At RPDP, we support educators through professional development. Professional development can occur in a variety of ways: Entire staff trainings, grade level meetings, one-on-one support, etc. We collaborate with administrators and teachers regarding the developing and strengthening math content knowledge, use of best practices in the classroom, we model lessons, and provide support for the use of quality instructional materials.

Providing educators with quality resources in regards to instructional materials is a continuous priority. We provide this support through math content overviews, the use of instructional materials, further practice/skill development materials, and through quality assessments/tasks. As we work to create these resources for educators, we may recommend other quality resources from time to time.

In recent years, some states have received funds to create quality instructional materials for ALL educators for ALL states to access. We have selected some of those materials that we believe support our vision of quality instructional materials that support teachers in providing a solid mathematical foundation for students. For more elementary math resources please visit Rpdp.net .

The Power of TEN

Grade 1 Mathematics

The focus of this unit is to develop students' understanding of whole number relationships and a strong foundation for place value. Students will benefit from grouping objects/materials into ones and tens and connecting their knowledge to standard numerals and symbols.

This unit reflects Critical Area 2 developing understanding of whole number relationships and place value, including grouping in tens and ones for grade 1 in the 2011 MA Frameworks for Mathematics: *developing understanding of whole number relationships and place value, including grouping in tens and ones*. It is estimated that it will take approximately two weeks to complete.

The suggested timeframe for implementation is Fall/Winter.

These Model Curriculum Units are designed to exemplify the expectations outlined in the MA Curriculum Frameworks for English Language Arts/Literacy and Mathematics incorporating the Common Core State Standards, as well as all other MA Curriculum Frameworks. These units include lesson plans, Curriculum Embedded Performance Assessments, and resources. In using these units, it is important to consider the variability of learners in your class and make adaptations as necessary.

Table of Content

Stage 1 Desired Results

ESTABLISHED GOALS

1.NBT.2. Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following special cases:

A)10 can be thought of a bundle of ten ones—called a "ten"

B) the numbers from 11-19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones

C) The numbers 10, 20, 30, 40, 50, 60,70, 80, 90 refer to one, two, three, four, five six, seven, eight, or nine tens (and 0 ones).

1.NBT.3. Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, = and <.

SMP2. Reason abstractly and quantitatively.

SMP3. Construct viable arguments

Transfer

Students will be able to independently use their learning to…

Apply mathematical knowledge to analyze and model mathematical relationships in the context of a situation in order to make decisions, draw conclusions, and solve problems.

Meaning

UNDERSTANDINGS **U**

Students will understand that...

U1. Our counting system is based on the number ten.

U2. Ten is a single entity as well as ten separate units.

U3. The position of digits in numbers determines whether the digit is representing ones or tens.

U4. Numbers can be represented in different ways.

ESSENTIAL QUESTIONS **Q**

Q1. How can you represent any quantity using only the digits 0-9?

Q2. In what ways are a ten and 10 ones different or the same?

Q3. How can you figure out if quantities are greater than, less than or equal to one another?

Acquisition

Students will know… **K**

K1. Ten ones is equivalent to one ten.

K2. That just as the "teen" counting numbers are composed of a ten and some ones (e.g. "thirteen" is one ten and three ones) the decade numbers are composed of sets of ten (e.g. thirty is 3 tens).

K3. In a two digit number, the digit in the tens place is the number of tens and the digit in the ones places is the number of

Students will be skilled at… **S**

S1. Composing and decomposing numbers into different combinations of tens and ones without changing the value.

S2. Representing a quantity of objects with its corresponding numeral up to 99.

S3. Efficiently organizing large collections by bundling in groups of ten in order to determine the total

and critique the reasoning of others. SMP4. Model with mathematics. SMP7. Look for and make use of structure. SMP8. Look for and express regularity in repeated reasoning Connection to additional mathematics standards: 1.MD.2 & 1.MD.4 Literacy Standards SL1.4. Describe people, places, things, and events with relevant details, expressing ideas and feelings clearly. SL1.5. Add drawings or other visual displays to descriptions when appropriate to clarify ideas and feelings clearly.	ones. K4. That when comparing two-digit numbers, the number with the most tens is the greatest. When the tens are equal the number with the most ones is greatest. K5. The symbols > (greater than), = (equal to) and < (less than). K6. Appropriate academic language, including vocabulary, when identifying place value components: quantity, object, collection, digit, ones place, tens place, place value, face value, ten ones, one ten, efficient/inefficient, standard/non-standard, greater than/more than/bigger than, less than/smaller than, least, greatest, decade, number line, number sentence, ten frame, hundreds chart	quantity. S4. Putting two-digit numbers in sequential order. S5. Using the correct symbols to represent equivalency or inequality. S6. How to articulate the value of differences between the tens and ones place.

Stage 2 - Evidence

Evaluative Criteria	Assessment Evidence
CEPA A: Do students consistently and accurately organize by tens?	**CURRICULUM EMBEDDED PERFORMANCE ASSESSMENT (PERFORMANCE TASKS)** **PT** TRANSFER TASK(S): CEPA A: Inventory 1. The principal needs to know how many school supplies we have in the closet because he/she needs to know how many to order for next year. You are going to be our inventory clerks. There are four supply boxes set up in the room with different supplies in them. (Note to teachers: some buckets should have large quantities of

