At RPDP, we support educators through professional development. Professional development can occur in a variety of ways: Entire staff trainings, grade level meetings, one-on-one support, etc. We collaborate with administrators and teachers regarding the developing and strengthening math content knowledge, use of best practices in the classroom, we model lessons, and provide support for the use of quality instructional materials.

Providing educators with quality resources in regards to instructional materials is a continuous priority. We provide this support through math content overviews, the use of instructional materials, further practice/skill development materials, and through quality assessments/tasks. As we work to create these resources for educators, we may recommend other quality resources from time to time.

In recent years, some states have received funds to create quality instructional materials for ALL educators for ALL states to access. We have selected some of those materials that we believe support our vision of quality instructional materials that support teachers in providing a solid mathematical foundation for students. For more elementary math resources please visit Rpdp.net .

Building Fractions from Unit Fractions

Applying and Extending Understandings of Operations on Whole Numbers

Grade 4 Fractions

The emphasis of this Grade 4 unit is developing proficiency with addition and subtraction of fractions with like denominators and multiplication of a fraction by a whole number. The emphasis is also on addition and subtraction of mixed numbers and multiplication of a mixed number by a whole number. Students will be able to compose and decompose non-unit fractions with like denominators, convert mixed numbers to improper fractions, and convert improper fractions to mixed numbers using the idea of unit fractions. Students should also be able to express addition and subtraction situations and whole number multiplication situations with unit fractions using equations and visual models.

Students will be asked to reason abstractly and quantitatively about fractions with like denominators and to construct viable arguments about the composition and decomposition of non-unit fractions with like denominators as they solve addition and subtraction problems. They will also be asked to solve a range of addition and subtraction situations using equations and visual models.

The standards as organized in the Massachusetts Curriculum Framework for Mathematics provide a logical sequence for the school year, beginning with Operations and Algebraic Thinking and Number in Operations in Base Ten and then moving to Number and Operations—Fractions. The fraction unit should be placed sometime after units teaching multiplication and division and factors and multiples. It assumes that students have experience working with a range of whole number addition, subtraction, and multiplication situations.

These Model Curriculum Units are designed to exemplify the expectations outlined in the MA Curriculum Frameworks for English Language Arts/Literacy and Mathematics incorporating the Common Core State Standards, as well as all other MA Curriculum Frameworks. These units include lesson plans, Curriculum Embedded Performance Assessments, and resources. In using these units, it is important to consider the variability of learners in your class and make adaptations as necessary.

Table of Contents

Stage 1 Desired Results

ESTABLISHED GOALS **G**

4.NF.3 Understand a fraction *a/b* with *a* > 1 as a sum of fractions 1/b.

a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.

b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. *Examples: 3/8 = 1/8 + 1/8 + 1/8; 3/8 = 1/8 + 2/8; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.*

c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.

d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.

Transfer

Students will be able to independently use their learning to… **T**

Meaning

UNDERSTANDINGS **U**

Students will understand that…

U1 the meaning of addition and subtraction are the same for fractions as they are for whole numbers.

U2 any whole number, mixed number, or fraction can be decomposed into the sum of its unit fractions.

U3 fractions can be composed or decomposed into unit fractions where the denominators are the same.

U4 when fractions are composed or decomposed, the result may be a mixed number, which is composed of whole number and a fraction.

U5 mixed numbers lie between whole numbers on a number line.

U6 the meaning of multiplication can be extended and applied to the multiplication of a fractions or a mixed number by a whole number.

ESSENTIAL QUESTIONS **Q**

Q1 What is the relationship between the addition and subtraction of whole numbers and the addition and subtraction of fractions?

Q2 What is the relationship between multiplication of whole numbers and the multiplication of a faction or mixed number by a whole number? [Combine these]

Q3 How do I know my answer makes sense?

Q4 How do models and strategies used for whole number computation play out in fraction computation?

4.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction by a whole number. a. Understand a fraction a/b as a multiple of $1/b$. *For example, use a visual fraction model to represent $5/4$ as the product $5 \times (1/4)$, recording the conclusion by the equation $5/4 = 5 \times (1/4)$.* b. Understand a multiple of a/b as a multiple of $1/b$, and use this understanding to multiply a fraction by a whole number. *For example, use a visual fraction model to express $3 \times (2/5)$ as $6 \times (1/5)$, recognizing this product as $6/5$.* (In general, $n \times (a/b) = (n \times a)/b$.) c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. *For example, if each person at a party will eat $3/8$ of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?* **SMP. 2** Reason abstractly and quantitatively **SMP. 3** Construct viable arguments and critique the reasoning of others. **SMP .4** Model with mathematics. **SMP. 6** Attend to precision. **4.RI.7** Interpret information presented	***Acquisition***	
	Students will know… **K**	***Students will be skilled at…*** **S**
	K1 which forms of fractions are most useful in problem solving (e.g., mixed number vs. improper fraction, equivalent fraction). **K2** that a mixed number has an equivalent improper fraction and vice versa. **K3** why the denominator stays the same when combining fractions or multiplying a fraction by a whole number. **K4** how to justify composition and decomposition of fractions using models when adding, subtracting, or multiplying by a whole number. **K5** how to explain solutions to problems that includes fractions. **K6** academic vocabulary – unit fraction, mixed number, improper fraction, proper fraction, equivalent fractions.	**S1** converting mixed numbers to improper fractions and improper fractions to mixed numbers. **S2** relating decomposition and composition of fractions and mixed numbers to visual models. **S3** creating visual models to show the addition and subtraction of fractions and mixed numbers with like denominators. **S4** solving addition and subtraction problems that include fractions and writing equations to represent them. **S5** creating visual models to show the multiplication of a fraction or a mixed number by a whole number. **S6** solving problems that include the multiplication of a fraction or a mixed number by a whole number and writing equations to represent them.

visually, orally or quantitatively and explain how the information contributes to an understanding of the text in which it appears. **NOTE:** Fractions in the 4th grade standards include only those with denominators of 2, 3,4,5,6,7,8,10,12 and 100.		

Stage 2 - Evidence

Evaluative Criteria	Assessment Evidence
Rubric for Snack Mix	**CURRICULUM EMBEDDED PERFORMANCE ASSESSMENT (PERFORMANCE TASKS) PT** **Snack Mix Problem** • **Students create 2 different recipes for snack mix by combining fractional amounts of ingredients to equal 5 cups.** • **Students create a shopping list to show how many packages of each ingredient they need to buy (packages contain different fractional and mixed number amounts)** • **Students answer questions and justify their responses for questions involving the addition and subtraction of fractions and mixed numbers, and the multiplication of a fraction by a whole number.**
	OTHER EVIDENCE: **OE** **Each lesson includes a short exit task.**

Stage 3 – Learning Plan

Summary of Key Learning Events and Instruction

It is assumed that this unit will be preceded by the unit on Comparison of Fractions. (Fractions: Size Matters covering Standards 4.NF.1 and 4.N.F.2). If not, a pre-assessment of the mastery of these concepts should be done.

Lesson 1: Decomposing Fractions (KC)

- **Given a "non-unit" fraction, how many ways can you decompose it?**
- **How close to 1 are you?**

- **Representing decomposition and re-composition of fractions using number lines, area models, and other representations.**
- **Using fraction equations**

Lesson 2: Adding and Subtracting Fractions with the Same Denominator (KC)

- **Adding and subtracting fractions with the same denominator involves adding numerators while keeping the same denominator.**
- **Strategically decomposing and recomposing fractions while adding and subtracting.**
- **Using fraction equations, number lines, area models, and other representations that reflect the addition and subtraction.**

Lesson 3: Fractions Greater Than One (CL)

- **Fractions greater than one can be written as whole numbers or mixed numbers using the idea of unit fractions. Mixed numbers can be written as fractions using the idea of unit fractions.**

Lesson 4: Addition and Subtraction of Mixed Numbers and Fractions (JZ)

- **Addition and subtraction of fractions and mixed numbers with the same denominators can be shown using number lines, area models, and other representations.**

Lesson 5: Multiplication of a Fraction by a Whole Number (JZ)

- **The multiplication of a fraction or mixed number by a whole number can be shown using number lines, area models, and other representations.**

Bank of Fraction Story Problems Addressing Fraction Addition, Subtraction, Multiplication by a Whole Number Using Identified Addition, Subtraction, and Multiplication Situations

CEPA

Adapted from Understanding by Design 2.0 © 2011 Grant Wiggins and Jay McTighe Used with Permission
July 2012

Lesson 1: Decomposing Fractions

Brief Overview of Lesson: This lesson extends the understanding of unit fractions used to compose larger fractions. Students will compose and decompose fractions into the sum of the parts. This lesson teaches the understanding of addition, subtraction, and multiplication (by a whole number) with unit fractions. It reinforces the concept of a whole being equal to any fraction a/a (for a > 0). Students use composition and decomposition of unit fractions as well as equivalent fractions to understand and operate with fractional parts. As you plan, consider the variability of learners in your class and make adaptations as necessary.

