fraction each of the two smaller bars is; what would the largest bar represent? When students realize that the whole can be defined by different size pieces, i.e., they are able to *look for and make use of structure* (SMP.7), proceed with the problems below. The teacher should pose problem #1, have students work on it in small groups, and then facilitate a whole group discussion. Based on the student work and the small group and whole group discussion, the teacher should reconstruct the small groups and have them work on problem #2.

Problem #1:

A family ordered two pizzas for dinner from a pizza place that made square pizzas. They ordered one large 12 in x 12 in pizza and one medium 8 in by 8 in pizza. Tawana ate ¼ of the large pizza and Jamilla ate ¼ of the small pizza. Their little brother Damian said they both ate the same amount because they each ate ¼ of the pizza. Do you agree or disagree with Damian? Why or why not?

Problem #2:

Every afternoon Erin runs a mile after school. One afternoon she needed to be at a friend's house for dinner so she only had time to do half of her run. Babs is training for a marathon and every afternoon she runs 4 miles. She also needed to be at a friend's house for dinner and had time to do only half of her run. While they were having dinner with their friends, Erin and Babs talked about being able to do only half of their runs that afternoon. Did they both run the same amount? Why or why not?

Using Pattern Blocks

After finishing these two problems, including the whole group discussion, shift to a pattern block activity having to do with identifying the whole. http://www.k-5mathteachingresources.com/support-files/pattern-block-fractions-4nf2.pdf

Note how flexibly students are able to identify the whole given a particular fractional part of a whole i.e., look for and make use of structure (SMP.7).

Extended Learning/Practice (homework):

Assign a homework problem that is similar to problem #1 or 2. Review the task to be sure the context of the homework problem is clear to all students.

Sample: Jack's Mom sent in two pans of brownies, a large one that was 12 inches by 12 inches, and a small one that was 12 inches by 6 inches. There were 12 brownies in each pan.

Each student got 1 brownie or 1/12 of the pan. Did all the students have the same amount? Draw a picture to show your answer. What else can you say about the sizes of the pieces in each pan of brownies?

Closure

Review outcomes of this lesson: Students understand that the fractional parts of wholes are equal only if the wholes are the same size.

Preview outcomes for the next lesson:

Be able to compare fractions when the wholes are the same size.

Lesson 2 – Comparison of Fractions

Brief Overview: This lesson guides students in learning how numerators and denominators are important to consider when comparing fractions. As you plan, consider the variability of learners in your class and make adaptations as necessary.

Estimated Time: 3 sessions, 60 minutes each

Resources for Lesson:

- Fraction Kits (manipulatives) provide a means for students to work with fractions on a concrete level and begin to relate these to the symbolic representations.
- Number Lines reinforce the concept that a fraction is a number and provide a representation that helps students to reason about the relative sizes of fractions; they assume that students are comfortable with the idea that a location further to the right on a number line means the value is larger.
- Fraction Cards : http://www.math-salamanders.com/free-printable-math-flash-cards.html
- Number Lines: Paper number lines marked 0, ¼, ½, ¾ and 1, and list of fractions. Transparencies of the number line with the fractions placed correctly. (Construction paper may used for number lines.)
- Worksheet: Fraction Comparisons

Content Area/Course: Mathematics, Grade 4

Unit: Fractions: Size Matters

Time (minutes): 3 sessions, 60 minutes each

Lesson 2: Comparison of Fractions

By the end of this lesson students will know and be able to:

Compare fractions using visual models, and reason about the size of the numerator and the denominator.

Be able to justify conclusions about relative fraction sizes.

Use the symbols >, < and =

Essential Question (s) addressed in this lesson:

How can I use models to compare fractions?

Standard(s)/Unit Goal(s) to be addressed in this lesson (type each standard/goal exactly as written in the framework):

4.NF2. Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model. (Denominators are limited to 2,3,4,5,6,8,10,12 and 100.)

SMP.3 Construct viable arguments and critique the reasoning of others.

Instructional Resources/Tools (list all materials needed for this lesson)

1. Fraction Kits (manipulatives) provide a means for students to work with fractions on a concrete level and begin to relate these to the symbolic representations
2. Number Lines reinforce the concept that a fraction is a number and provide a representation that helps students to reason about the relative sizes of fractions; they assume that students are comfortable with the idea that a location further to the right on a number line means the value is larger.
3. Fraction Cards: http://www.math-salamanders.com/free-printable-math-flash-cards.html
4. Number Line - Paper number lines from 0 to 1.0 with ¼, ½ and ¾ marked, and a list of fractions (The teacher may wish to use construction paper.) Transparencies of the number line with the fractions placed correctly.
5. Fraction Comparison Worksheet (Included)

Anticipated Student Preconceptions/Misconceptions

Fractions with larger denominators are greater than those with smaller denominators.

Instructional Tips/Strategies/Suggestions:

Allow students to use concrete manipulatives and diagrams until they are comfortable with the numerical representations.

In the Number Line Activity, the list of fractions can be differentiated for groups of students based on the need for more time or more challenging work.

What students need to know and are able to do coming into this lesson (including language needs):

How to use models and number lines to represent fractions; A location further to the right on a number line means the value is larger.

Vocabulary: Numerator, Denominator, Compare

Lesson Sequence

DAY 1

Engagement Activity: (Activate Prior Knowledge) (10 mins.)

I baked a chocolate cake and a banana cake for my sister's birthday. Both were the same size. I sliced the chocolate cake into 8 slices and the banana cake into 5 slices. After dinner my family ate 4 slices of the chocolate cake and 3 slices of the banana cake. Did the family eat more of the chocolate cake or the banana cake? How do you know? Each group should be prepared to present their visual model explaining their answer.

Teacher Guiding Questions: How did you decide what each fraction looked like? How did you decide which fraction is larger?

Note: Students should have access to Fraction Kits (manipulatives).

Discussions, be they with a partner or whole class facilitate students' construction and communication of *viable arguments and critiquing the reasoning of others*. (SMP.3)

Discussion: (partner, table, etc.) (10 mins.)

What did you notice? Evidence?
Students can post responses on chart paper with diagrams.
Quick carousel share of all chart papers.

Whole Group Discussion: (10 mins.)

What conclusions can we draw regarding fraction sizes?

Teacher should record these for later use.

Guided Practice:

With partners, students use fraction models (bars, circles, number lines, towers, etc.) to respond to the following comparisons deciding whether it is greater than, less than, or equal to. They should be prepared to justify their answers.

Begin with fraction comparisons involving the same numerator or denominator such as:

3/5 and 3/8 or 4/10 and 4/12
2/5 and 2/6 or 1/12 and 1/10

Guide students to reason about the numerators and denominators and the relationship between two fractions when the numerators or denominators are the same. This should activate students' prior knowledge of fractions.

Move to more difficult comparisons that require students to reason about the size relative to other common fractions. Students should be asked to explain their numerical reasoning such as I know 3/7 is smaller than 5/8 because 3/7 is less than ½ while 5/8 is a bit more than ½.