CEPA B: Can students place a two-digit numbers consistently and accurately on a blank number line?	supplies and some have small quantities between 10 and 100.) Count the supplies in at least three "boxes." For each box, use this form from the principal to write how many supplies are in the box. Draw a picture so the principal can check your work without counting them one-by-one. **Criteria for exemplary rating** **Accurately count & write the quantities and accurately represent groupings of Ten.** CEPA **2**: Ages Task 1: Help Mrs. Smith place her family members' ages on a number line. Step one: Place the ages where they belong on the blank number line. Task 2: On a handout showing a horizontal line with zero on one end and one hundred on the other, indicate where each person's age lies. Task 3: (Individual interview) Provide a decade number line (same number line with decade increments marked). Ask child it he/she is still satisfied with the placements they chose for each dot. *Explain how you knew where to put the dot.* Backup question: *What if you had one more person and they went right here (on a landmark marker) how old do you think this person is? How did you know?* **Criteria for exemplary rating: In creating their number lines:** **Do they accurately approximate the position of numbers 1-99 on a blank number line? Do they use the decade number line to more precisely place the numbers 1-99?** **In the interview, can they provide a place-value rationale for their placement of numbers on the number line?**
Do students understand that the value of a digit depends on its position within a number? Can children name numbers in more	**OTHER EVIDENCE:** **OE** **Pre Assessment** OE1. Show a child 16 counters and ask him/her to make a drawing. Ask the child how many there are. Ask the child to write 16 with numbers. Circle the 6 in the 16

than one way?	and ask the child to indicate in the drawing what the 6 means. Circle the numeral 1 in the 16 and ask the child to indicate what it means in the drawing.. Circle the whole numeral 16. Ask the child what it means. (this is a one-to-one interview) Progress Monitoring **Exit Slips, Teacher Observation, Student Work Samples, Reflective Journals**

Stage 3 – Learning Plan

Summary of Key Learning Events and Instruction

Sequence of Lessons- 2 to 3 week unit

1. Bundling Lesson 1
   1.NBT.2, SMP7. In this lesson children come to grips with the dual nature of ten – that it is simultaneously a single entity and ten separate objects – and these two identities are equivalent. Because our counting system is base-10, creating sets of tens provides us with an efficient shortcut in identifying large quantities without counting every item in the set.
2. Bundling Lesson 2
   1.NBT.2, SMP8. In this lesson, children become comfortable with the bundling strategy as not changing the overall quantity. In other words, they understand that it is always possible to count the entire set (or clustering using numbers other than 10, such as 2 or 5), but that using base-ten structures allows the quantity to be known much more efficiently, because the number of tens corresponds to the digit in the tens place of the numeral, and the number of leftover units corresponds to the digit in the units place of the numeral.
3. How Long AreWe?
   1.NBT.2, 1.MD.2, SMP5, SMP8. In this lesson, children get a chance to see that this place-value concept can be applied to a measurement task; namely, that breaking a Unifix train into bundles of ten allows them to quickly determine how many Unifix cubes tall they are. Students also place their recorded heights on a number line, another tool that facilitates comparison of numbers.

CEPA A (Formative)

1.NBT.2, SMP2 , SMP7. In this activity, children have to choose for themselves how to make a large quantity

knowable quickly and accurately. It is hoped that their experiences with bundling will help them realize that arranging materials in groups of tens and ones is the most efficient method.

4. Making Standard and Non-Standard Groupings
   1.NBT.3, SMP.7. Look for and make use of structure. In this lesson students see the equivalence of quantities, no matter how many of the objects are bundled into sets of ten. Therefore none can be considered illegitimate or wrong. By seeing there are multiple ways that base-ten bundling can be done, they better understand that the traditional method is the most efficient model, but not necessarily the only option.
5. Making Greater Than/Less Than Judgments
   1.NBT. 3, SMP3. Students will be working together to make greater than/less than judgments. They will each have to explain their reasoning, using place-value vocabulary and concepts and consider carefully the reasoning of their partner in order to come to consensus.
6. Ten Frame Compare
   1.NTB.3, SMP2In this lesson the children need to first make a judgment of equality or inequality using a concrete place-value model and then translate that understanding to the appropriate symbol system. They must also interpret the inequality symbols in two directions: 57 > 39 means both that 57 is greater than thirty-nine and that thirty-nine is less than fifty-seven.
7. Comparing Numerals on a Decade Number line
   1.NBT.3, SMP4, SMP3. In this lesson children will justify their judgments of greater than/less than using various mathematical place-value models– bean sticks, ten-frames, a blank hundreds board, and a decades number line. By making a connection between their conceptual understanding and various physical models, they will get different perspectives on the concept, thereby making their understanding broader and deeper.
8. Prove It!
   1.NBT.3, SMP2. (In this lesson, students are expected to be able to make judgments of relative quantity without the aid of visuals or manipulatives. They must rely solely on their understanding of place value and the relative impact on quantity of changes in the units place versus changes in the tens place to make their judgments.
9. Ordering Numbers from Least to Greatest
   1.NBT.3, SMP2.In this lesson, children must make greater than/less than judgments relative to random benchmark numbers and to the other numbers they are placing. The task requires them not just to differentiate the tens from the units digit but to strategize when the tens digit alone gives them all the information they need to make a greater than/less than judgment and when they must also take the units digit into account.

CEPA B (Summative)

1.NBT.2, 1.NBT.3, SMP2 (Reason abstractly and quantitatively). This task required students to make judgments of relative quantity for two-digit numbers, with a minimal set of cues provided. They must draw heavily on their understanding of numbers in 1-100 space, differentiating between tens digit information and ones digit information.

Lesson 1: Bundling 1

Brief Overview: In this lesson children come to grips with the dual nature of ten – that it is simultaneously a single entity and ten separate objects – and these two identities are equivalent. Because our counting system is base-10, creating sets of tens provides us with an efficient shortcut in identifying large quantities without counting every item in the set. As you plan, consider the variability of learners in your class and make adaptations as necessary.

Prior Knowledge Required:

- Working with numbers 11-19 to gain foundation for place value
- Extend the counting sequence

Estimated Time: 45-65 minutes

Resources for Lesson:

- 16 of some classroom object (crayons or blocks etc.)
- Collections of 32 counters in baggies- one baggie per two children

Content Area/Course: Math/ Place Value

Grade: 1

Time: 40-65 minutes

Unit Title: *The Power of Ten*

Lesson 1: Bundling 1

Essential Question(s) to be addressed in this lesson:
In what ways are a ten and 10 ones different or the same?

Standard(s)/Unit Goal(s) to be addressed in this lesson:
1.NBT.2 Understand that the two-digits of a 2-digit number represent amounts of tens and ones. Understand the special cases a, b, & c.

SMP7. Look for and make use of structure.

Students should have had previous experience:
Working with numbers 11-19 to gain foundation for place value

Extend the counting sequence

By the end of this lesson students will know and be able to:
Understand and show that ten is a single entity as well as ten separate units.

Understand that ten ones is equivalent to one ten.