Prior Knowledge Required:

- Understand that a fraction 1/b is a quantity formed by 1 part when a whole is partitioned into b *equal* parts and that a/b is the quantity formed by a parts of size 1/b.
- Understand how to compare and order fractions
- Understand how to generate equivalent fractions

Estimated Time (minutes): approximately 60 minutes

Resources for Lesson:

- Manipulatives - fraction sets of your choice
- Five cards, labeled with 1/5 on each and masking tape
- Homework sheet

Unit: Building Fractions from Unit Fractions -

Applying and Extending Understandings of Operations on Whole Numbers
Content Area/Course: Mathematics Grade 4
Lesson 1: Decomposing Fractions
Time (minutes): approximately 60 minutes

By the end of this lesson students will know and be able to (write out clear and concise objectives for this lesson):

- Locate unit fractions on a number line.
- Represent fractions as visual representations, including area models and number lines
- Understand that fractions can be represented as numbers, part of a whole, and a point on a number line.
- Be able to compose and decompose fractions fluently.
- Understand that a fraction is made up of its unit parts.
- Use fraction equations to represent fraction situations.

Essential Question(s) addressed in this lesson:
What is the relationship between the addition and subtraction of whole numbers and the addition and subtraction of fractions?

Standard(s)/Unit Goal(s) to be addressed in this lesson (type each standard/goal exactly as written in the framework):

4.NF.3.a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.

SMP. 3 Construct viable arguments and critique the reasoning of others.

SMP. 4 Model with mathematics.

Instructional Tips/ Strategies/Notes for Teacher

The lessons in this unit are designed to follow this basic sequence:

1) Pre-assessment to identify what students understand about the mathematical focus of the lesson
2) Problem solving and discussion of the mathematical focus of the lesson including exploring a range of strategies and visual representations including area models and number lines
3) Reflective practice using strategies, manipulatives, and visual representations to develop fluency
4) Integration of these understandings and fluencies using two story problems

At the beginning of this lesson, be sure that students understand the numerator counts the equal parts of fractions and the denominator names the number of equal parts that the whole is divided into. (i.e. the numerator is the adjective, and the denominator the noun, 5 sixths is analogous to 5 apples).

Use concrete and representational models before abstract work.

Students need to continually see different representations of fraction models.

A common error is for students to add the denominators in as well as the numerators.

Encourage students to show their work in an organized way.

Students need to be able to support their reasoning Focus on using precise mathematical language and constructing viable arguments when writing and speaking (SMP.3 Construct a viable argument and critique the reasoning of others.).

Anticipated Student Preconceptions/Misconceptions

- Only numbers less than one are fractions.
- Students may try to add denominators as well as numerators.
- Students think of fractions as parts of wholes and do not understand where to locate fractions on a number line.

Lesson Sequence

1) Pre-assessment to identify what students understand about the mathematical focus of the lesson
2) Problem solving and discussion of the mathematical focus of the lesson with attention to a range of strategies and visual representations including area models and number lines
3) Reflective practice using strategies, manipulatives, and visual representations to develop fluency
4) Formative assessment: integration of these understandings and fluencies using two story problems

Pre-Assessment/Warm-up:

Prior knowledge: name fractional parts of a whole. Have each student draw a diagram of three-fourths of a region on an individual whiteboard. The students should also identify the numerator and the denominator and explain what each represents.

Show fractions on a number line by making a human number line. Tape cards with $\frac{1}{5}$ written on them to the left shoulders of 5 students. Refer to the children as "unit fractions". Have one of the "unit fractions" children stand with his shoulder facing the crowd in front of a number line drawn on the board. Place a mark along with the fraction $\frac{1}{5}$, on the line where the first "unit fraction" child is standing. Have the next $\frac{1}{5}$ "unit fraction" come up to the board and put his hands on the shoulders of the other $\frac{1}{5}$. Mark the length of the two $\frac{1}{5}$'s as $\frac{2}{5}$ on the board.

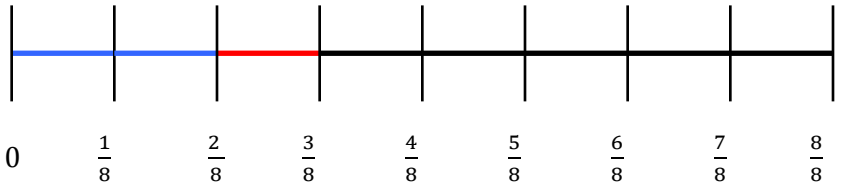
Do the same with the next three $\frac{1}{5}$' s and mark them consecutively the number line. Students are then asked to make observations.

Instructional Activities

Students will decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition with an equation. They will justify the decompositions using an area model and a number line model. SMP.3 (Construct viable arguments an critique the reasoning of others.)

Examples: $\frac{3}{8} = \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$; $\frac{3}{8} = \frac{1}{8} + \frac{2}{8}$; $2\frac{1}{8} = 1 + 1 + \frac{1}{8} = \frac{8}{8} + \frac{8}{8} + \frac{1}{8}$

Possible Number Line Model for 3/8 = 2/8 + 1/8:



Students will be given sets of fractions. They will have one whole, two halves, three thirds, four fourths, six sixths, and eight eighths that they will then work with a partner to decompose.

The group will reconvene to compare their answers as the teacher writes the results on chart paper. The results should be

$\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{4}{4}$; $\frac{1}{4} + \frac{1}{4} + \frac{2}{4} = \frac{4}{4}$; $\frac{1}{4} + \frac{3}{4} = \frac{4}{4}$; $\frac{2}{4} + \frac{2}{4} = \frac{4}{4}$.

Students will be asked to discuss the patterns they see in the decompositions.

Students will then be asked to work with a partner to find all of the ways to decompose 6/8. Point out that this time we are decomposing a fraction that is not a whole to start out but the process is the same.

Students will use manipulatives, as needed, and write the equations on paper. Students will meet again to share their answers. The teacher will add this information to the chart paper in an organized manner. The teacher will then ask students how all of this would look as fractions on a number line. She will draw a number line on the chart paper and have students show how these decompositions would look. (SMP.2: Reason abstractly and quantitatively.)

Formative Assessment:

Present problems that also assess the Standards of Mathematical Practice together with the content standard(s). For example, in the following problem when students draw a model or number line and write the equation using their knowledge of decomposition of fractions, SMP.4 (Model with mathematics) is also being assessed.

I have $\frac{6}{10}$ of a candy bar, how many different ways can I break this into pieces? Write as many equations as you can to show the sums of fractional pieces that equal $\frac{6}{10}$.

For example: $\frac{2}{10} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10} = \frac{6}{10}$

Draw an area model or number line representation for each equation.

Homework: Building Fractions Lesson 1 Homework Sheet

Lesson 2 Adding and Subtracting Fractions With the Same Denominator

Brief Overview of Lesson Adding and Subtracting Fractions with the Same Denominator

As you plan, consider the variability of learners in your class and make adaptations as necessary.

Prior Knowledge Required: Unit fractions

Estimated Time (minutes): 60 minutes

Resources for Lesson (list resources and materials):

- Fraction circles divided into 8 equal pieces
- Fraction manipulatives
- Pre-assessment worksheet
- Classwork problems
- Homework Sheet

Unit: Building Fractions
Content Area/Course: Grade 4 Mathematics
Lesson 2 Adding and Subtracting Fractions with the Same Denominator
Time (minutes): 60

By the end of this lesson students will know and be able to:
Add and subtract fractions with like denominators, understanding why they are performing these operations on only the numerators.
Use manipulatives and multiple representations to demonstrate this understanding.

Essential Question(s) addressed in this lesson:
What is the relationship between the addition and subtraction of whole numbers and the addition and subtraction of fractions?

Standard(s)/Unit Goal(s) to be addressed in this lesson:

4.NF.3.a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.
SMP. 3 Construct viable arguments and critique the reasoning of others.
SMP.6 Attend to precision.

Instructional Tips/ Strategies/Notes for Teacher

It is important to emphasize that a fraction 1/b is a quantity formed by 1 part when a whole is partitioned into b *equal* parts and that a/b is the quantity formed by a parts of size 1/b and how this connects to what is a numerator and what is a denominator. .

Use different fraction models and representations throughout so that students understand there are a number of ways to model or represent fractional quantities and how they are added and subtracted.

It is important to explore both take away *and* comparison subtraction examples with connections to whole number subtraction strategies for each.

The Grade 4 standards call for students being able to add and subtract fractions with like denominators. However, it would be helpful for students to understand that fractions with unlike denominators can also be added.

It's crucial for students to understand that when fractions are added or subtracted the denominators don't combine.

Students need to be able to support their reasoning about fractions. Focus on using precise mathematical language and constructing viable arguments when writing and speaking (SMP. 3, SMP. 6).

Anticipated Student Preconceptions/Misconceptions

When adding and subtracting fractions, both the numerators and the denominators are added or subtracted. You can't add or subtract fractions unless they have the same denominator. Students may forget to pay attention to what the whole is when you are working with more than one fraction.

Instructional Model

1) Pre-assessment /Warm-up to identify what students understand about the mathematical focus of the lesson
2) Problem solving and discussion of the mathematical focus of the lesson with attention to a range of strategies and visual representations including area models and number lines
3) Reflective practice using strategies, manipulatives, and visual representations to develop fluency
4) Post-assessment exploring connections between fraction addition and subtraction and whole number addition and subtraction.