Independent Practice: (homework, exit slip, individual work in class)

Note: this depends on length of instructional block.

Would you rather have 1/2, or 6/10 of a chocolate cream pie? Use words and pictures to explain your reasoning.

This also serves as a formative assessment.

Day 2

Review/Assess Understandings: (15 mins.)

Select students to share Independent Practice from Day 1. Review with the whole class. This is a good opportunity to have the students explain what the denominator means and why the larger the denominator the smaller the piece. The teacher should use additional examples if students are showing any confusion here.

Engagement Activity (reactivate prior knowledge): (15 mins.)

Fraction Flash Game –

Materials – Fraction Cards

Resource: http://www.math-salamanders.com/free-printable-math-flash-cards.html

- Students work with partners
- Each student draws a card from the pile,
- Students compare the sizes of fractions on their cards,
- The student with the largest fraction keeps the 2 cards.
- Students continue to draw one card each and compare them.
- If the fractions are equivalent, each student keeps one card.
- Play continues until all cards are used or time is called.
- The player with the most cards wins.

Optional Computer Activity

- Students use >, < or = signs to compare fractions, computer provides feedback with number lines for each fraction. This could be especially helpful for those struggling with comparisons. www.visualfractions.com/CompareL/comparel.html

Guided Practice: (15 mins.)

Ask students to use pieces of their Fraction Kits (manipulatives) to make and record comparing statements (e.g., 1/3> 1/4). Alternatively, the teacher may be able to use an interactive whiteboard here. One example of a template can be found here: http://www.printable-math-worksheets.com/fraction-circles.html

After students appear to understand comparing with simple fractions, ask them to combine fraction parts and make comparisons (e.g. combine 1/4 and 1/4 to get 2/4, then compare that to 1/3, using pieces of their Fraction Kit (manipulatives).

Independent Practice: (15 mins.)

- Students complete Lesson 3 Independent Practice worksheet.
- Students should have access to manipulatives.
- Worksheet contains a listing of fractions and students determine correct symbols
  >, < and =
- Worksheet should include the requirement for a visual representation for each response.

Homework:

- Students pick a fraction out of a hat
- Students write a story (5 sentences) about their fraction. They must explain the meaning of the numerator and the denominator in their story.

Day 3

Lesson Continuation/Review: (15 mins.)

- Students place their fraction (from homework) on a clothespin number line (Benchmarks fractions are already placed)
- Have half the students find a "fraction friend" that is less than they are
- Students work with their partners
  *share stories, justify with a model or picture their relationship with their partners and explain the importance of the numerators and denominators

Extension: Write a story including both fractions

Independent Practice: **(**15 mins.) *Fraction Compare*

Note: This activity provides multiple opportunities for students to employ SMP.3 (Construct viable arguments and critique the reasoning of others.) For example, by using visual models to justify conclusions they are able to construct and communicate their reasoning to others.

Fraction Cards – numbers only

1. Turn over two fraction cards. Use >, <, or = to compare the fractions.
2. Justify your conclusion using a visual fraction model.
3. Repeat with 10 pairs of fraction cards.
4. Number Line - Paper number lines from 0 to 1.0 with ¼, ½ and ¾ marked, and list of fractions. Transparencies of the number line with the fractions placed correctly.
1. Students write the fractions on the list on their number lines
2. When they are finished, they come to the front of the room and get an answer transparency to check their answers.
3. They note any fractions which are not correct and go back to their seats, place them correctly and explain why the fraction is less than or more than another.

Early Finishers?

Working at their desks…

Students make up word problems that require comparison of at least two fractions in order to answer them.

Homework:

Students make up word problems as above, or solve the word problems made up by their classmates.

Closure

Review outcomes of this lesson: Students will be able to compare fractions using visual representations, number lines, etc. Students will be able to explain the relationship between two fractions by reasoning about the sizes of the numerators and denominators.

Summative Assessment:
(15 – 30 mins)
Melissa gave the following answer on a quiz 1/5 is greater than 2/8. Do you agree? Why or why not? Prove your answer using numerical reasoning and a visual model.

Preview outcomes for the next lesson:
Students will be able to understand how to use benchmark fractions for comparisons.

Lesson 2: Fraction Comparisons

Use >, <, or = to complete the number sentences. Use a visual model to justify your answers.

Number Sentence	Visual Model
2/6 ___ 3/6	
3/5 ___ 3/8	
3/6 ___ 1/4	
4/8 ___ 5/10	
7/8 ___ 9/10	

Lesson 3 – Benchmarks

Brief Overview: This lesson explores "benchmark fractions", those that are typical in measurement (1/4, 1/3, 1/2, 2/3, 3/4) and how they can be used when estimating and measuring. Students learn to compare fractions by *looking for and making use of structure* (SMP.7). As you plan, consider the variability of learners in your class and make adaptations as necessary.

Estimated Time: 2 sessions, 60 minutes each.

Resources for Lesson:

- Manipulatives should be available: fraction circles, bars, rulers, construction paper and any other manipulatives students are accustomed to using. These will help to provide access to the lesson for students needing more concrete representations.
- Two worksheets (included)

Content Area/Course: Mathematics, Grade 4

Unit: Fractions: Size Matters

Time (minutes): 2 sessions, 60 minutes each

Lesson 3: Benchmarks

By the end of this lesson students will know and be able to:

Order benchmark fractions

Place benchmark fractions on a number line

Use benchmark fractions to determine the relative size of other fractions

Essential Question (s) addressed in this lesson:

How can I use models to compare fractions and how do I know my answer makes sense?

Standard(s)/Unit Goal(s) to be addressed in this lesson (type each standard/goal exactly as written in the framework):

4.NF2. Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model. (Denominators are limited to 2,3,4,5,6,8,10,12 and 100.)

SMP.7 Look for and make use of structure.

Instructional Resources/Tools (list all materials needed for this lesson)

Manipulatives should be available: fraction circles, bars, rulers, construction paper and any other manipulatives students are accustomed to using. These will help to provide access to the lesson for students needing more concrete representations.

Anticipated Student Preconceptions/Misconceptions

A larger denominator means a larger piece

Fractions don't live on the number line like whole numbers

Pre-Assessment

Have students represent benchmark fractions with manipulatives.

Instructional Tips/Strategies/Suggestions:

Activities can be differentiated by broadening or narrowing the categories into which the fractions are sorted. For example, the Sorting Fractions activity can be differentiated by making fewer jars (eliminate the fourths) or more jars (include thirds or include mixed numbers).

What students need to know and are able to do coming into this lesson (including language needs):

Fractions represent part of a whole that has been divided into equal parts;

That a fraction is a number and can be located on a number line;

The meaning of numerator and denominator.

When fractions have the same denominator, the one with the larger numerator is the larger fraction.

When fractions have the same numerator, the one with the larger denominator is the smaller fraction.

Information for the Teacher:

For the purposes of this unit benchmark fractions are defined as:
1/4, 1/3, 1/2, 2/3, 3/4

The warm-up activity is to get students thinking about estimation and how this helps to compare values.