Efficiently organize large collections by bundling groups of ten in order to determine the total quantity.

Instructional Resources/Tools
16 of some classroom object (crayons or blocks etc..)

Collections of 32 counters in baggies- one baggie per two children

Anticipated Student Preconceptions/Misconceptions

Students may over rely on unitary counts to understand quantity.

Students may take a while to agree that grouping by ten is the best way to group (over 2s, 5s, etc.)

Students may not be able to keep the concepts that ten is a single entity and that it is ten separate units in mind simultaneously, and may therefore sometimes treat ten as one.

Assessment

Pre-assessment/ Formative
Place a pile of 16 blocks on a table. Pose the question, "I am wondering how many blocks are on this table? How can I find out? Turn to your partner and discuss how you would determine how many blocks there are on the table." Listen to students' discussions about how they might count the number of books.

Summative (optional)
Exit Ticket. See description in Lesson Sequence and Description.

Lesson Launch (15 minutes) whole group. **Work time**- counting collections (20-40 minutes)

Closing reflections including exit ticket (10 minutes)

Lesson Sequence and Description
Demonstrate counting the pile of blocks described above using the student suggestions (by ones, twos, fives, etc.) Discuss why counting by twos is a more efficient (quicker) way of counting than counting by ones, why five is more efficient than twos, etc.

Have students work in pairs to group their counters. Pass out one paper bag with 32 counters to each pair of students. Explain that you are going to go around and count everyone's counters, but you want to count them fast! How can they organize their counters so they can be counted quickly?

After students have organized their counters, the teacher should walk around the room counting the counters of each pair of students.

This lesson is intended to be an anchor lesson. Students may not understand grouping. They might not understand why grouping in ten is the most efficient way to group (SMP7: *Look for and make use of structure*). Practice will be needed. This activity can easily transfer to centers. Please note: the term "counters" could refer to any set of small objects (teddy bears, dinosaurs, plastic chips, etc.)

It is important to be explicit with our ELL learners. They can be asked to restate the thinking used by others in the class. They should be encouraged to use math words. They should have many opportunities to talk about the math they are doing. Support students by using visuals and being explicit about language.

Vocabulary: *organize, collection, objects, group (as a verb), counters, efficient, tens, ones.*

Collections can be differentiated by size according to learning needs.

Suggested Questions:

- ☐ How did you group the counters?
- ☐ Are the counters easy to count?
- ☐ Why did you choose to organize them in that way?
- ☐ Is there a more efficient way to organize your counters? (SMP7: Look for and make use of structure).

Next, students walk around observing how each pair of their classmates has grouped the 32 counters and count them according to the way they are grouped.

Students return to their pairs. Ask students to turn and talk to their partner.

Suggested Questions:

- ☐ What ways did your classmates organize their counters?
- ☐ Which ways do you think were the best?
- ☐ Would you change how you organized your counters?

Introduce students the paper version on the overhead- "I saw Jack count by 2s" Show students picture of objects circled by two"

Pose as a whole-group discussion, " I heard Jack say the best way to count was by groups of 10. Do you agree? Why?" Show objects circled by 10

Teacher then explains why grouping by 10s is the most efficient way of grouping (SMP7: Look for and make use of structure). "I was able to determine how many counters Jack had very quickly. I counted 10, 20, 30, 31, 32."

Closure

Review outcomes of this lesson:

- Some ways of counting can make the job go faster than counting by ones.
- By organizing the counters first, we can count them faster.

Preview outcomes for the next lesson:
We can make groupings of ten when counting large groups of objects.

Exit Ticket
Give students worksheet that has a picture of 26 counters. Tell students to circle as many tens as possible. Label how many tens and how many ones.

Additional practice may be needed before continuing on to lesson 2. Activity can be set up in a center for additional practice.

Extended Learning/Practice (homework)
Similar activities for counting and grouping by tens and ones can be conducted at home.

Journal papers can record what was counted, how it was grouped (in at least two ways), and a math picture to show the count.

Children may bring in collections from home to count in school.

Suggested resources for reinforcement

An interactive shark game in which students represent a number using tens and ones place value tools.
http://www.ictgames.com/sharknumbers.html

Two-digit place value flip book. Students can use their flip books through-out the unit to represent two-digit numbers:

1. Use half a piece of copy paper (5 ½ by 8 ½ inches).
2. Fold in half to make a shape that is almost a square.
3. Cut a slit towards the fold in the middle of the paper leaving about a ¼ inch. One side will be your ones column and the other side will be your tens column.
4. Repeat 5 more times.
5. Assemble the pages like a book so that the slits line up and staple the pages together.
6. Write tens on the left side and ones on the right side.
7. Flip up the first page and write 0 on both sides.
8. Continue flipping up pages and writing digits until you get to 9.
9. Students can use their flip books throughout the unit to show numbers they have created

Additional uses for the flip book: *Guess my Number* (I'm thinking of a number that has 9 in the ones place and 1 in the tens place. What is it?)

Literature connection: *More than One* by Miriam Schlein

Teacher Reflection (to be completed after lesson)
What went well in this lesson?
Did all students accomplish the outcome(s))? What evidence do I have? What would I do differently next time?

Lesson 2: Bundling 2

Brief Overview: In this lesson, children become comfortable with the bundling strategy as not changing the overall quantity. In other words, they understand that it is always possible to count the entire set (or clustering using numbers other than 10, such as 2 or 5), but that using base-ten structures allows the quantity to be known much more efficiently, because the number of tens corresponds to the digit in the tens place of the numeral, and the number of leftover units corresponds to the digit in the units place of the numeral. As you plan, consider the variability of learners in your class and make adaptations as necessary.

Prior Knowledge Required:

- Students must have cardinality in place.
- Student should have a solid foundation of working with numbers 11-19
- 1.NBT.1 Extend the counting sequence to 120.

Estimated Time: 40-65 minutes

Resources for Lesson:

- Collections of counters in baggies
- Recording forms printed for each student

Content Area/Course: Math/Place Value

Grade: 1

Time: 40-65 minutes

Unit Title: The *Power of Ten*

Lesson 2: Bundling 2

Essential Question to be addressed in this lesson:
In what ways are a ten and 10 ones different and the same?