Pre-Assessment/Warm-up:

Students form groups of three. Each group has 2 circles divided into eight slices. First, the first student colors 2 parts of the pie on the first circle and writes the fraction he colored. The second child colors 3 parts of the same pie and writes the fraction she colored. Student 3 then writes the total fraction for all the shaded parts. Next, the first student colors in any amount of fractional parts he wants between 1 and 7 pieces on the second pie. He then writes down the fraction he colored. The second child colors any remaining pieces of the same pie that she chooses. She also writes down the fractional parts she colored. Student three writes the total fraction for all of the shaded parts. Together they write an equation for all of it. Students then share their ideas with the class. See worksheet below.(a)

Lesson Sequencs

The focus of the lessons is on joining and separating fractions. Students add and subtract fractions as joining and separating parts referring to the same whole.

Problem One: Pose the first part of this two-part problem:

a) For Juan's birthday party his parents bought a large pepperoni pizza and a large cheese pizza for the guests to enjoy. Both were the same size and each was cut into eight equal pieces. On Saturday night after Juan's birthday party was over, there was $\frac{3}{8}$ *of a large pepperoni pizza and* $\frac{2}{8}$ *of a large cheese pizza left. Juan's mom put it all in one box. How much pizza was left altogether?*

Ask students to work with a partner to solve this problem. They should be sure to use a model or representation to find or justify their solution. They should also write an equation that represents their work.

As students work on the problem, circulate with the following questions in mind:

- What's the action in the problem?
- What seems like a reasonable answer?
- How can you justify your solution?
- How does your model or representation show your solution?
- How does your equation show your solution?
- Why doesn't the denominator change when we add fractions?

Note: These questions are posed such that students construct viable arguments and critique the reasoning of others (SMP.3).

Strategically identify several partners to share their strategies and representations keeping in mind the range of approaches you want students to consider, particularly those using area models and number lines. Discuss how they are the same and how they are different.

Then pose the second part of the problem:

b) On Sunday morning Juan snuck downstairs and ate a $\frac{1}{8}$ *piece from the leftover pizza. How much was left after he had his morning snack?*

Briefly check in with students about their solutions and how they found them.

Problem 2: *Melissa was swimming. She swam* $\frac{2}{6}$ *of a mile and took a break. Then she swam* $\frac{3}{6}$ *of a mile. How far did she swim in all?*

Ask students to demonstrate their thinking for this problem on a number line. Show then that they can jump on a number line when adding and subtracting fractions the same way they can jump on a number line when adding and subtracting whole numbers. Have students write an equation to match the story problem.

Pose the attached additional story problems involving the addition and subtraction of fractions with the same denominators. Ask students to use a model or representation to find or justify their solution and write an equation that represents their work. Encourage students to try strategies, models, or representations that were shared and discussed that they found helpful. Also provide some practice adding and subtracting fraction equations. Some students may need a number line or manipulative to help them with this. Students should be encouraged to use strategies from whole number computation to explore fraction computation. Examples: decomposing and recomposing fractions to make the computation easier or using addition to solve fraction subtraction problems.

Formative Assessment:

Quick write: have student describe how the addition and subtraction of fractions compares to the addition and subtraction of whole numbers.

Exit Ticket: To monitor their fluency, as students leave the room, have them add or subtract the fractions you say to them.

Homework: Building Fractions Lesson 2 Homework Sheet

Building Fractions –Grade 4 Lesson 2 Pre-assessment Worksheet

Student #1: Color 2 parts of the pie in blue and name the fraction.

Student #2: Color 3 parts of the same pie in red and name the fraction.

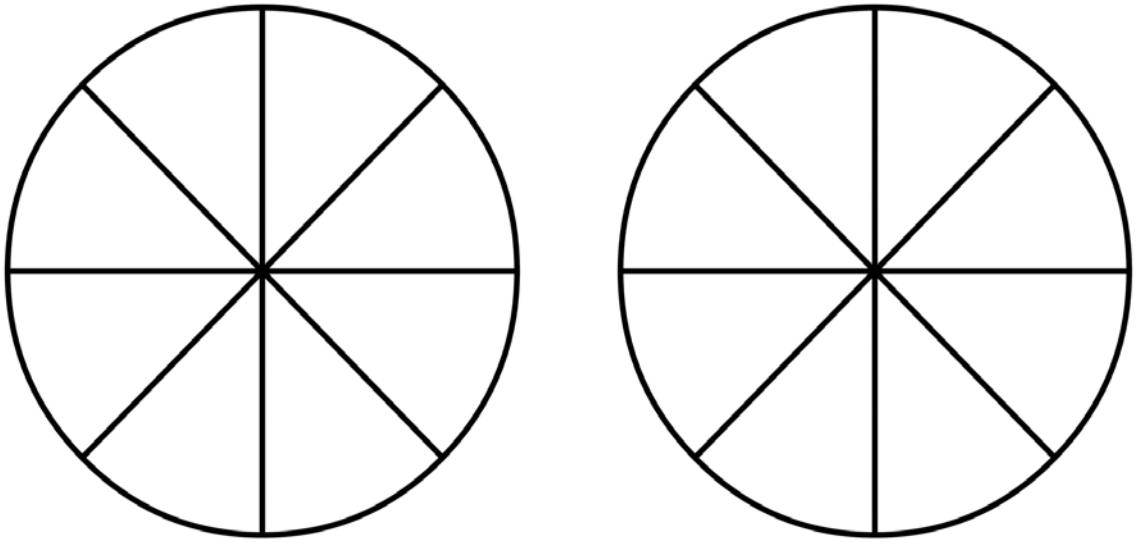
Student #3. Write an equation for the total fraction of the pie that is now colored.

Student #1 : You choose. Color any number of the parts of the other pie from one to seven in blue and name the fraction.

Student #2. You choose. Color any number of leftover parts of the same pie in red that student #1 did not color and name the fraction.

Student #3. Write an equation for the total fraction of the pie that is now colored.

All students: What did you notice about the equation that was written? Write what you notice.



Name_____ Date_____

Homework

Solve each story problem. Use a model or representation to find or justify your solution and write an equation that represents your work.

1. Sarah and Mary are going to the deli for their mom. She wants them to buy $\frac{5}{8}$ of a pound of ham and $\frac{2}{8}$ of a pound of turkey. How much meat will they buy in all at the deli? Write your answer in its simplest form.

2. When they get home their mom makes sandwiches and uses $\frac{3}{8}$ of a pound of the meat. How much is left? Write your answer in its simplest form.

3. Sarah says she bought more than a half of a pound of meat. Mary said Sarah is wrong. Who is correct and why? Explain how you know.

4. How much more meat did Sarah buy than Mary? Write you answer in its simplest form.

Classwork

Name_____

1. For each of these problems do the following.
   a. Create a model or representation to find or justify your solution.
   b. Write an equation that represents your work.

Jane has $\frac{3}{6}$ of a pound of butter and her mom has $\frac{1}{6}$ of a pound of butter to make cookies. How much do they have in all? **	Joey runs $\frac{3}{5}$ of a mile on Monday and another $\frac{1}{5}$ of a mile on Tuesday. How far did he run all? *
There is $\frac{7}{8}$ of a pie left in the refrigerator. Dad comes along and eats $\frac{2}{8}$ of it. How much is left? ***	Alice has $\frac{3}{4}$ her Hershey's candy bar left. Kathy has $\frac{1}{4}$ of hers. How much more does Alice have than Kathy? ****

Jose has $\frac{1}{3}$ acre smaller plot of land than Juan. Juan has $1\frac{2}{3}$ acre plot of land. How large is Jose's plot of land? *****	There was $\frac{5}{6}$ of a pizza left on the counter. I ate some of it. There was $\frac{2}{6}$ left. How much did I eat? ******
There was an orange on the table that was originally cut into eight equal pieces. Now it is gone. If Luis and Sheldon each had some, write all of the possible fractional ways they could have split it. #	Mrs. Rivera split the rest of the brownies that were in a pan with the children at lunch. Table #1 got $\frac{5}{12}$ of the pan of brownies. Table #4 got $\frac{6}{12}$ of the brownies. Now they are all gone. How much of the brownie pan was filled when she started to give them out? # #

Julie had $\frac{4}{5}$ more feet of ribbon to make a bow than Ashley. They have $2\frac{1}{5}$ feet altogether. How many feet of ribbon does Ashley have? ###	Joanne has $\frac{4}{3}$ of a yard of rope. Linda has an equal amount. Carol has the same amount that Joanne and Linda have combined. How much do they have altogether? ####
On Monday morning the bucket in the backyard had $\frac{1}{8}$ inch of rainwater in it. At the end of the week it was $\frac{7}{8}$ inches deep with water. How much rain fell during the week? #####	Georgiana has a dozen roses she got from her mom and dad for graduation. If she puts $\frac{4}{12}$ of the roses in her pink vase. How could she split the rest between her purple vase and her yellow vase? Is there more than one way? ######

Teacher Notes for the Classwork Question Bank.