Students should understand that two fractions are equivalent (equal) if they are the same size, or the same point on a number line.

Students should be able to compare two fractions with the same numerator or the same denominator by reasoning about their size.

Lesson Sequence:

Warm-up:

I went to the store to buy some stickers. I can buy a package of 4 stickers for 12 cents or a package of 5 stickers for 13 cents. How can I tell which is the better buy?

How much does one sticker cost if I buy the package of 4?

How much does one cost if I buy the package of 5?

Guide students to realize that they don't need to know the exact amount that each sticker in the package of 5 costs as long as they know that it is more than 2 cents but less than 3 cents. In the package of 4, each sticker costs 3 cents; in the package of 5, one sticker costs >2 cents and <3 cents.

Use the warm-up to remind students that there are times when we don't need an exact answer but can just get "close enough." Ask students to share examples of this. When is it important to get an exact answer and when is it not?

Ask students to work in pairs or tables to discuss benchmark ideas and then share with the class.

Teacher Questions to Guide Discussion:

• How close is close enough?
• What numbers do we usually use when we need just "close enough"? (Students should be guided to realize that we use both numbers that are easy to manipulate like 10s and 100s and numbers or concepts that have meaning to us like 25 cents or a week)
• Are there fractions that have more meaning to us than other fractions? Are there fractions that you can picture in your head without drawing them or making a model?

Discuss the meaning of benchmark—something we know or understand that can be used to help us estimate something—and that sometimes we can associate a number with a benchmark and sometimes it's just a comparison.

Ask them how they would explain to a little kid who doesn't yet understand what a day is how soon something is going to happen (by referencing something they do know—after lunch, after you wake up twice, after summer starts but before your birthday)

Ask if students can think of any benchmarks they use. Possible suggestions to get them started are comparison benchmarks (about waist high, just over my head, a little longer than my foot, a lot heavier than my cat) and more specific benchmarks (a centimeter is about the width of your little finger, an inch is about the length of your last thumb joint, a yard is about the distance from the tip of your fingers to your nose).
Explain that we use fractions that are commonly used as benchmarks. Ask for examples of fractions we use more commonly. (Some possible

teacher questions are "Why are so many people suggesting ½? Why haven't I heard 1/19? Can you picture what ¼ looks like in your head? What are you picturing—fraction circles? A ruler? A measuring cup?)

Define the fractions that we are using as benchmarks and why.

Activities:

Ordering Benchmarks
Give each table a card with the benchmark fractions on them. Ask students to use manipulatives to build a model of each fraction and then order the fractions from smallest to largest.

Benchmark Measuring Cup
Give students an empty cup and individual measuring cups (1/4, 1/3). Have students measure and mark the measurements for ¼, 1/3, ½, 2/3, ¾ and 1 cup on their empty cup to make a benchmark measuring cup. Have the children use the cup to measure amounts and express their measurements as relative to benchmarks (there is about 1/3 cup of oatmeal, there is between 2/3 and ¾ of a cup of popcorn).

Sorting Fractions
Give students a set of fraction cards. Have them sort the fractions relative to the benchmarks, using manipulatives to build the fractions for comparison if necessary, and place the fractions in the appropriate jars:

- between 0 and 1/4,
- equal to ¼,
- between ¼ and ½,
- equal to ½,
- between ½ and ¾,
- equal to ¾,
- between ¾ and 1 whole.

Benchmark Number Line
Give students fraction strips. Ask students to fold the strips to show fourths, thirds and halves. Mark fraction strips with each fraction shown (1/4, 2/4, ¾, 1/3, 2/3, ½) as well as 0 and 1. Use fraction strips to find equivalents and order benchmarks. Fold a sentence strip to show all benchmark fractions and mark benchmarks on the sentence strip as well as 0 and 1 to make a fraction number line.
Exit Ticket: Mr. Sullivan's Class (see Summative Assessment)

Homework:

"Fractions in Boxes" (worksheet included)

Closure

Review outcomes of this lesson: Students understand the concept of a benchmark, its use as a way to compare fractions, to estimate or check the reasonableness of an answer, and how to order the benchmark fractions on a number line.

Summative Assessment:
Mr. Sullivan's Class (worksheet included)
Using benchmarks to determine fractions of a set.
Students may use the line at the bottom of the worksheet to make a number line.

Preview outcomes for the next lesson:
Students will be making equivalent fractions by multiplying or dividing the numerator and denominator by the same number.

Lesson 3: Fractions in Boxes

Place the fractions below in the correct box in the chart.

$$1/7, 1/3, 7/8, \quad 2/4, \quad 2/3, \quad 1/5, 4/8, 5/8, 9/10, 3/8$$

How did you decide where to place 1/7?

How did you decide where to place 5/8?

Between 0 and ¼	Between ¼ and ½	Equal to ½	Between ½ and ¾	Between ¾ and 1

Lesson 3: Mr. Sullivan's Class Activity

Students in Mr. Sullivan's fourth grade class can earn stickers for completed homework. Mr. Sullivan has a list of the number of stickers different students have earned this week but the sticky notes with the students' names that were attached have gotten lost. The list of the number of stickers is shown in the bottom row in the chart below. Use the clues to figure out how many stickers each student has earned. Explain or show how you decided which box to put each student's name in.

Each package of stickers contains 24 stickers.

1. Tamisha has earned more than 1/2 of a package of stickers but less than 3/4 of a pack.
2. Fernanda has earned more than 1/4 of a package but less than 1/3 of a pack.
3. Kevin has earned more than 1/12 of a package but less than 1/6.
4. Mandy has earned exactly ¼ of a package
5. Alex has earned more than 1/3 of a package but less than ½ of a package.
6. Sophie has earned more than 3/4 of a package.
7. Joe earned more than 1/6 of a package but less than ¼ of a package.
8. Emily earned less than 1/12 of a package

Name								
Number of Stickers	1	3	5	6	7	11	14	19

Lesson 4 – Generating Equivalent Fractions

Brief Overview: In this lesson, students use manipulatives, number lines and reasoning to explore equivalent fractions. As you plan, consider the variability of learners in your class and make adaptations as necessary.
Estimated Time: 2 sessions, 60-90 minutes each

Resources for Lesson:

- Short video clip on Orange crystals from CyberChase http://www.teachersdomain.org/resource/vt107.math.number.fra.orangecrys
- NLVM http://illuminations.nctm.org/ActivityDetail.aspx?ID=80
- Chart of Fraction Number Lines
- Graph paper
- Coins (optional)
- Preceding lessons include standard fraction terminology (numerator, denominator, equivalent).
- Recording Sheet for Illuminations Activity
- Worksheet for Generating Your Own Equivalent Fractions (fraction, equivalent ones, diagram)
- Money Worksheet- Convert these Cents to Fractions of a dollar
- Fraction Cards and Coins or Tally Sheets
- Quilt Grid and Table (Adapted from http://www.eworkshop.on.ca/edu/pdf/Mod27_lesson_summary.pdf
- Candy Store Problem
- Optional Literature Connection: Eight Hands Round: A Patchwork Alphabet by Ann Whitford Paul

Content Area/Course: Mathematics, Grade 4
Unit: Fractions: Size Matters

Time (minutes): 3 sessions, 60-90 minutes each

Lesson 4: Generating Equivalent Fractions

By the end of this lesson students will know and be able to:

Make equivalent fractions by multiplying or dividing the numerator and denominator of a fraction by the same number. Use visual representations to demonstrate that this is true; apply the concept to money, (i.e. ½ dollar = 2 quarters = 5 dimes) and other real-world situations.