Standard(s)/Unit Goal(s) to be addressed in this lesson:
1.NBT.2. Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following special cases:

a) 10 can be thought of as a bundle of ten ones—called a "ten."

b) The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones.

c) The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five six, seven, eight, or nine tens (and zero ones).

SMP8. Look for and express regularity in repeated reasoning.

Assumptions about what students know and are able to do coming into this lesson (including language needs):
Students must have cardinality in place. Student should have a solid foundation of working with numbers 11-19 and 1.NBT.1
Extend the counting sequence to 120.

By the end of this lesson students will know and be able to:
Efficiently organize collections by bundling in groups of ten in order to determine the total quantity.

Connect the given quantity with the number of tens and ones in the quantity. Students will know that in a two-digit numeral, the digit in the tens place is the number of tens and the digit in the ones place is the number of ones.

Use the precise language when identifying place value components (tens place, ones place, digit in the tens place, digit in the ones place).

Instructional Resources
Collections of counters in baggies

Recording forms printed for each student.

Teacher note: If using partners put similar learners together. Provide small group support to groups with higher need.

Anticipated Student Preconceptions/Misconceptions

Students may think of a bundle of ten as the number 1.

Students may count, "ten, twenty, thirty, thirty-one, thirty-two" but not understand why we write the number 32 as we do.

Students may think about numbers in a "unitary" way; they may not easily adjust to the idea of creating groups of ten.

It is likely that students will count a full set without stopping at tens and without any effort to group by tens.

Assessment

Pre-assessment/ Formative

Many classrooms count the days of school with sticks and bundles of ten. Show the students the bundles of sticks to date and ask, "*How many days have we been in school?" How do you know*? Instruct your students to add one stick to the collection, and count the bundles of "tens" and "ones" sticks to name the number. Ask the students: *How many do we have now?* Have the students prove their count is accurate. Next, teacher unbundles sticks and puts them in a pile. Ask, "*How many do we have now?"* Wrap up the pre-assessment by asking the students: *How many days have we been in school?*

Summative (optional)

On an overhead, have two partners fill in the tens frame with the appropriate number of counters. A ten-frame is a visual organizer for grouping by ten. Have students determine the number of objects for each partner. Record on exit slip.

Please note that this pre-assessment needs to occur prior to day 99 to avoid bundling in hundreds.

Launch

Lesson Sequence and Description

Teacher begins the lesson by examining the thinking from the previous lesson. "Yesterday, we were thinking about efficient ways of counting our collections. Turn and talk to your partner about what that math word *efficient* means, and how your counters looked."

Pause while discussion ensues. Teacher listens for examples of tens thinking. Teacher highlights a few cases. (e.g, ."I heard Jack say that he placed his counters in groups of ten. He thought that tens were a fast way to count.")

Exploration

"Today we are going learn about why counting by tens is an easy and fast way to count."

Pass out bags of counters (11-40) and ten-frame recording sheet. Model use of the recording sheet.

Tell the children that they are to organize their collections into the 10-frames on the recording sheet. Students write their quantity in numerical form.

Guiding Questions

- ☐ How are you using the frames to organize your counters
- ☐ How did the ten-frames help you write your number?

This activity can be duplicated into a center.

Reflection

Bring students back into whole group. Using examples of student work, teacher helps students to make the connection between the bundle of tens (full tens) and the tens place. Teacher helps students make connection between the "extra ones" and the ones place (SMP8: *Look for and express regularity in repeated reasoning.)*

Using the above guiding questions, model the connection between a bundle of tens and the tens place. Model the connection between the "extra ones" and the ones place.

On an overhead, show two partners completed tens frame. Have students determine the number of objects for each partner. Students record on an exit slip and pass it to the teacher.

It is important to be explicit with our ELL learners. They can be asked to restate the thinking used by others in the class. They can be encouraged to use math words. They should have many opportunities to talk about the math they are doing. Support students (use visuals etc..) for *ten-frame, efficient, bundle, organize, collection, objects*.

Collections can be differentiated by size according to learning needs.

Lesson Launch (15 minutes) whole group.

Work time- counting collections (20-40 minutes) Closing reflections including exit ticket(10 minutes)

This lesson is intended to be an anchor lesson. Teachers may need to scaffold lessons based on student need. Example, the students may not be ready to write numerals the first time through. Practice will be needed. This activity can easily transfer to centers.

Suggested resources to review and reinforce building numbers up to 29:

Grouping and Grazing: Interactive game to practice/reinforce the efficiency of grouping by tens. **Note to teachers:** Select: count by tens. It is recommended that you try this game before introducing it to children. (10 cows must be turned over before they can be grouped.) http://illuminations.nctm.org/ActivityDetail.aspx?I D=218

Interactive ten frame: this timed activity gives students practice quickly reading ten frame http://www.fuelthebrain.com/Interactives/app.ph p?ID=29

Illuminations ten frame: this activity reinforces reading two-digit numbers on the ten frame (select the "add" feature to build frames larger than ten.) http://illuminations.nctm.org/ActivityDetail.aspx?I D=75

Interactive dinosaur game: this game focuses on selecting from tens and ones to create two digit numbershttp://www.ictgames.com/dinoplacevalue.html