This is a bank of questions for your use. You may judge some questions more difficult than others. You may use as many as you choose or you may choose to differentiate your instruction as you see fit in your classroom.

These are the addition and subtractions situations for each problem. The situations should be varied so students develop fluency with the range of problem situations.

* Add to-result unknown

**Put together- total unknown

***Take from- result unknown

****Compare- difference unknown

*****Compare- smaller unknown

******Take from –change unknown

Both addends unknown

##Take from- start unknown

###Compare – larger unknown

####Add to – result unknown and some addends unknown

#####Add to – change unknown

######Take apart- both addends unknown

Lesson 3: Mixed Numbers

Brief Overview: This lesson extends the understanding of operations with unit fractions to mixed numbers; it reinforces the concept of a whole being equal to any fraction $\frac{a}{a}$ (for a > 0). As you plan, consider the variability of learners in your class and make adaptations as necessary.

Estimated Time: approximately 60 minutes

Resources for Lesson:

Manipulatives including Cuisinaire rods
Handout on Mixed Numbers
Index cards or Mixed Number and Fraction matching cards.
Building Fractions Lesson 3 Homework Sheet

Content Area/Course: Grade 4 Mathematics

Unit: Building Fractions from Unit Fractions

Lesson #3: Mixed Numbers

Brief Overview: This lesson extends the understanding of operations with unit fractions to mixed numbers; it reinforces the concept of a whole being equal to any fraction $\frac{a}{a}$ (for a > 0). Students use composition and decomposition of unit fractions as well as equivalent fractions to understand and manipulate mixed numbers.

Estimated Time: approximately 60 minutes

Resources:

Manipulatives – Cuisinaire Rods, Construction paper rods or NLVM Fraction Bars
http://nlvm.usu.edu/en/nav/frames_asid_203_g_2_t_1.html?from=category_g_2_t_1.html
Worksheet **L3-1**
Mixed Number Cards

By the end of this lesson students will know and be able to:

- Understand what we mean by a mixed number
- Represent mixed numbers as visual diagrams, on a number line, and as a fraction with a numerator greater than its denominator.
- Understand why any fraction with a numerator greater than a denominator is greater than 1.
- Be able to fluently move back and forth between mixed numbers and fractions.

Essential Question addressed in this lesson:

What is the relationship between the addition and subtraction of whole numbers and the addition and subtraction of fractions?

Standard(s)/Unit Goal(s) to be addressed in this lesson:

4. NF.3.b Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition

by an equation. Justify decompositions, e.g., by using a visual fraction model. *Examples:* $\frac{3}{8} = \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$; $\frac{3}{8} = \frac{1}{8} + \frac{2}{8}$; $2\frac{1}{8} = 1 + 1 + \frac{1}{8} = \frac{8}{8} + \frac{8}{8} + \frac{1}{8}$

SMP.3 Construct viable arguments and critique the reasoning of others
SMP.6: Attend to precision

Anticipated Student Preconceptions/Misconceptions

- Only numbers less than 1 are fractions.
- To convert a mixed number to a fraction, one adds the whole number to the numerator.

Instructional Model

1) Pre-assessment /Warm-up to identify what students understand about the mathematical focus of the lesson
2) Problem solving and discussion of the mathematical focus of the lesson with attention to a range of strategies and visual representations including area models and number lines
3) Reflective practice using strategies, manipulatives, and visual representations to develop fluency
4) Post-assessment exploring connections between fraction addition and subtraction and whole number addition and subtraction.

Instructional Tips/Strategies/Suggestions:

It is important to ask students to use the idea of a unit fraction when moving back and forth between mixed numbers and fractions. An important question to address is how many unit fractions there are in a whole. It should be noted that the framework does not use the term improper fraction to describe fractions greater than one. They are just called fractions.

Students need to be able to support their reasoning about fractions. Focus on using precise mathematical language (SMP.6) and constructing viable arguments (SMP.3) when writing and speaking.

Pre-Assessment

- Draw a picture of $\frac{4}{4} + \frac{1}{4}$.
- Is the sum greater than or less than 1? Explain your answer.

- Place the sum on a number line. Is it to the right or left of 1? How do you know?

What students need to know and are able to do coming into this lesson (including language needs):

- Understand a fraction $\frac{a}{b}$ with $a > 1$ as a sum of fractions $\frac{1}{b}$.
- Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.
- Understand how to decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation, and how to justify these decompositions, e.g., by using a visual fraction model. *Examples:* $\frac{3}{8} = \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$; $\frac{3}{8} = \frac{1}{8} + \frac{2}{8}$; $2\frac{1}{8} = 1 + 1 + \frac{1}{8} = \frac{8}{8} + \frac{8}{8} + \frac{1}{8}$

Pre-Assessment/Warm-up:

- Draw an area model of $\frac{4}{4} + \frac{1}{4}$.
- Is the sum greater than or less than 1? Explain your answer. For instance, someone might claim that the area model shows 5/8. Could this be right? Why or why not? Be sure to include a discussion of what is your whole.
- Place the sum on a number line. How did you decide where to place it?

Whole class discussion: How do we know where to place fractions on a number line? What are some ways to name fractions that are greater than one?

Lesson Sequence

Ask students to create an area model for $\frac{7}{4}$ and then place it on a number line. What mixed number is equivalent to $\frac{7}{4}$? How do we know? Students may work with Cuisinaire rods, construction paper bars, or the website as they construction their area model or think about placement on a number line.

Students work with several more fractions: $\frac{6}{3}, \frac{7}{3}, \frac{4}{2}, \frac{5}{2}$ using resources as needed until they develop some fluency moving back and forth between fractions and mixed numbers. Ask how these numbers compare to $\frac{3}{6}$, $\frac{3}{7}$, $\frac{2}{4}$, $\frac{2}{5}$.

Now provide students with the **Worksheet 4 NF.3. L3-1**, to complete individually. After students have worked on the problems for a while, ask them to work with a partner to share and discuss their strategies and solutions. After students have spent some time talking together about their approaches, ask them to continue to work on the problems individually, drawing on what they learned as a result of talking with their partner.

To provide opportunities for thoughtful practice, set some time aside at the end of class for students to play the Mixed Number Card Game described below.

Mixed Number Card Game (Similar to Go Fish)

Give each student 8 index cards. Have them write a different mixed number on four separate cards and the equivalent fraction on the four remaining card. Each student has someone check their cards. Students play in groups of 3 or 4. One child deals each player 3 cards and the rest in a stack in the middle face down. Players check their hands for matches; they remove those cards from their hand and set them face-up. Any player with matching cards picks another card(s) from the deck to replace the ones set down.

Player 1 converts a mixed number to a fraction or vice versa (Students can do this mentally and/or with paper) Player 1 then asks the other players if they have that card. If so, player 1 is given the matching card and the pair is now set face-up by him/her. Player 1 draws another card converts it and asks again. If no one has a match to it, then it is Player 2's turn. The one with the most cards in the end wins.

*To differentiate this activity make sets of 32 fraction cards for each group. (some with easy fractions and others challenging).

Formative Assessment: I bought 7 pizzas for my class. Each pizza is cut into 5 equal pieces.

a. How many students can get a piece of pizza? Use pictures, numbers or words to explain your answer.
b. Write the total number of pieces of pizza as a fraction and a mixed number.

Homework:

1. The cafeteria cuts each orange into four equal pieces to serve them. There are 6 oranges left for Mrs. Smith's Kindergarten class.
    a. Draw a model or representation and write the total number of orange pieces as a fraction if all 6 oranges are cut up.
    b. Today there are only 17 students in the class and each student will get a piece of orange ($\frac{1}{4}$). Mrs. Café wants to cut the smallest number of oranges, how many oranges should cut into four pieces?

c. What fraction of the oranges will be left after the orange sections have been given out? Give your answer as a fraction and as a mixed number.

Challenge:

2. Sarah and Ruby are making favors for a party. Sarah needs $\frac{32}{6}$ feet of ribbon for her baskets and Ruby needs $\frac{8}{6}$ feet for her bunches of flowers. They have 7 feet of ribbon. Is that enough for all the party favors?

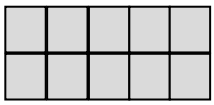
   Explain your answer. Use a model or representation to justify your solution.

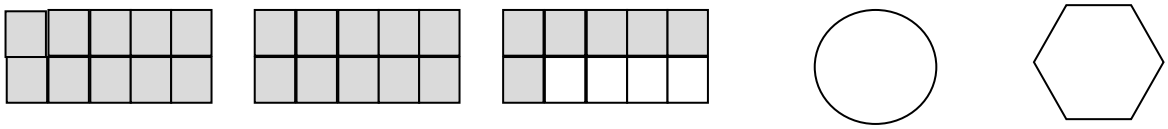
MIXED NUMBERS Handout

Building Fractions Grade 4 Lesson 3

Directions: Write fractions for the shaded area of each of these pictures. Write the fraction in the circle to the right of the picture. Write the mixed number in the hexagon. Write the fraction on the number line. Be sure to put the scale on the bottom of the number line.

Key

1 whole =  or 1 whole = 

1. 

2. 