Essential Question (s) addressed in this lesson:

How do you know what the size of a fraction is?

How does equivalence help us solve problems?

Standard(s)/Unit Goal(s) to be addressed in this lesson (type each standard/goal exactly as written in the framework):

4.NF.1. Explain why a fraction a/b is equivalent to a fraction (n x a)/(n x b) by using visual fraction models, with attention to how the numbers and sizes of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

SMP.2 Reason abstractly and quantitatively.

SMP.3 Construct viable arguments and critique the reasoning of others.

SMP.7 Look for and make use of structure.

Instructional Resources/Tools (list all materials needed for this lesson)

Short video clip on Orange crystals from CyberChase
http://www.teachersdomain.org/resource/vt107.math.number.fra.orangecrys
NLVM http://illuminations.nctm.org/ActivityDetail.aspx?ID=80
Graph paper
Coins (optional)Recording Sheet for Illuminations Activity
Worksheet for Generating Your Own Equivalent Fractions (fraction, equivalent ones, diagram)
Chart of Fraction Number Lines
Money Worksheet- Convert these Cents to Fractions of a dollar
Fraction Cards and Coins or Tally Sheets
Quilt Grid and Table (Adapted from http://www.eworkshop.on.ca/edu/pdf/Mod27_lesson_summary.pdf)
Candy Store Problem
All 6 of the above are included at the end of this lesson.

Anticipated Student Preconceptions/Misconceptions

A larger denominator means a larger piece.
If either numerators or denominators are the same, the fractions are equal.
You can increase the size of either the numerator only or the denominator only and still have an equivalent fraction.

Pre-Assessment

List as many fractions as you can that are equal to ½.

Draw a diagram to show that they are equal.

Instructional Tips/Strategies/Suggestions:

Be sure students already know:

- that fractions are numbers and can be represented on the number line;
- fraction terminology from preceding lessons (numerator, denominator, equivalent);
- the value of coins and that 100 cents = 1 dollar.

Possible modifications for differentiated instruction:

Provide the Chart of Fraction Number Lines for students, as needed. This chart displays a number line for each of the fractions with denominators from 2 through 10. Equivalent fractions align vertically.

Allow students to use fraction manipulatives as needed, or the teacher may wish to provide this for all students.

Word wall posters of fraction terms giving the term, a definition, and an example with visual.

What students need to know and are able to do coming into this lesson (including language needs):

Students understand that a fraction results only when the whole is divided into equal parts. Students understand the meaning of numerator and denominator. Students have knowledge of the value of coins and that 100 cents = 1 dollar.

Information for the Teacher:

Timeline:
Day 1 (90 minutes):
initial short diagnostic
Introduction with Cyberchase Orange Crystals
Guided Practice with the Illuminations Applet.
The Cyberchase activity gets students thinking about comparing fractions with unlike denominators.

In the Illuminations activity, students build equivalent fractions and the number line below the figures provide feedback to let them know they are equal when they occupy the same place on the number line. This activity is also appropriate for the interactive white board.
Independent practice: Worksheet: Generating Equivalent Fractions is intended to continue what the students had been doing on the computer.

Day 2 (90 minutes)
Quick review of homework as formative assessment..
Extend fractions to those with 100 in the denominator.
And extend multiplication of numerator and denominator to division of numerator and denominator by the same number.
Discussion of coins as fractions of a dollar, make equivalent fractions for each (use play money as scaffolding for students)
Day 3
Students should work individually first, then they can compare answers with a partner or in a group.

Lesson Sequence:

DAY 1

Pre-assessment: Have students write as many fractions equal to ½ and give a visual representation for each. This short pre-assessment will tell if the students have the expected pre-requisite knowledge.

Introductory Instruction: Take students to the Cyberchase **Orange Crystal lesson** in Teacher's Domain. Note: In this activity students engage in SMP.7 (*Look for and make use of structure*).

Frame: Sometimes you might have to compare fractions to see which is bigger. Do you know which is more, ½ or ¼? How do you know? How about comparing 1/8 and 2/8? Which is bigger, and how do you know? Are denominators important in comparing fractions likes these? Why? What if fractions have different denominators, like 1/8 and 3/4? How do you go about comparing fractions like that?

Focus: As you watch this video segment, think about how the CyberSquad compares the different amounts of orange on the various crystals. Consider how their knowledge about fractions helps them. Are denominators important? How does the CyberSquad compare fractions with different denominators?

Show the short video.
http://www.teachersdomain.org/resource/vt107.math.number.fra.orangecrys

Follow Up: How did the CyberSquad know which crystal had the most orange and thus, the most power? How can you compare fractions that have unlike denominators, like 5/8 and 2/4 for example? What does the denominator in a fraction actually tell us? Can you name (write) three fractions equivalent to 1/2? (write on white boards would be useful here to do a quick formative check) How could knowing about equivalent fractions be helpful when food shopping?

Guided Practice:

Students go to the Illuminations site, Equivalent Fractions at http://illuminations.nctm.org/ActivityDetail.aspx?ID=80; Under *Build Your Own* in squares, demonstrate how to make ½; then ask students to make ½ another way and explicitly point out the correlation with the number line in the center. Then break the third square into 16 small squares (4 x 4) and color in 4 squares in two of the diagonally opposite corners. Ask students if this is also ½. And how can they be sure?

Provide Equivalent Fractions Illuminations Recording Sheet (a list of fractions for student to make equivalents for with space for them to record their equivalents).

Can you do these with the circles? Do you get the same answers? Which is easier? And Why?

Extensions:

Create a fraction whose denominator is a prime number.

How many equivalent fractions can you create?

Is it possible to create a fraction such that no equivalent fractions could be created with this Equivalent Fractions applet? Find at least one.

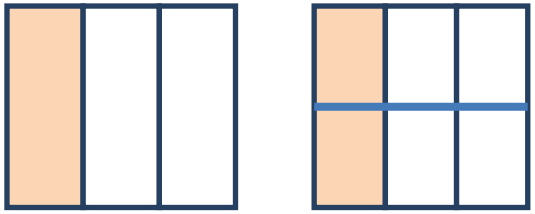
Can you describe the fractions for which this is possible?

Find the Pattern: **As a whole class discussion, the teacher** lists sets of equivalent fractions starting with the smallest numerator. In this activity, lead students to see that the numerator and denominator are multiplied by the same number to get to the next one (to derive the multiplicative relationship of equivalent fractions). Students should understand the relationship that a/a = 1 and so you are multiplying by 1 when you multiply the numerator and denominator by the same number.