Extended Learning/Practice (homework)
Place activity in centers

Students may use recording sheet for home activities

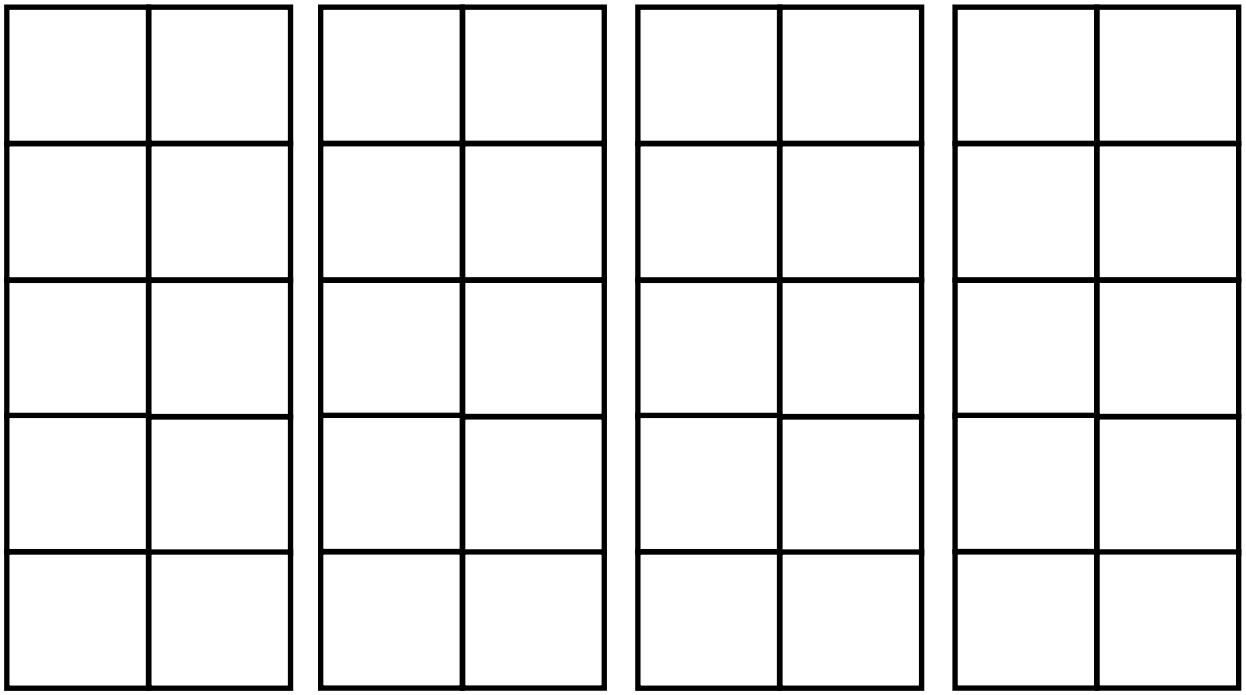
First to 100 Game: This game reinforces the value of the tens and ones places.

1. Materials: 2 place value mats, ones, tens and one hundred place value materials, dice.
2. Two students sit next to each other. Each student has a place value in front of them. A third student is the "banker."
3. Player 1 rolls a die. The "banker" gives Player 1 the number of ones indicated on the die. Player 1 puts them in the ones place on the place value mat.
4. Repeat step 1 for Player 2.
5. The students continue step one until one of them has 10 ones. The child turns in the 10 ones to the bank for one ten and places it in the tens place. On the next turn, this child will roll the die twice. The first time will indicate how many ones they can get and the second will indicate how many tens they can get.
6. Repeat until one player reaches or passes 100. This player gets to hold the 100 place value piece as a trophy (for a few minutes) and becomes the banker for the next game.

Literature connection:
From One to One Hundred by Teri Sloat

Teacher Reflection (to be completed after lesson)
What went well in this lesson?
Did all students accomplish the outcome(s)? What evidence do I have?
What would I do differently next time?

Lesson 2 Recording Sheet



Lesson 3: How Long Are We?

Brief Overview: In this lesson, children get a chance to see that this place-value concept can be applied to a measurement task; namely, that breaking a Unifix train into bundles of ten allows them to quickly determine how many Unifix cubes tall they are. Students also place their recorded heights on a number line, another tool that facilitates comparison of numbers. As you plan, consider the variability of learners in your class and make adaptations as necessary.

Prior Knowledge Required:

- Work with numbers 11-19 to gain foundation for place value
- Extend the counting sequence
- Bundling Lesson 1 & 2

Estimated Time: 40-65 minutes

Resources for Lesson:

- Unifix cubes
- Post-it notes
- Chart paper with large decade number line from 0-100

Content Area/Course: Math/ Place Value

Grade: 1

Time: 40-65 minutes

Unit Title: *The Power of Ten*

Lesson 3: How Long Are We?

Essential Question(s) to be addressed in this lesson:
In what ways are a ten and 10 ones different or the same?

Standard(s)/Unit Goal(s) to be addressed in this lesson:

1.NBT.2 Understand that the two-digits of a 2-digit number represent amounts of tens and ones. Understand the following special cases:

a) 10 can be thought of as a bundle of ten ones—called a "ten."

b) The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones.

c) The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five six, seven, eight, or nine tens (and zero ones).

Connects to **1.MD.2**:

Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. Limit to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlaps.

SMP5. Use appropriate tools strategically and
SMP8. Look for and express regularity in repeated reasoning.

Assumptions about what students know and are able to do coming into this lesson (including language needs):
Work with numbers 11-19 to gain foundation for place value.

Extend the counting sequence

Bundling Lesson 1 & 2 (previously taught)

By the end of this lesson students will know and be able to:
Demonstrate that ten is a single entity as well as ten separate units.

Explain that ten ones is *equal to* (i.e., *equivalent to*, or *the same as*) one ten.

Efficiently organize large collections by bundling groups of ten in order to determine the total quantity.

Express the length of an object as a whole number of units.

Instructional Resources/Tools
Unifix cubes

Post-it notes

Chart paper with large decade number line from 0-100

Anticipated Student Preconceptions/Misconceptions:
Students may over rely on unitary counts to understand quantity.

Students may take a while to agree that grouping by ten is the best way to group (over 2s, 5s etc.)

Students may not transfer the concept that ten is a single entity as well as ten separate units.

Assessment
Pre-assessment/ Formative Summative (optional)
What are some tools we can use to measure objects?

If we used _____how long do you think I would be?

If we used Unifix cubes, how long do you think I would be?

Summative assessment
Exit Ticket- placing post-it on decade number line (see Exit ticket instructions below)

Lesson Launch (5-10 minutes) whole group.
Work time- counting collections (20-40 minutes)
Closing reflections including exit ticket (10 minutes)

Lesson Sequence and Description
Launch- students work in partners

"I wonder, how tall/long each of you is? Today you are going to measure each other using Unifix cubes."

"One partner is going to lie down on the rug while the other connects Unifix cubes together in a line until the Unifix cubes are as long as your partner. Then count …how many Unifix cubes did it take to measure your partner?