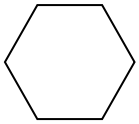
3. 

4. 

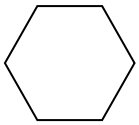
5. 

Draw a picture for each of these fractions. Write the fraction as a mixed number in the hexagon. Show the fraction on the number line.

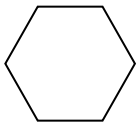
6. $\frac{7}{4}$



7. $\frac{5}{2}$



8. $\frac{2}{8}$



9. How can you tell if a fraction is greater than 1?

10. What can you say about any fraction that is to the right of 1 on the number line?

$1\frac{1}{2}$		$1\frac{3}{4}$
$1\frac{1}{3}$	$1\frac{2}{3}$	$2\frac{2}{5}$
$2\frac{3}{7}$	$2\frac{5}{8}$	$2\frac{7}{12}$
$2\frac{1}{7}$	$1\frac{6}{7}$	$2\frac{4}{5}$

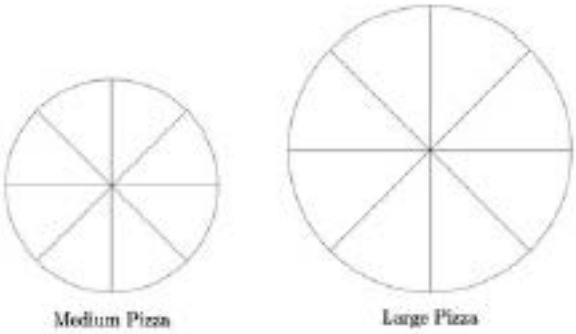
$1\frac{3}{12}$	$4\frac{1}{5}$	$3\frac{1}{6}$
$2\frac{1}{5}$	$3\frac{1}{8}$	$2\frac{1}{9}$
$4\frac{1}{10}$	$1\frac{1}{12}$	$3\frac{2}{7}$
$2\frac{2}{10}$	$3\frac{3}{5}$	$2\frac{5}{6}$

$4\frac{2}{7}$	$1\frac{9}{10}$	$2\frac{4}{10}$
$1\frac{5}{8}$	$1\frac{7}{8}$	$2\frac{4}{7}$

Name _____ Date _____

Building Fractions Lesson 3 Homework

Eva and her friends ordered two pizzas. One is a medium pizza and one is a large pizza.



Eva ate two slices of the medium pizza. Her friend Ana ate two slices of the large pizza.

They say that $1\frac{1}{2}$ of a pizza is left for everyone else.

Decide if you agree or disagree with this statement.

Agree Disagree

Explain in words why you agree or disagree with this statement.

Use diagrams to justify your reasoning.

Lesson 4: Adding and Subtracting Mixed Numbers

Brief Overview: Students will demonstrate the concept of adding and subtracting mixed numbers with a variety of visual models and representations as well as strategies that build on and extend whole number addition and subtraction. They will support their reasoning about fractions through a focus on using precise mathematical language and constructing viable arguments when writing and speaking (SMP. 3: Construct viable arguments and critique the reasoning of others, SMP. 6: Attend to precision). Finally, students will be apply whole number computation strategies to explore fraction computation, e.g., decomposing and recomposing fractions to make the computation easier or using addition to solve fraction subtraction problems. As you plan, consider the variability of learners in your class and make adaptations as necessary.

Prior Knowledge Required: Students should know how to add and subtract fractions with like denominators, how to express fractions greater than one as mixed numbers, and how to express mixed numbers as fractions.

Estimated Time: Two 60-minute lessons

Resources for Lesson:

http://mass.pbslearningmedia.org/resource/vtl07.math.number.fra.crumpetrec/crumpets-recipe/

- Rulers
- Number Lines
- Fraction Circles, Fraction Bars
- Crayons or Colored Pencils
- Sheets of Fourths and Eighths

Content Area/Course: Math Grade 4
Unit: Building Fractions from Unit Fractions
Time (minutes): Two 60 minute class periods
Lesson #4: Adding & Subtracting Mixed Numbers

Overview: Students will learn to add and subtract mixed numbers

By the end of this lesson students will know and be able to:

- Add and subtract mixed numbers
- How to express a fraction greater than one as a mixed number.
- How to express a mixed number as a fraction

Essential Question addressed in this lesson:

What is the relationship between the addition and subtraction of whole numbers and the addition and subtraction of fractions?

Standard(s)/Unit Goal(s) to be addressed in this lesson:

4.NF.3c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.
SMP.3 Constrcut viable arguments and critique the reasoning of others
SMP.6 Attend to precision

Instructional Resources/Tools

http://www.teachersdomain.org/resource/vtl07.math.number.fra.crumpetrec/

- Fraction circles
- Fractions bars
- Rulers
- Number Lines

- Printouts of fraction bars and circles

Anticipated Student Preconceptions/Misconceptions

- It is always appropriate to convert mixed numbers to fractions in order to add or subtract
- When adding or subtracting mixed numbers,

Instructional Model

1. Pre-assessment /Warm-up to identify what students understand about the mathematical focus of the lesson
2. Problem solving and discussion of the mathematical focus of the lesson with attention to a range of strategies and visual representations including area models and number lines
3. Reflective practice using strategies, manipulatives, and visual representations to develop fluency
4. Post-assessment exploring connections between fraction addition and subtraction and whole number addition and subtraction.

Instructional Tips/Strategies/Suggestions:

Students should be able to demonstrate the concept of adding and subtracting mixed numbers with a variety of visual models and representations as well as strategies that build on and extend whole number addition and subtraction.

Students need to be able to support their reasoning about fractions. Focus on using precise mathematical language and constructing viable arguments when writing and speaking (SMP. 3: Construct viable arguments and critique the reasoning of others, SMP. 6: Attend to precision)

Students should be encouraged to use strategies from whole number computation to explore fraction computation. Examples: decomposing and recomposing fractions to make the computation easier or using addition to solve fraction subtraction problems.

Pre-Assessment

Question: George, Dylan and Eli were hungry for dessert. Eli ate 2/8 of a pie, George ate $\frac{3}{8}$ of a pie and Dylan ate $\frac{4}{8}$ of a pie. How much pie did the boys eat altogether? How many whole pies did I need to feed the boys? How do you know? Justify your answer using pictures, numbers or words.

Lesson Sequence

Addition (Day 1)

Warmup:

Students will independently attempt this question: George, Dylan and Eli were hungry for dessert. Eli ate $\frac{2}{8}$ of a pie, George ate $\frac{3}{8}$ of a pie and Dylan ate $\frac{4}{8}$ of a pie. How much pie did the boys eat altogether? How many whole pies do I need to feed the boys? How do you know? Justify your answer using pictures, numbers or words.

Questions for Discussion:

- How did you determine how much pie was eaten?
- Are all the answers the same?
  - Can $\frac{9}{8}$ and $1\frac{1}{8}$ both be right?
  - How do you know that $\frac{9}{8}$ is more than one?
  - How can you write 1 whole piece as a fraction?
- How do you know how many whole pies I need?
- Will there be any pie left?

Lesson:

Watch Cyberchase video clip to demonstrate how unit fractions can be combined to make mixed numbers.
http://mass.pbslearningmedia.org/resource/vtl07.math.number.fra.crumpetrec/crumpets-recipe/

Ask students to use different models or representations to show how mixed numbers can be combined in the following problem:

We need 6 quarts of cider for a picnic. Brandon brought $2\frac{1}{4}$ quarts, Jorge brought $1\frac{2}{4}$ quarts and Mei brought $2\frac{3}{4}$ quarts. Do we have enough cider? How do you know?

Share and discuss solutions: How are they the same? How are they different? How do they all make sense? What equation can you write to go with each of these solutions? Be sure area models and

number line models are included in the discussion. What are some different ways you decomposed and recomposed the fractions to make computation easier?
Ask students to work in pairs to model and solve this problem:

Antonio Ant was crawling along a table looking for food. He crawled 3-3/4 inches and found a tasty cookie crumb for breakfast. Then he crawled $4\ \frac{2}{4}$ inches farther and found a yummy apple slice for lunch. Finally, he crawled another 2-1/4 inches and found a delicious lettuce leaf for dinner. How far had he crawled in all?

Share and discuss solutions: How are they the same? How are they different? How do they all make sense? What equation can you write to go with each of these solutions? Be sure area models and number line models are included in the discussion.

Give students this number sentence to solve: $4\frac{4}{8} + 2\frac{3}{8} = ?$

Discuss their solutions (this should be quick as the eighths combine to be less than one whole).

Next, have students solve $3\frac{4}{6} + 2\frac{3}{6} = ?$ Discuss solutions.

Have students complete a few more similar problems.

Exit Ticket (Formative Assessment): I am making several different pies for a bake sale on Saturday. I need $2\frac{7}{8}$ cups of sugar for the apple pies, $3\frac{3}{8}$ cups of sugar for the pumpkin pies and $4\frac{5}{8}$ cups of sugar for the pecan pies. How much sugar do I need in all? Use visual models or representations to show your work and write an equation that reflects your solution.

(Day 2)

Warmup: I have $4\frac{3}{4}$ cups of sugar. I use $2\frac{1}{4}$ cups. How much is left?