For Example: 1/3 = 2/6 = 3/9 = 4/12

$$\frac{1 \times 2}{3 \times 2} = \frac{2}{6}$$

It is helpful to show the picture for each so the students see that as the denominator gets larger, each piece gets smaller but the same part of the whole is shaded.



The Chart of Fraction Number Lines may be used here and in subsequent parts of this lesson.

Independent Practice (homework): Generating Equivalent Fractions Worksheet (List of fractions, generate as many equivalent fractions for each and give one visual model i.e. double number line, shaded figures, etc.)

The teacher may administer the CEPA Equal Shares – Party Day here or at the end of this unit.

DAY 2

Warm-up: How do you know that 2 quarters, 1 half dollar, and 5 dimes are equivalent amounts of money?

(Optional Hint: write each as a fraction of a dollar.)

Suggested Guiding Questions:

If a dollar is one whole, what fraction is a penny? A nickel?

Use the Worksheet: Money to Fractions (convert cents to fractions of a dollar, give as many equivalent fractions for each). Students should work independently first, then in pairs or groups and review any values that students struggle with as a whole class.

Guided Practice:

Fractions to coins card game.

Give students fraction cards and each student picks a card in turn and comes up with 1/3,) you say so and pick again. If you can make the fraction with more than one type of coin (i.e. ¼ you can count both 5 nickels and 1 quarter).

The student with the most money at the end wins.

Tally sheets or actual coins can be used.

Homework Question:

What fractions cannot be made into coins? Why?

DAY 3

Formative assessment on the Concept of Generating Equivalent Fractions by multiplying or dividing the numerator and denominator by the same number.

Ask students if these equations are correct. Have them explain how you get the next fraction from the first one if the fractions are equivalent:

¼ = 2/8 = 25/100

50/100 = 25/50 = 5/10 = ½

2/6 = 1/12 = 1/3

Review the Concept of Coins as Fractions with a denominator of 100.

Ask for a coin or combination that equals a given fraction. Students can represent coins as circles of relative sizes with P, N, D, Q written in them for penny, nickel, dime and quarter. Why is it easier to do this for fractions with a denominator of 100?

Discuss the Homework Question**.** Talk about problem-solving strategies the students used, and encourage them to construct viable arguments and critique the reasoning of others.

The Quilt Problem

Give each student a sheet with 4 quilt designs, each showing shaded areas. Have them figure out what fraction of the whole is shaded for each quilt. They need to write at least one equivalent fraction for each.

In whole class discussion, be sure they understand that 1/10 = 10/100; 1/5 = 2/10 = 20/100; 40/100 = 4/10 = 2/5. Ask them to justify their reasoning.

Extend this discussion to show that both operations (multiplication and division can be used to generate equivalent fractions). For example, 1/5 = 2/10 (multiply numerator and denominator by 2). Ask if 2/10 = 1/5. Then how do you get that? (division of numerator and denominator by 2). Continue with other examples.

Challenge Problem:

Candy Store: You have exactly $1 to spend (no more and no less). You can buy any assortment of candies from the ones in the jars. You want at least 3 different kinds of candies. How many of each will you get? What fraction of the $ did you spend for each kind of candy?

Closure Review outcomes of this lesson: Students understand what equivalent fractions are, and why multiplying or dividing the numerator and denominator by the same number generates these. They can represent these with visual models and that the visual representations of equivalent fractional parts of a whole need to refer to the same portion of the whole but may be arranged differently; they understand that they occupy the same place on the number line or fill the same amount of total space in an area model. They can relate fractions to parts of a dollar and to $1/100^{th}$ such that they are positioned to begin decimals.

Preview outcomes for the next lesson:

Extend the idea of generating equivalent fractions to using this as a strategy to compare fractions with unlike numerators and denominators.

Lesson 4: Equivalent Fractions Illuminations Activity Recording Sheet

Students go to the Illuminations site http://illuminations.nctm.org/ActivityDetail.aspx?ID=80;
Be sure you are in *Build Your Own*
Make the fractions below in the top square by using the sliders. Then make equivalent fractions in each of the two squares at the bottom and record these fractions here. Be sure to check the number line to check your work. Then write all the equivalent fractions on the line next to the one you are given.

1/4

_____

1/3

2/3

3/4

1/9

2/5

4/16

Can you do these with the circles? Do you get the same answers?

Which is easier, why?

Look at one set of equivalent fractions. Can you find a way to tell how the numbers in the numerator and denominator are related using only numbers?

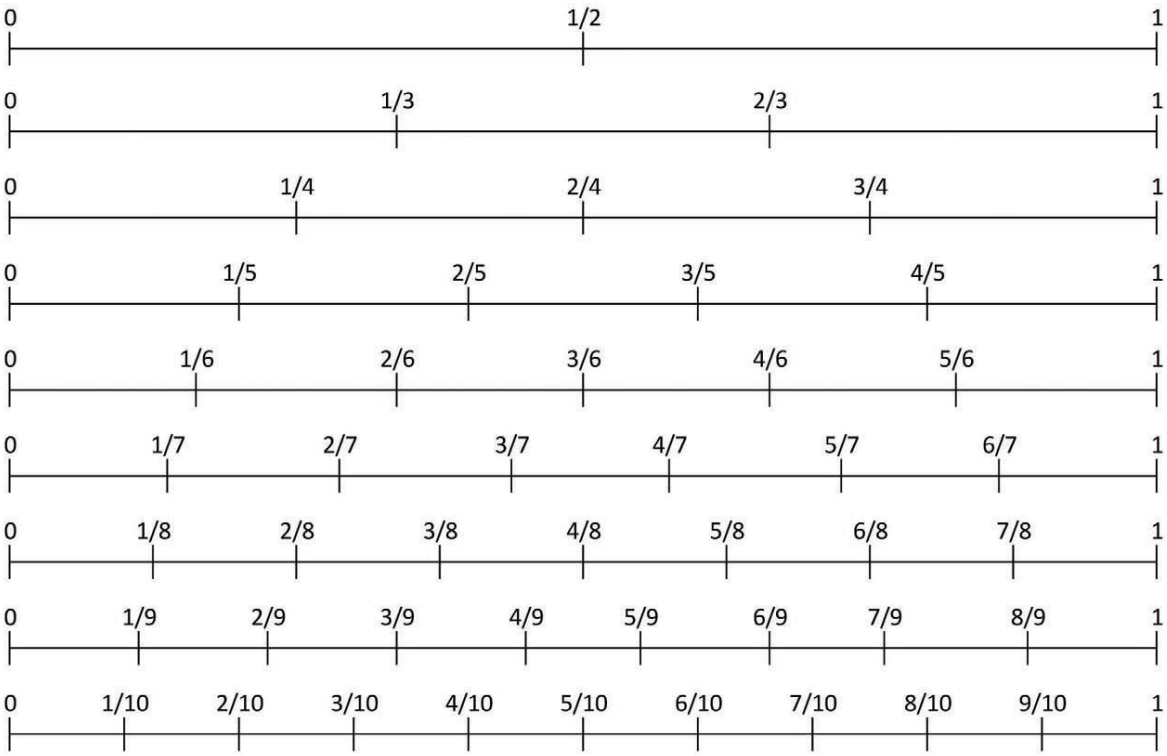
Is this true for all sets of equivalent fractions?