Suggested Guiding Questions

- ☐ How long is your partner?
- ☐ How did you count the Unifix cubes
- ☐ Why did you choose to organize them in that way?
- ☐ Why is your method of counting the fastest way to count all of those Unifx cubes?
- ☐ For students who did not group by tens, ask, "Is there another way to group your cubes?

Next, students record the number of Unifix cubes on a post-it note:

46 cubes long

Exit Ticket

Teacher says, "Let see how tall everyone in the class is by displaying our numbers"

Student place their post-it numbers on the large decade number line.

If time allows, discuss with students their findings and their placement on the number line. Not all post-it notes need be discussed.

If time is constrained, follow-up can be done at calendar time or morning meeting time the following day.

Probing Questions:

- ☐ *Based on the number line who is the tallest student in our class? (John) How do you know?*
- ☐ *Why was (John) placed here on our number line?* – give students time to defend their placements
- ☐ Go over a few more correct placements and have students link their placements back to the tens and ones
- ☐ *Is there anyone who wants to move their post-it?* Why
- ☐ Go over a few incorrect placements and discuss reasoning.

This lesson is intended to be a follow-up, formative lesson to *Bundling 1 & Bundling 2.* It is meant to introduce decade number lines in the context of bundling. This lesson also provides a connection to measurement (1.MD.2), but isn't intended to replace a measurement unit. Although data is being generated, 1.MD.4 is not explicitly taught.

It is important to be specific with our ELL learners. They can be asked to restate the thinking used by others in the class. They can be encouraged to use math words. They should have many opportunities to talk about the math they are doing. Support students (use visuals etc..) for*, organize, collection, objects*, *long/tall, number line, greater than, less than, equal to.*

Note: as students are justifying their placements, encourage them to use precise mathematical comparative language and the appropriate measurement units.

For example:
"John is 51 cubes long."
"I grouped the cubes into bundles of 10 and had 1 cube left over. I counted the bundles by 10. I counted 10, 20, 30, 40, 50 and I added 1 more to 50 to make 51 cubes long."

Extended Learning/Practice (homework)
Similar "experiments" can be conducted at center with other objects (e.g., giant footprints, books, desks, etc…).

Teachers can create response sheets that include:

- ☐ Object measured
- ☐ Number of cubes
- ☐ Tens and ones

Closure

Review outcomes of this lesson:
Some ways of counting can make the job go faster than counting by ones.

By organizing the cubes first, we can count them faster.

The number of groups of tens and ones shows how to write a number and where we place it on a number line.

Preview outcomes for the next lesson:
We can make groupings of ten when counting large groups of objects.

We can start to make comparisons and quantity discriminations based on place value.

Embedded Performance Assessment (CEPA) A: Counting a Collection

(NOTE: administer CEPA A after Lesson 3)

Content Area: Math Grade: 1

Massachusetts Standards

1.NBT.2. Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following special cases:

a) 10 can be thought of a bundle of ten ones—called a "ten"

b) The numbers from 11-19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones

c) The numbers 10, 20, 30, 40, 50, 60,70, 80, 90 refer to one, two, three, four, five six, seven, eight, or nine tens (and 0 ones).

SMP2. Reason abstractly and quantitatively and
SMP7. Look for and make use of structure.

SL1.5. Add drawings or other visual displays to descriptions when appropriate to clarify ideas and feelings clearly.

Note to Teacher:
In this assessment, children have to choose for themselves how to make a large quantity quickly and accurately. It is hoped that their experiences with bundling will help them realize that arranging materials in groups of tens and ones is the most efficient method.

These tasks are intended to assess student understanding. Not all students need to take this CEPA at the same time. During center time, students who have demonstrated readiness, can go to the assessment stations and complete the tasks.

In addition, teachers should have a conversation with students about what a "math picture" looks like and how that is different from an art picture. Math pictures are circles, x's, dots, etc… This is not art.

Purpose	Time Required	When to Administer	Room Setup	Activity 1 Setup	Activity 2 Setup
The purpose of this formative assessment is to assess the understanding that the two digits in a two-digit number represent amounts of Tens and Ones.	Each child will take approximately 15-20 minutes to complete all three stations.	Administer the activities in this assessment at the end of the unit.	Assessments are done individually. All students go to the same three stations. Assessment stations can be duplicated around the room if necessary.	A container with 23 items (Station A) A container with 56 items (Station B)	A total of 79 unifix cubes is needed Seven, brown, stapled, bags stapled should contain 10 cubes in each 9 cubes are loose(Station C)

Goal: To find out how many items they have in a collection.

Role: Counter

Audience: Principal

Directions and Handouts for Students

Activity 1: The principal needs to know how many school supplies we have in the closet because he/she needs to know how many to order for next year. You are going to help us count. There are two supply boxes set up in the room with different supplies in them (Station A & B). Count the supplies in each "box." For each box, use this form from the principal to write how many supplies are in the box. Count the supplies. Do not count by ones! Draw a picture so your principal can see how you counted.

Activity 2: Another group was in charge of counting how many Unifix cubes were in a bin. They forgot to write the total number of cubes. They put 10 cubes in each brown bag and sealed the bag closed. Here are the full bags and some extra cubes (Station C). What is the total number of cubes? How did you decide (students may write or answer orally).

Criteria for Evaluating Student Products and Performances:

0 – Inaccurately count & write the quantities; no evidence of grouping.

1 – Accurately count & write the quantities, but grouped other than by tens OR accurately grouped by tens, but did not correctly count or write quantity.

2 – Accurately count & write the quantities and accurately represent groupings of ten.

Name _____

Station A (Activity 1)

How many items? _____

Draw a math picture so the principal can check your work without counting them one-by-one.

Station B- (Activity 1)

How many items? _____

Draw a math picture so the principal can check your work without counting them one by one.

Station C- (Activity 2)

Total number of cubes? _____

How did you decide?

Lesson 4: Making Standard and Non-Standard Groupings

Brief Overview: In this lesson students see the equivalence of quantities, no matter how many of the objects are bundled into sets of ten. Therefore none can be considered illegitimate or wrong. By seeing there are multiple ways that base-ten bundling can be done, they better understand that the traditional method is the most efficient model, but not necessarily the only option. As you plan, consider the variability of learners in your class and make adaptations as necessary.