Share and discuss solutions: How are they the same? How are they different? How do they all make sense? What equation can you write to go with each of these solutions? Be sure area models and number line models are included in the discussion.

Lesson

Give students several subtraction problems to solve as well as some addition problems from the day before. Allow them to use any manipulative materials they choose to help them think about the problem. Have fraction circles, bars, paper copies of bars and circles, rulers and number lines available. Ask them to draw visual models or representations and include equations that reflect their solutions.

Some types of problem to include:

1. I have $4\frac{1}{4}$ cups of cat food. My cat eats $\frac{2}{4}$ of a cup. How much cat food is left?

2. A carpenter has a board that is 6 $\frac{1}{3}$ feet long. He cut off $2\frac{2}{3}$ feet. How much is left?

3. My friend walked $1\frac{2}{5}$ miles. I walked $\frac{4}{5}$ miles less. How far did I walk?

4. I need $2\frac{3}{8}$ pounds of strawberries for a pie. I have $1\frac{5}{8}$ pounds. How many more pounds of strawberries do I need?

Subtracting without drawing a model

Ask students to solve number sentences.

$3\frac{4}{5} - 1\frac{2}{5} =$

$5\frac{5}{6} - 2\frac{4}{6} =$

Exit Ticket (Formative Assessment): Everyone in my house wanted to eat cereal this morning. We started with $6\frac{2}{4}$ cups of cereal. My brother ate $1\frac{1}{4}$ cups, my sister ate $\frac{3}{4}$ cup and I ate $1\frac{3}{4}$ cups. How much cereal was left after breakfast? Make sure your answer includes an equation and a visual representation.

Lesson 5: Multiplication of Fractions by Whole Numbers

Brief Overview: This lesson extends the understanding of addition and subtraction with fractions and mixed numbers to multiplication of fractions and mixed numbers by a whole number. As you plan, consider the variability of learners in your class and make adaptations as necessary.

Prior Knowledge Required:

- Understand how to add fractions with like denominators
- Understand how to visually represent combining fractions using number lines, rulers, fraction bars and circles.
- Understand how to express a quantity greater than one as a fraction or a mixed number

Estimated Time: 60 minutes

Resources for Lesson:

- Rulers
- Fraction bars, fraction circles, sheets of visual models of fraction circles/bars

Content Area/Course: Math

Unit: Building Fractions from Unit Fractions: Applying and Extending Understandings on Whole Numbers

Time (minutes): 60 minutes

Lesson # 5: Multiplication of Fractions by Whole Numbers

Overview: This lesson extends the understanding of addition and subtraction with fractions to multiplication of fractions and mixed numbers by whole numbers. Students will apply their knowledge of multiplication as repeated addition to multiplication of fractions and mixed numbers by whole numbers.

By the end of this lesson students will know and be able to:

- Combine groups of like fractions and mixed numbers using repeated addition
- Combine groups of like fractions and mixed numbers using multiplication by a whole number
- Understand the relationship between repeated addition and multiplication as it applies to fractions
- Represent these combinations using one or more visual models or representations
- Solve word problems involving combining groups of like fractions and mixed numbers
- Understand how to use the distributive property to multiply mixed numbers by a whole number

Essential Question addressed in this lesson:

What is the relationship between multiplication of whole numbers and the multiplication of a fraction or mixed number by a whole number?

Standard(s)/Unit Goal(s) to be addressed in this lesson:

4.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.

a. Understand a fraction $\boldsymbol{a/b}$ as a multiple of $\mathbf{1/b}$. *For example, use a visual fraction model to represent* $\boldsymbol{5/4}$ *as the product* $5 \times (\frac{1}{4})$, *recording the conclusion by the equation* $\frac{5}{4} = 5 \times (\frac{1}{4})$.

c. Understand a multiple of $\boldsymbol{a/b}$ as a multiple of $\mathbf{1/b}$, and use this understanding to multiply a fraction by a whole number. *For example, use a visual fraction model to express* $3 \times (\frac{2}{5})$ *as* $6 \times (\frac{1}{5})$, *recognizing this product as* $\boldsymbol{6/5}$. (In general, $n \times (\boldsymbol{a/b}) = (n \times a)/b$.)

SMP.3 Construct viable arguments and critique the reasoning of others
SMP.6 Attend to precision

Instructional Resources/Tools

- Fraction circles
- Fractions bars
- Rulers
- Sheets of visual models of fraction bars and circles

Anticipated Student Preconceptions/Misconceptions

- Multiplication always has an answer that is larger than the two parts being multiplied
- Like addition, multiplication requires common denominators

Instructional Model

1) Pre-assessment /Warm-up to identify what students understand about the mathematical focus of the lesson
2) Problem solving and discussion of the mathematical focus of the lesson with attention to a range of strategies and visual representations including area models and number lines
3) Reflective practice using strategies, manipulatives, and visual representations to develop fluency
4) Post-assessment exploring connections between fraction addition and subtraction and whole number addition and subtraction.

Instructional Tips/Strategies/Suggestions:

Students should be able to demonstrate the concept of multiplication of fractions and mixed numbers as repeated addition with a variety of visual models and representations and use equations to communicate their solutions.

Students need to be able to support their reasoning about fractions. Focus on using precise mathematical language (SMP.6) and constructing viable arguments when writing and speaking (SMP.3).

Pre-Assessment

Question: I had a party with five people. Everyone ate $\frac{1}{2}$ of a pizza. How much pizza was eaten? How many pizzas did I need to order? Use a visual model or representation and an equation to show your solution.

What students need to know and are able to do coming into this lesson (including language needs):

- Understand how to add fractions with the same denominator
- Understand that multiplication is repeated addition

Lesson Sequence

Warmup

Students will independently attempt this question: I had a party with five people. Everyone ate $\frac{1}{2}$ of a pizza. How much pizza was eaten? How many pizzas did I need to order? Ask students to use a visual model or representation and an equation to explain their thinking and show their work.
Share and discuss solutions: How are they the same? How are they different? How do they all make sense? What equation can you write to go with each of these solutions? Be sure area models and number line models are included in the discussion.

Lesson:

1. Using different visual models and representations, discuss how groups of like fractions can be combined.

- Use fractions circles and bars to combine pieces
- Have students color pieces on paper fractions circles to show total

- Use number lines and rulers to combine measurements

Make connections to whole number multiplication. Discuss how the multiplication of fractions and mixed numbers is repeated addition.

Students should individually attempt the Fruit Salad Problem that follows.

Fruit Salad

I like to make fruit salad for parties. My recipe is very easy to make—it only uses apples, bananas and grapes. My recipe makes enough for 3 people. Sometimes I need to make enough for a larger group. Figure out how much of each fruit I need if I increase the recipe.

	Amount (in cups)			
Fruit	**3 People**	**6 People**	**9 People**	**15 People**
Apple	$\frac{1}{2}$ cup			
Banana	$\frac{3}{4}$ cup			
Grapes	$\frac{2}{3}$ cup			

Draft 8/2013

Formative Assessment: Here are the ingredients for my super secret recipe for Sugar Cookies. People like my cookies so much that I have to make twice as many. I need your help figuring out how much of each ingredient I need to double the recipe.

$\frac{1}{2}$ cup butter

$\frac{1}{3}$ cup shortening

$3\frac{1}{2}$ cups flour

$\frac{2}{3}$ cup sugar

$1\frac{3}{4}$ teaspoon cinnamon

1 teaspoon salt

2 eggs

Share and discuss solutions: How are they the same? How are they different? How do they all make sense? What equation can you write to go with each of these solutions? Be sure area models and number line models are included in the discussion.
Then pose the following extensions to the problem:

1. Rewrite my recipe on the recipe card so that you have enough ingredients to make twice as many cookies.

2. Draw a visual model and write an equation to show your thinking for how much sugar you need and how much cinnamon you need. (You need two visual models or representations and two equations, one for sugar and one for cinnamon).

Curriculum Embedded Performance Assessment (CEPA)

Snack Mix

This CEPA is designed to be sure students understand that fractions can be composed or decomposed into unit fractions where the denominators are the same, and that when fractions are composed or decomposed, the result may be a mixed number composed of whole number and a fraction.

Students will also understand that fractions with numerators larger than their denominators are greater than one and can be written as mixed numbers.

Further, students will understand what it means to add and subtract fractions with like denominators and what it means to multiply a fraction by a whole number, building on their understanding of these operations with whole numbers.

Student understanding will be demonstrated through the use of visual representations, including area models and number lines, and through the use of equations that show their work.

Snack Mix

Your class is going on a hike and needs some snacks for the trip. You decide to mix some of your favorite foods together and make snack mix. Your class chooses 6 different possible ingredients: peanuts, chocolate candy, oat cereal, dried cranberries, raisins and pretzels. The ingredients are sold in different sized packages and each group needs 5 cups of trail mix.

1. Create two different recipes for snack mix. Each recipe must include at least 4 different foods and the total amount must be exactly 5 cups. You must use whole packages of food. The sizes of the packages are shown below. Your recipe must list the total amount of each ingredient on your recipe cards.