Can you explain why this is mathematically correct?

Extensions:
Create a fraction whose denominator is a prime number.
How many equivalent fractions can you create?

Is it possible to create a fraction such that no equivalent fractions could be created with this applet? Find at least one.
Can you describe the fractions for which this is possible?

Lesson 4: Chart of Fraction Number Lines



Lesson 4: Generating Equivalent Fractions Independent Practice

FRACTION	EQUIVALENT FRACTIONS	VISUAL REPRESENTATION PICTURE
1/2		
1/3		
4/5		
2/8		
4/10		
3/12		

Lesson 4: Money as Fractions

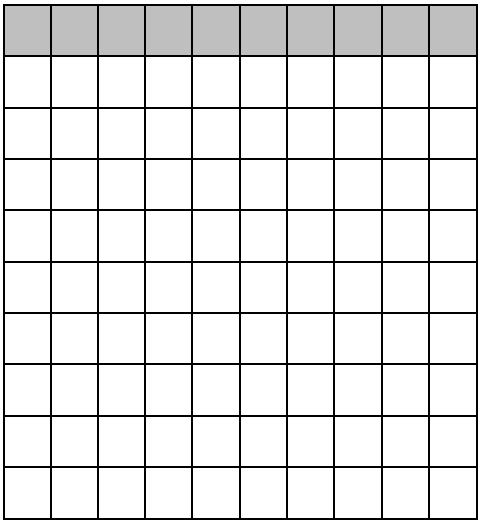
Convert these Coins or Cents to Fraction of a Dollar

List as many equivalent fractions as you can in the second column.

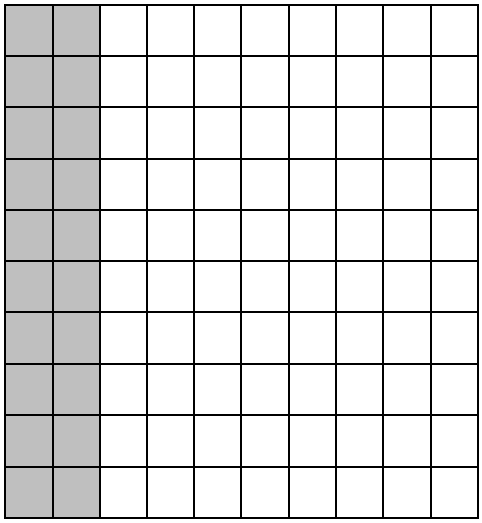
MONEY	FRACTION OF A DOLLAR
25 Cents	
75 Cents Three Quarters	
5 Cents One Nickel	
10 Cents	
Two Dimes and a Nickel	
5 Dimes	
2 Dimes and 10 Pennies	
2 Quarters and 3 Dimes	
1 Quarter, 1 Nickel and 10 Pennies	
3 Nickels and 3 Pennies	
3 Dimes and 3 Pennies	
6 Dimes, 3 Nickels and 5 Pennies	

Lesson 4: QUILT DESIGNS

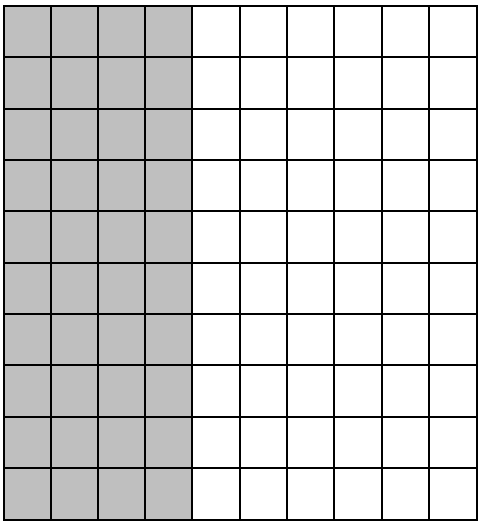
Each of the four large squares is a design for a quilt. On the line below each design, write the fraction for the part of each quilt that is shaded.



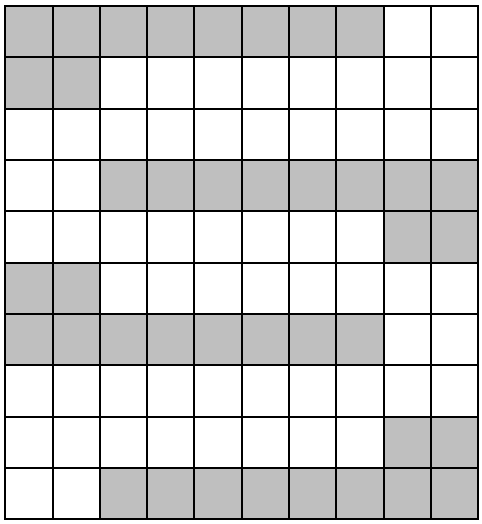
1_____



2_____



3_____



4_____

Does the same fraction of a whole always have to look the same?

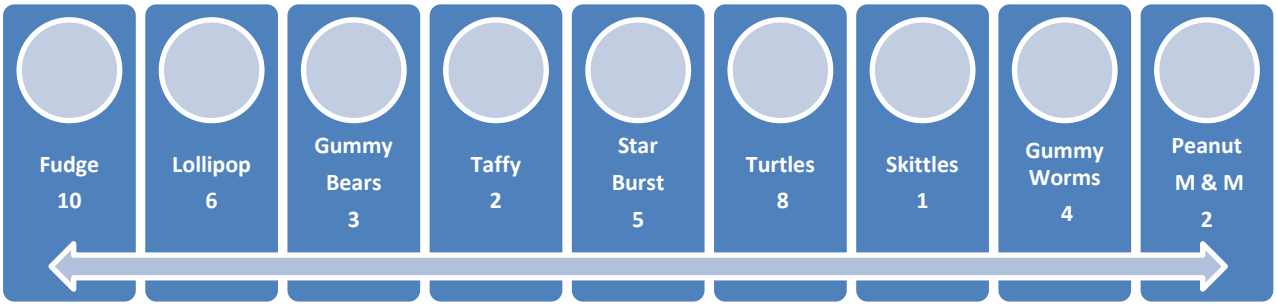
For which of the quilts was it easiest to find the fraction that was shaded? Why?

Lesson 4: Candy Store

You have exactly $1 to spend (no more and no less). You can buy any assortment of candies from the ones in the jars. You want at least 3 different kinds of candies.

How many of each will you get?
What fraction of your dollar did you spend for each kind of candy? (Be sure to list as many equivalent fractions as you can.)

Show your thinking.



Lesson 5 – Comparing Fractions with Like Denominators or Numerators

Brief Overview: In this lesson students engage in activities that help develop a deeper understanding of the value of numbers based upon their numerators and denominators. As you plan, consider the variability of learners in your class and make adaptations as necessary.