Prior Knowledge Required: Grouping ones into bundles of ten make it more efficient to count

Estimated Time: 40-65 minutes

Resources for Lesson:

- Place value materials (blocks, bean sticks, etc.)

Content Area/Course: Math

Grade: 1

Time: 40 – 65 minutes

Unit Title: *The Power of Ten*

Lesson 4: Making Standard and Non-Standard Groupings

Essential Question to be addressed in this lesson:
Numbers can be represented in different ways.

Standard(s)/Unit Goal(s) to be addressed in this lesson:
1.NBT.3. Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, = and <.

SMP.7. Look for and make use of structure.

By the end of this lesson students will know and be able to:
Identify non-standard representations of numbers as valid representations, but That the standard representation is the most efficient.

Instructional Resources/Tools
Place value materials (blocks, bean sticks, etc.)

Please note: The Curriculum Embedded Performance Assessment (CEPA) A should be administered before this lesson begins.

Anticipated Student Preconceptions/Misconceptions:
Thinking that a quantity that is represented with place value materials in standard and non-standard grouping are not *equivalent* (i.e., equal to the same amount). For example, children might think that 2 tens and 16 ones is more than 3 tens and 6 ones.

Thinking it is not okay or desirable to group materials in non-standard place-value configurations.

Vocabulary:
ones, tens
most efficient
quick and correct
equivalent/equal to/the same as

Assumptions about what students know and are able to do coming into this lesson (including language needs):
Grouping ones into bundles of ten make it more efficient to count.

Assessment
Pre-assessment/ Formative (CEPA A):

- ☐ Given tens and ones place value materials, ask students to show 35 all the ways they can think of.
- ☐ Of all the ways of showing 35 you have come up with, which is the most efficient way? Have students explain why.

Lesson Sequence and Description:
At a teacher table with base-ten blocks, bean sticks or Digi-blocks.

Say, "Show me the quantity 24."

Have students represent 24 using manipulatives.

"Turn to your partner and talk…"

"Did you all come up with the same way?"

Explore optional representations. Make sure 24 units, 1 ten and 14 units, and 2 tens and 4 units are amongst the displays considered.

Guiding questions:
Write the numeral 24: "Why do you think we write twenty-four this way?" Help students see connection of place-value digits to tens and ones in the quantity.

"Which of the ways of showing twenty-four do you think is the most efficient way?" Make sure students make a connection between the most efficient representation and the digits in the number 24.

Write the first three numbers below on the board. Have students work with a partner and use place-value materials to show each number in as many ways as they can (SMP7: *Look for and make use of structure*). Circulate to make sure each partner is contributing and that non-traditional arrangements are included. If some pairs finish early, have them work with the last two numbers below.

- ☐ 29
- ☐ 41
- ☐ 36
- ☐ 82
- ☐ 55

Summary:
Students share their solutions to 41 with the group.

- ☐ Did the group find all of the solutions?
- ☐ How do you know?
- ☐ Are all of these solutions equivalent/equal?
- ☐ What is the most efficient way?

After the launch, students will work in pairs. Teachers should look for multiple solutions. Look for all of the possible solutions to 41 to display for the summary.

The teacher should circulate to look at students' solutions. Possible questions to ask:

- ☐ Are there more ways to show this number?
- ☐ Have you shown all of the possible ways?
- ☐ How do you know?
- ☐ What is the most efficient way to show this number?

Exit Ticket
Teacher shows 3 tens and 19 ones. Children write the number shown.

Closure
Review outcomes of this lesson
Preview outcomes for the next lesson

Teacher Reflection (to be completed after lesson)
What went well in this lesson?
Did all students accomplish the outcome(s))? What evidence do I have?
What would I do differently next time?

Lesson 5: Making Greater Than/Less Than Judgments?

Brief Overview: Students will be working together to make greater than/less than judgments. They will each have to explain their reasoning, using place-value vocabulary and concepts and consider carefully the reasoning of their partner in order to come to consensus. As you plan, consider the variability of learners in your class and make adaptations as necessary.

Prior Knowledge Required:

- Understand that the two digits of a two-digit number represent amounts of tens and ones.

Estimated Time: 40-65 minutes

Resources for Lesson:

- Collections of counters in baggies.
- Recording forms printed for each student.

Content Area/Course: Math /Place Value

Grade: 1

Time: 40 – 65 minutes

Unit Title: The Power of Ten

Lesson 5: Making Greater Than/Less Than Judgments

Essential Question(s) to be addressed in this lesson:
How can you figure out if quantities are greater than, less than or equal to one another?

Standard(s)/Unit Goal(s) to be addressed in this lesson:
1.NBT. 3 Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, =, and <.

SMP3 Construct viable arguments and critique the reasoning of others.

Assumptions about what students know and are able to do coming into this lesson (including language needs):
Understand that the two digits of a two-digit number represent amounts of tens and ones.

By the end of this lesson students will know and be able to:
Justify when quantities are greater than, less than or equal to one another, using place-value reasoning.

Compare two quantities 40 or less that are bundled into tens and loose units and determine the number with more tens is greater, and when the tens are equal the number with more ones is greater.

Instructional Resources/Tools
Collections of counters in baggies.

Recording forms printed for each student.

Teacher Note: If using partners, put similar learners together.

Provide small group support to groups with higher need.

Anticipated Student Preconceptions/Misconceptions
Students may think of a bundle of ten as the number 1.

Students may count, "ten, twenty, thirty, thirty-one, thirty-two" but not understand why we write the numeral 32 as we do.

Students may still use a "unitary" way of thinking about numbers they may not easily adjust to the idea of creating groups of ten.

Students may not understand the vocabulary *more than* and *less than*.

Students may struggle to make the connection between the numerical form and/or the ten-frame model with the quantity.

Assessment

Formative:

Exit Ticket: On an overhead, show two ten-frame representations - 27 and 34. Record on exit slip, the numerals for each representation, and write which is larger and how they know. (Look for: Numerals are correct and use of place value to determine the larger quantity)

Lesson Sequence and Description

Lesson Launch- (15 minutes) whole group.