2. You must justify your solution by showing and explaining all your work including visual models and equations that include addition, subtraction, and multiplication of fractions and mixed numbers.

3. You also need to make a shopping list showing how many packages of each ingredient you need to buy.

Peanuts $\frac{1}{5}$cup	Chocolate Candy $\frac{2}{5}$ cup	Oat Cereal $\frac{3}{5}$ cup
Raisins $\frac{4}{5}$ cup	Pretzels $1\frac{2}{5}$ cup	Dried Cranberries $1\frac{4}{5}$ cup

4. The class hiked for $1\frac{5}{8}$ miles before they stopped for a snack. Then, they hiked for $1\frac{7}{8}$ miles before they stopped for lunch. Finally, they hiked 6/8 mile to the end of the trail. How far did they hike altogether? Use a number line model to show your work.

5. Your group arrives at the hike really hungry and they eat 3/5 of a cup of the snack mix before the hike even begins. At the first break the group eats another $1\frac{4}{5}$ of a cup more. How much snack mix does the group have left then? Draw a picture to explain your work. (SMP.4: Model with mathematics.)

6. Six members of your group haven't had any snack mix yet. Is there enough left for them to each have $\frac{2}{5}$ of a cup? If yes, how much is left over? If no, how much more do you need? Use pictures, numbers or words to show your thinking. (SMP.3: Construct viable arguments and critique the reasoning of others.)

Challenge #1: Some of the children in your class are allergic to peanuts. Create one peanut free recipe.

Challenge #2: Make a recipe that includes half cups of particular ingredients.

CEPA Teacher Instructions:

Day 1: Complete questions 1, 2 and 3 of the CEPA. By the end of day 1, students should have 2 recipe cards, 1 shopping list, and the visual models and equations that show and explain their work. These equations should show addition, subtraction, and multiplication depending on the visual models used and the computations needed.

Day 2: Finish any work from day 1. Break students into small groups and have each student present his or her work to the rest of the group. Ask a few students to present to the whole group. Be sure to highlight the addition, subtraction, and multiplication situations with fractions and mixed numbers and how that computation makes sense.

Day 3: Finish any presentations that are incomplete. Complete questions 4, 5 and 6 of the CEPA.

Rubric for Snack Mix (CEPA)

	4: Advanced	3: Proficient	2: Needs Improvement	1: Warning
Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers. ***4.NF.3. Understand a fraction a/b with a > 1 as a sum of fractions 1/b.*** a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole. b. Decompose fractions into a sum of fractions with the same denominator in more than one way, recording each decomposition and composition as an equation, and justifying decompositions and compositions using visual fraction models. c. Add and subtract mixed numbers with like denominators, e.g., by replacing each number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction. d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.	Student work demonstrates an exemplary understanding of joining and separating fractions and mixed numbers in ways that communicate an understanding of these quantities and what it means to join or separate them across a range of addition and subtraction situations (e.g., combining ingredients to make a trail mix recipe and determining how close the recipe is getting to the 5 cups needed) while taking into account the same whole (e.g., all measurements are in cups). Student work also demonstrates an exemplary understanding of the links between whole number computation and fraction computation, using properties of number (e.g., decomposing and recomposing factions and mixed numbers strategically to make the computations more efficient) and the relationships between addition and subtraction to compute fluently and efficiently (e.g., adding up makes the computation easier than subtracting). Finally, student work demonstrates an exemplary understanding of how to make sense of story problems using visual representations (e.g., area models and number lines to show the addition of ingredients and how close the recipe is getting to 5 cups) and equations that represent	Student work demonstrates a good understanding of joining and separating fractions and mixed numbers in ways that communicate an understanding of these quantities and what it means to join or separate them across a range of addition and subtraction situations (e.g., combining ingredients to make a trail mix recipe and determining how close the recipe is getting to the 5 cups needed) while taking into account the same whole (e.g., all measurements are in cups). Student work also demonstrates a good understanding of the links between whole number computation and fraction computation, using properties of number (e.g., decomposing and recomposing factions and mixed numbers strategically to make the computations more efficient) and the relationships between addition and subtraction to compute fluently and efficiently (e.g., adding up makes the computation easier than subtracting). Finally, student work demonstrates a good understanding of how to make sense of story problems using visual representations (e.g., area models and number lines to show the addition of ingredients and how close the recipe is getting to 5 cups) and equations that represent what is	Student work demonstrates a fair understanding of joining and separating fractions and mixed numbers in ways that communicate an understanding of these quantities and what it means to join or separate them across a range of addition and subtraction situations (e.g., combining ingredients to make a trail mix recipe and determining how close the recipe is getting to the 5 cups needed) while taking into account the same whole (e.g., all measurements are in cups). Student work also demonstrates a fair understanding of the links between whole number computation and fraction computation, using properties of number (e.g., decomposing and recomposing factions and mixed numbers strategically to make the computations more efficient) and the relationships between addition and subtraction to compute fluently and efficiently (e.g., adding up makes the computation easier than subtracting). Finally, student work demonstrates a fair understanding of how to make sense of story problems using visual representations (e.g., area models and number lines to show the addition of ingredients and how close the recipe is getting to 5 cups) and equations that represent what is	Student work demonstrates a minimal understanding of joining and separating fractions and mixed numbers in ways that communicate an understanding of these quantities and what it means to join or separate them across a range of addition and subtraction situations (e.g., combining ingredients to make a trail mix recipe and determining how close the recipe is getting to the 5 cups needed) while taking into account the same whole (e.g., all measurements are in cups). Student work also demonstrates a minimal understanding of the links between whole number computation and fraction computation, using properties of number (e.g., decomposing and recomposing factions and mixed numbers strategically to make the computations more efficient) and the relationships between addition and subtraction to compute fluently and efficiently (e.g., adding up makes the computation easier than subtracting). Finally, student work demonstrates a minimal understanding of how to make sense of story problems using visual representations (e.g., area models and number lines to show the addition of ingredients and how close the recipe is getting to 5 cups) and equations that represent what is

	what is shown by these visual representations.	shown by these visual representations.	shown by these visual representations.	shown by these visual representations.
Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers. ***4.NF.4. Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.*** a. Understand a fraction a/b as a multiple of 1/b. For example, use a visual fraction model to represent 5/4 as the product of 5 x (¼), recording the conclusion by the equation 5/4 = 5 x (1/4). b. Understand a multiple of a/b as a multiple of 1/b, and use this understanding to multiply a fraction by a whole number; For example, use a visual fraction model to express 3 x (2/5) as 6 x (1/5), recognizing the product as 6/5. c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. For example, if each person at a party will each 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?	The student work demonstrates an exemplary understanding of how multiplication by a whole number can be used to capture the computation required when fractional amounts need to be added repeatedly (e.g., when adding together multiple cups of a particular ingredient for a recipe). The student work also demonstrates an exemplary understanding of the links between whole number computation and fraction computation, using properties of number (e.g., decomposing and recomposing factions and mixed numbers strategically to make the computations more efficient) and the relationships between addition and multiplication to compute fluently and efficiently. Finally, the student work demonstrates an exemplary understanding of how to make sense of story problems using visual representations and equations (e.g., area models and number lines to show the repeated addition or whole number multiplication of ingredients) to represent the problem situation.	The student work demonstrates a good understanding of how multiplication by a whole number can be used to capture the computation required when fractional amounts need to be added repeatedly (e.g., when adding together multiple cups of a particular ingredient for a recipe). The student work also demonstrates a good understanding of the links between whole number computation and fraction computation, using properties of number (e.g., decomposing and recomposing factions and mixed numbers strategically to make the computations more efficient) and the relationships between addition and multiplication to compute fluently and efficiently. Finally, the student work demonstrates a good understanding of how to make sense of story problems using visual representations and equations (e.g., area models and number lines to show the repeated addition or whole number multiplication of ingredients) to represent the problem situation.	The student work demonstrates a fair understanding of how multiplication by a whole number can be used to capture the computation required when fractional amounts need to be added repeatedly (e.g., when adding together multiple cups of a particular ingredient for a recipe). The student work also demonstrates a fair understanding of the links between whole number computation and fraction computation, using properties of number (e.g., decomposing and recomposing factions and mixed numbers strategically to make the computations more efficient) and the relationships between addition and multiplication to compute fluently and efficiently. Finally, the student work demonstrates a fair understanding of how to make sense of story problems using visual representations and equations (e.g., area models and number lines to show the repeated addition or whole number multiplication of ingredients) to represent the problem situation.	The student work demonstrates a minimal understanding of how multiplication by a whole number can be used to capture the computation required when fractional amounts need to be added repeatedly (e.g., when adding together multiple cups of a particular ingredient for a recipe). The student work also demonstrates a minimal understanding of the links between whole number computation and fraction computation, using properties of number (e.g., decomposing and recomposing factions and mixed numbers strategically to make the computations more efficient) and the relationships between addition and multiplication to compute fluently and efficiently. Finally, the student work demonstrates a minimal understanding of how to make sense of story problems using visual representations and equations (e.g., area models and number lines to show the repeated addition or whole number multiplication of ingredients) to represent the problem situation.