Estimated Time: 4 sessions, 60 minutes each

Resources for Lesson:

- Manipulatives
- Handout: Fractions-Finding Their Places on the Number Line
- For the CEPA: The Handout and paper for students to make a number line (sentence strip, construction paper, etc.)

Content Area/Course: Mathematics, Grade 4

Unit: Fractions: Size Matters

Time (minutes): 4 sessions, 60 minutes each

Lesson 5: Comparing Fractions with Like Denominators or Numerators

By the end of this lesson students will know and be able to:

Tell whether fractions are greater, lesser or equal to other fractions based on comparing the size of numerator or denominator; order fractions based on comparing the size of the numerator or denominator; use strategies such as creating a common denominator or numerator to compare fractions.

Essential Question (s) addressed in this lesson:

How do you know the size of a fraction and how its size compares to other fractions and whole numbers? How can I use models to compare fractions? How can I prove that a fraction is greater than, less than or equal to another fraction?

Standard(s)/Unit Goal(s) to be addressed in this lesson (type each standard/goal exactly as written in the framework):

4.NF2. Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model. (Denominators are limited to 2,3,4,5,6,8,10,12 and 100.)

SMP.2 Reason abstractly and quantitatively.

SMP.3 Construct viable arguments and critique the reasoning of others.

Instructional Resources/Tools (list all materials needed for this lesson)
Preceding lessons include standard fraction terminology (numerator, denominator, equivalent).

Anticipated Student Preconceptions/Misconceptions
A larger denominator means a larger piece.
We can compare the number of pieces without considering the size of the pieces.
We can compare the size of the pieces without considering the number of pieces.

Instructional Tips/Strategies/Suggestions:

Students should already know: how to identify and create equivalent fractions by multiplying the numerator and denominator by the same number, how to use fraction manipulatives to demonstrate the size of fractions.

Possible modifications for differentiated instruction:

- Choose fractions that only include halves, fourths, thirds and sixths
- Compare only two fractions at a time
- Include mixed numbers and improper fractions
- Compare more than three fractions at a time
- Include denominators that require a greater familiarity with multiplication facts to find a common denominator

Pre-Assessment

Students will choose three fractions from a list and put them in order from least to greatest. Then they will use pictures, numbers or words to explain how they know they are correct. Multiple representations to explain how they know they are correct helps students demonstrate communicate their own reasoning as well as understand the reasoning of others. (SMP.3: *Construct a viable argument and critique the reasoning of others*)

What students need to know and are able to do coming into this lesson (including language needs):

Students understand the meaning of numerator and denominator. Students know how to make equivalent fractions.

Information for the Teacher:

Timeline:

- Allow students to work independently for 5-10 minutes on each guiding question.
- Students may then check in with a partner for help with specific questions or just to see if they're on the right track.
- Students should continue to work independently for another 5-7 minutes until they have a solution and a model.
- Share out with partner or table (5-7 min)
- Share out with class (7-10)

Lesson Sequence:

Pre-Assessment:

5/8 4/5 8/10 2/3 5/100 4/6 10/12

Choose three fractions from the list above. Put your fractions in order from least to greatest. Use pictures, numbers or words to explain how you know you are correct.

Warm-up: Robbie is making cookies for dessert. Robbie's recipe uses 3/4 cup of milk. However, Robbie only has a 1/8 cup measuring cup. How can Robbie use his measuring cup to measure 3/4 cup of milk? Show how you solve this problem with fractions. Use pictures or words to support your answer.

Guided Practice

Teacher will use guiding questions to allow students to explore comparing fractions which have like denominators or like numerators, or which can easily be converted into fractions with like denominators.

Students will use manipulatives (fraction circles, fraction bars, pieces of construction paper) to explore fraction size.

Typical format:

Note: This format provides students opportunities to construct, communicate, demonstrate and understand each other's reasoning (SMP.3: Construct a viable argument and critique the reasoning others) as they reason abstractly and quantitatively (SMP. 2) to solve fraction problems.

- Teacher presents guiding question (examples are given on the next page).
- Students work individually, using manipulatives of their choice or paper and pencil to answer the guiding question.
- Students will create a model, either by using manipulatives or drawing a picture, to provide evidence that their conclusion is correct.
- Students will write a comparison number sentence, using correct fraction notation and a greater than, less than or equal sign.
- Students will demonstrate to a partner or small group, by using their model, that their conclusion is correct.
- Students will share selected solutions with the whole group.
- Teacher circulates among students while they are working. Some teacher questions to guide exploration might be:
    - Can you build these fractions?
    - Can you draw a picture of what's going on?
    - Can you show me with a model why you think that's true?
    - What do you notice about the size of the pieces?
    - What do you notice about the number of pieces?
    - How can you use that to help you solve the problem?
    - What do you notice about the denominators in the two fractions that you are comparing?
    - What do you notice about the numerators in the two fractions that you are comparing?
    - What do you know about equivalent fractions that might help you compare these fractions?
    - What do you know about putting these fractions on a number line that might help you compare these fractions?

Suggested Guiding Questions:

Comparing fractions with common denominators:

- Some children are eating pizza at a party. All pizzas are the same size. Ana eats 3/8 of a pizza and James ate 5/8 of a pizza. Who ate more pizza?
- Three children ran a road race to see how far they could run in 5 minutes. Franklin ran 2/6 of a mile, Karen ran 5/6 of a mile and Jose ran 4/6 of a mile. Who ran the farthest?

Comparing Fractions by Making a Common Denominator:

- Carlos and Karina had some pizza, too. Carlos ate ¾ of a pizza and Karina ate 5/8. Who ate more pizza?
- In another 5-minute race, Gabriel ran 5/6 of a mile, Sara ran 2/3 of a mile, and Stephanie ran 7/9 of a mile. Who ran the farthest?

Comparing Fractions with a Common Numerator:

- Cassidy ate ¾ of a pizza and Jayci ate 3/5. Who ate more pizza?
- In a 3-minute race, Rebecca ran 3/8 of a mile, Oscar ran 3/6, and Stephanie ran 3/12 of a mile. Who ran the farthest?

Comparing Fractions by Making a Common Numerator:

- George ate 4/7and Luis ate 2/5. Who ate more pizza?
  In 6 minutes, Hunter ran 6/7 of a mile, Meagan ran 2/3 of a mile and Mac ran 3/5 of a mile. Who ran the farthest?

Fractions _Finding Their Places on the Number Line

This worksheet is designed for students to use a variety of methods to compare fractions: benchmarks, generating equivalent fractions, reasoning about the size based on the denominator, etc..

This activity can be done as a "Think, Pair, Share" or as an assessment. The teacher may want to review this as a whole class, with students presenting their reasoning.
(incorporating Math Practices 2 and 3).

Closure

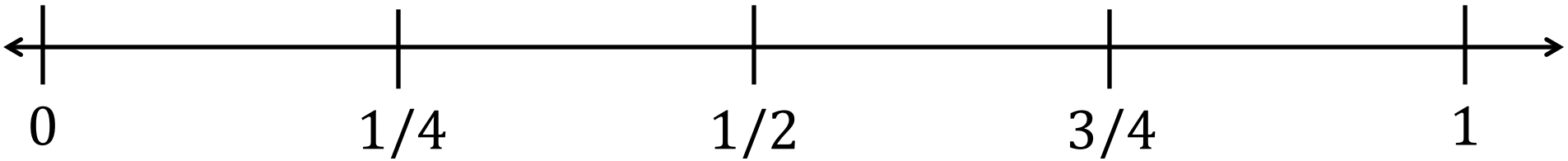
Review outcomes of this lesson: Students are able to compare fractions with unlike numerators and denominators by generating an equivalent fraction with either same numerator or denominator. Students can justify their answers in multiple ways with visual representations, relationships to benchmark fractions, etc.

Summative Assessment

CEPA Picture Day at Insect Street School

Lesson 5: Fractions – Finding Their Places on a Number Line

Place the fractions in the boxes below on the number line. Use the space in the boxes to tell how you know where they go. Be specific. Use words like: "exactly halfway between . . . ; a little closer to . . ."



1/8	3/5	2/3	11/12

Curriculum Embedded Performance Assessments and Rubrics for "Fractions: Size Matters"

CEPA 1: Equal Shares – Party Day

Part 1

Your group has won the attendance award for the month of March and the principal is giving you a party with cake. There are 12 students in the class. They are seated at 4 tables. Each table seats 3 children. Each table has 1 cake and all of the cakes are the same size. No more cuts can be made to any pieces of cake.

- The blue table's cake and the green table's cakes are cut into 3 equal pieces.
- The red table's cake is cut into 6 equal pieces and
- The yellow table's cake is cut into 12 equal pieces.

No cake can be given to any other table and all the cake must be shared.

You need to tell what fraction of a cake each child will get and whether each child gets an equal portion. You must use a visual model to prove your answer.

Part 2

Zachary sat at the blue table. He says that he got fewer pieces of cake than Sam did at the red table and that's not fair because more pieces of cake must be more cake. Convince Zachary that it is fair.

Part 3 –Challenge

Your class also won the award for April. However, this time the green table's cake was cut into 8 pieces. The teacher claims there is still a way to share the cakes so that each student in the class gets an equal portion of cake. Some of your classmates disagree. It is your job to determine who is correct and prove it to the class and the teacher. **Note: No more cuts can be made to any cakes.**

Rubric for Equal Shares Assessment (CEPA 1)

	4	3	2	1
Visual Representation	Accurately shows that the cakes are the same size and each fractional piece within each cake is equivalent to the others. The cakes are divided into the correct number of pieces for each table. The student's diagram shows how many pieces each student gets at each table 1/6 = 2/12.	Sizes and partitions of the cakes are accurate, and the representation shows the portion of the cake each student gets. How the student determined the number of pieces for each student is not clear nor is the equivalence of 1/6 and 1/12 clear.	The cakes are divided into equal parts; the amounts for each student at each table are not clear and/or accurate.	The pieces of the cakes are not equal and/or do not represent the fractions described in the problem.
Comparison of the fractions	Describes a method for demonstrating that the fractions are equal, e.g. making common denominators, explaining that the numbers and sizes of the parts differ and can be related for example 1/12 is half the size of 1/6.	Show a clear understanding that 1/6 = 2/12, and that the number of pieces is relevant only if the cakes are all the same size and the pieces are the same size. The larger the denominator the smaller the piece.	Explanation does show an understanding of the equivalence of 1/6 and 2/12. May not include a statement of the importance of the cakes and the pieces in each cake being equal.	Does not provide an explanation of the equivalence of the two fractions.
Uses precise language and vocabulary	Demonstrates an understanding of the meaning of numerator, denominator, equivalent, and greater than.	Explanation makes use of vocabulary involving fractions. Explanation makes sense but is not related to the diagrams.	Explanation and language are given, but lack precision.	Explanation lacks precision and accuracy. Vocabulary is not used correctly.
Reasoning and arguments presented demonstrate an understanding of the comparison of fractions.	Comparison is explained in a logical way and is related to the visual representation. The challenge problem is discussed and the student's reasoning is evident.	Comparison is explained in a logical way and is related to the visual representation for parts 1 and 2.	Some understanding is evident in the explanation, but it is incomplete.	Thinking and reasoning are not evident.

CEPA 2: Fraction Comparisons - Picture Day at Insect Street School

Insect	Length (in inches)
Angela Aphid	5/8
Bruno Bee	9/10
Caitlyn Cicada	2/4
Draco Dragonfly	1/8
Ella Earwig	6/12
Francisco Firefly	1/3
Gus Grasshopper	3/8
Hana Hornet	4/6
Iverson Inchworm	4/4
Jaklyn Jewel-Beetle	1/4
Kelvin Katydid	11/12
Lizette Lacewing	2/3
Marcos Mayfly	3/4
Neville Gnat	7/8
Orla Owlfly	5/6

It's Picture Day at Insect Street School. The photographer has asked the student insects to line up in order of length. He wants the shortest insect first and the tallest insect last. He's busy taking pictures so it's your job to have the insects in order so he can take pictures as quickly as possible. Here is a list of insects and their lengths. Your job is to arrange the insects by length from least to greatest.

A. Make a list of insects in order from least to greatest. Note: Some students may be the same length.

B. Using a blank sentence strip make a number line. Fold the number line to place the numbers and benchmark fractions 0, ¼, ½, ¾, and 1 on the number line. Place the insects in the right place on the number line.

C. Explain how you know where to place Gus Grasshopper on the number line.

D. Phillip Praying Mantis (Phillip is 7/10 inches long) arrives just after the photographer has taken Ella Earwig's picture. The photographer says Phillip has missed his chance because he's shorter than Ella. Do you agree or disagree? Justify your answer.

Rubric for Picture Day at Insect Street School (CEPA 2)

	4	3	2	1
Ordering of Fractions	All fractions are correctly ordered	Most (9-12) fractions are correctly ordered	Some (7-8) fractions are correctly ordered	Fewer than 6 fractions are correctly ordered
Visual Representation	Number line is accurate and fractions are correctly spaced.	Number line is accurate and most fractions are correctly spaced.	Number line is only partly accurate and some fractions are correctly spaced	The number line and/or spacing of fractions is totally or mostly inaccurate
Uses precise language and vocabulary	Explanation makes use of vocabulary involving fractions that is precise, accurate and complete.	Explanation makes use of vocabulary involving fractions but may lack some precision or completeness.	Explanation and language are given, but lack precision or completion.	Explanation lacks precision, completion and accuracy. Vocabulary is not used correctly.
Reasoning and arguments presented demonstrate an understanding of the comparison of fractions.	Comparison is complete, explained in a logical way and is related to the visual representation, benchmarks and equivalents. Student shows more than one way of explaining his or her thinking.	Comparison is explained in a logical way and is related to the visual representation, benchmarks and/or equivalents.	Some understanding is evident in the explanation, but it is incomplete.	Thinking and reasoning are not evident.