MP 2: Reason abstractly and quantitatively.	The student work demonstrates an exemplary understanding of what it means to reason abstractly and quantitatively, making sense of the quantities involved in the problem, decontextualizing these quantities in order to operate with them, and then returning to the context of the problem to ensure that the solution makes sense.	The student work demonstrates a good understanding of what it means to reason abstractly and quantitatively, making sense of the quantities involved in the problem, decontextualizing these quantities in order to operate with them, and then returning to the context of the problem to ensure that the solution makes sense.	The student work demonstrates a fair understanding of what it means to reason abstractly and quantitatively, making sense of the quantities involved in the problem, decontextualizing these quantities in order to operate with them, and then returning to the context of the problem to ensure that the solution makes sense.	The student work demonstrates a minimal understanding of what it means to reason abstractly and quantitatively, making sense of the quantities involved in the problem, decontextualizing these quantities in order to operate with them, and then returning to the context of the problem to ensure that the solution makes sense.
MP 3: Construct viable arguments and critique the reasoning of others.	The student work demonstrates an exemplary understanding of what it means to use stated assumptions, definitions, and previously established results in constructing arguments and can communicate these arguments to others.	The student work demonstrates a good understanding of what it means to use stated assumptions, definitions, and previously established results in constructing arguments and can communicate these arguments to others.	The student work demonstrates a fair understanding of what it means to use stated assumptions, definitions, and previously established results in constructing arguments and can communicate these arguments to others.	The student work demonstrates a minimal understanding of what it means to use stated assumptions, definitions, and previously established results in constructing arguments and can communicate these arguments to others.
MP 6: Attend to precision.	The student work demonstrates an exemplary understanding of what it means to communicate with precision, providing carefully formulated explanations, using appropriate symbols accurately, and being careful to specify the units in all their measurements.	The student work demonstrates a good understanding of what it means to communicate with precision, providing carefully formulated explanations, using appropriate symbols accurately, and being careful to specify the units in all their measurements.	The student work demonstrates a fair understanding of what it means to communicate with precision, providing carefully formulated explanations, using appropriate symbols accurately, and being careful to specify the units in all their measurements.	The student work demonstrates a minimal understanding of what it means to communicate with precision, providing carefully formulated explanations, using appropriate symbols accurately, and being careful to specify the units in all their measurements.

Bank of Fraction Story Problems Addressing Fraction Addition, Subtraction and Multiplication by a Whole Number Using Identified Addition, Subtraction, and Multiplication Situations

1. Add to problem – start unknown

I have a bottle of orange juice. I drank $\frac{5}{8}$ of a cup of orange juice. My brother came home and drank $\frac{7}{8}$ of a cup of orange juice. Then my sister drank $1\frac{2}{8}$ of a cup of orange juice and finished the bottle. How much juice did the bottle start with?

2. Add to problem- total unknown:

Ms. Jones is putting up a border on her bulletin board. She needs 14 feet of border altogether. She has one piece that is $3\frac{2}{4}$ feet, two pieces that are $4\frac{1}{4}$ feet each and one piece that is $5\frac{3}{4}$ feet. Does she have enough border for her bulletin board?

3. Add to problem –change unknown:

Jenny saw that there were 2 boxes of cereal in the cabinet. The box of rice cereal weighed $10\frac{6}{10}$ ounces. Together the boxes weigh $20\frac{7}{10}$ ounces. How much does the box of corn cereal weigh?

4. Add to problem – result unknown or equal groups problem – unknown product:

Mr. Gomez planted a rectangular garden. The dimensions of his garden are $6\frac{3}{5}$ feet by $8\frac{4}{5}$ feet. What is the perimeter of his garden?

5. Comparison problem – difference unknown

My friend's house is $9\frac{3}{5}$ miles away. I biked $5\frac{4}{5}$ miles and then stopped to rest. How much farther do I need to bike to get to my friend's house?

6. Comparison problem- difference unknown:

I bought $4\frac{3}{4}$ gallons of paint to paint my room. I found out I really needed $6\frac{1}{4}$ gallons of paint. How much more paint do I need to buy?

7. Compare problem – smaller unknown

I am cooking Thanksgiving dinner. The turkey will take $4\frac{3}{4}$ hours to cook. I need to put the pies in when the turkey has cooked for $3\frac{1}{4}$ hours. How long do the pies need to cook?

8. Put together problem- Result unknown:

I am baking bread. My bread recipe asks for ¾ cup of whole wheat flour and $4\frac{1}{4}$ cups of white flour. How much flour do I need?

9. Take from problem- Change unknown

Julia's mom has $\frac{7}{8}$ of the pumpkin pie left in the pan. Her kids came by and ate some. She saw that now there was only $\frac{3}{8}$ of a pie. How much did her kids eat?

10. Take from problem – addend unknown:

I have $4\frac{3}{8}$ gallons of paint. I used some of it to paint my wall. Now I have $2\frac{5}{8}$ gallons of paint. How much paint did I use?

11. Take from problem- start unknown:

Uncle Joe used $2\frac{3}{5}$ cups of sugar to bake a cake. Auntie Jane used the rest which was $3\frac{2}{5}$ cups. How much sugar was in the canister before they started baking?

12. Put Together/Take apart problem- addend unknown:

The hose in the backyard measures $18\frac{2}{4}$ feet. The distance from the faucet to the pool is $26\frac{3}{4}$ feet. How much longer would the hose have to be to reach the pool?

13. Take apart- both addends unknown:

My aunt has $26\frac{1}{2}$ feet of rope. How could she cut it up to make two equal jump ropes using all of the rope ?

14. Equal groups problem- product unknown

Mary has a piano concert coming up. She practiced for ¾ hour each day for five days. How many hours did she practice?

15. Equal groups problem – product unknown or Add to problem/result unknown:

Rosa walked to her aunt's house and back. Her aunt lives $4\frac{7}{6}$ miles away. How far did she walk?

16. Equal groups problem- result unknown:

My brownie recipe makes 12 brownies and uses $\frac{2}{3}$ cup of sugar. I need to make 48 brownies. How much sugar do I need?

17. Equal groups problem- result unknown

Popcorn comes in 2 cup bags. I want to serve popcorn to 9 of my friends. Each friend will get ½ cup of popcorn. If I buy 2 bags of popcorn will I have enough to serve all of my friends?

18. Comparison problem – difference unknown:

One of the events at field day was a long jump. Ana jumped $7\frac{5}{12}$ feet and Daniel jumped $9\frac{3}{12}$ feet. How much farther did Daniel jump than Ana?

19. Comparison problem – bigger unknown:

David is $54\frac{1}{1}$ inches tall. Ashley is $2\frac{3}{4}$ inches taller. How tall is Ashley?

20. Comparison problem – unknown product

Briana has a piece of ribbon that is $2\frac{3}{4}$ feet long. Cindy's ribbon is three times as long. How long is Cindy's ribbon?

21. Comparison problem – unknown product

Ashley has a giraffe stuffed animal that is $1\frac{3}{4}$ feet tall. Chloe has a leopard stuffed animal that is 3 times as tall. How tall is the leopard stuffed animal?

22. Comparison problem – Number of groups unknown

Aunt Nellie was knitting a scarf for her granddaughter. At the beginning of the week, her scarf was 11 ½ inches long. At the end of the week it was 46 inches longer. How many times longer was it at the end of the week than it was at the beginning of the week?

23. Area problem – unknown product

Ms. Murphy planted a rectangular garden. The dimensions of her garden are $6\frac{3}{8}$ feet by 9 feet. What is the area of her garden?

24. Array problem – unknown product:

I am building wooden shelves for my room. Each shelf is $8\frac{2}{5}$ inches long. I want 7 shelves. How long a piece of wood do I need to build the shelves?

25. Array problem- group size unknown:

I have $12\frac{3}{8}$ pizzas to share with the fourth grade. There are 3 fourth grade classes. How many pizzas should each class get?

List of Unit Resources

List and include resources by lesson sequence

Lesson 1

- Manipulatives - fraction sets of your choice
- Five cards, labeled with 1/5 on each and masking tape
- Homework sheet

Lesson 2

- Fraction circles divided into 8 equal pieces
- Fraction manipulatives
- Pre-assessment worksheet
- Classwork problems
- Homework

Lesson 3

- Manipulatives- Cuisinaire Rods, construction paper bars or virtual manipulatives web sit:e http://nlvm.usu.edu/en/nav/frames_asid_203_g_2_t_1.html?from=category_g_2_t_1.html.
- Handout on Mixed Numbers,
- Index cards or Mixed Number and Fraction matching cards.

Lesson 4

- Computer and projector for WGBH site: http://mass.pbslearningmedia.org/resource/vtl07.math.number.fra.crumpetrec/crumpets-recipe/
- Rulers, Number Lines
- Fraction Circles, Fraction Bars
- Crayons or Colored Pencils
- Sheets of Fourths and Eighths

Lesson 5

- Fraction bars, Fraction circles
- Rulers
- Sheets of visual models of fraction circles/bars

CEPA

- Recipe cards (2 per student)
- Presentation option (projector with document camera, large paper, etc.)

Question Bank of Addition, Subtraction and Multiplication Word Problems Arranged by Type