Work time- counting collections and interviewing pairs (20-40 minutes)

Closing reflections, including exit ticket (10 minutes)

Launch

Teacher begins lesson by examining the thinking from the previous lesson. "Earlier, we were practicing ways of counting our collections by putting counters into bundles of ten."

Pose the scenario: "Amber and Jack are counting pencils. Amber counts two pencils. Jack counts ten pencils and bundles them with a rubber band. Then he counts ten more and bundles them with another rubber band. Amber says they both have two pencils. Is she correct? Turn to your partner and talk about who has more pencils. Defend your answer." (SMP3: *Construct viable arguments and critique the reasoning of others*).

Students share their thinking. Teacher highlights responses that justify why two bundles of ten is more than two single objects (ones).

Exploration

Teacher introduces the essential question: *"How can we figure out* if quantities are greater than or less than one another?" Would putting the objects into ten frames help? If so, how? Turn and talk to your partner and justify your thinking." (SMP3: *Construct viable arguments and critique the reasoning of others*).

Students work in groups of two. Pass out a bag of counters (20-40) and a ten-frame recording sheet to each student.

Tell the children that they are to organize their collection into the 10-frames on their recording sheet and write their quantity in numerical form.

Students then compare their worksheet with their partner and determine who has more and who has less.

Teacher circulates to different groups and asks each of the partners: "Do you have more than or less than ____?" Response should be either "I have more than ___, because …" or "I have less than ____, because …". Students should justify their reasoning. (SMP3: *Construct viable arguments and critique the reasoning of others*).

Guiding Questions (SMP3: *Construct viable arguments and critique the reasoning of others*)

- ☐ How are you using the frames to organize your counters? Why?
- ☐ How did the ten-frames help you write your number?
- ☐ Who has more counters? How do you know?
- ☐ How did you determine who has more?

May be duplicated in a center for practice. Also, see *cube comparison* as additional center practice.

Reflection

Bring students back into whole group. Using examples of student work, teacher helps students to make the connection between the bundle of tens (full tens) and the tens place in the written numeral. Teacher helps students make connection between the "extra ones" and the ones place in the written numeral. Teacher helps students make the connection between their bundles, written numerals and the total quantity of their objects.

Using the above guiding questions, have the students share their thinking about the connection of the digits and numerals with the ten frames.

Model both accurate thinking and inaccurate thinking for students about object quantities and what the number represents. The discussion can focus on reversals – i.e., comparing 24 to 42. Have students justify thinking.

Have students complete the exit ticket.

Learning experiences can be conducted at home.

Journal papers can record what was counted, how it was counted (in at least two ways), and a math picture to show the count. Children may bring in collections from home to count in school.

This lesson is intended to be an anchor lesson. Teachers may need to scaffold lessons based on student need. More practice with Bundling Lesson 2 may be needed. This activity can easily transfer to centers.

If a student claims that Amber and Jack have "two" (without taking into account place value (tens and ones), use this as a *teachable moment*. The

teacher should ask the student to clarify the difference between place value and *face value*: "Two what? Are two tens and two ones the same amount?" Show ten-frame models if necessary.

It is important to be explicit with our ELL learners. They can be asked to restate the thinking used by others in the class. They can be encouraged to use math words. They should have many opportunities to talk about the math they are doing.

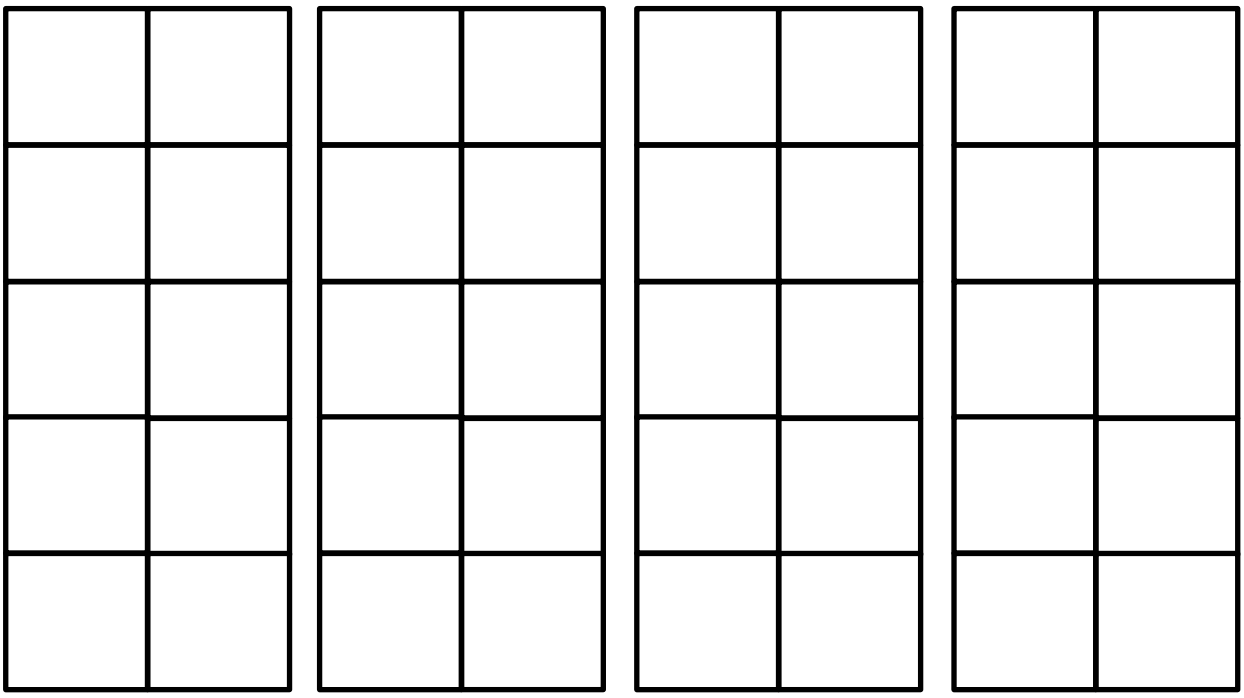
Support students (use visuals etc..) for:
ten- frame, efficient, bundle, organize, collection, objects, quantity, more, less, greater, equal, same

Collections can be differentiated by size according to learning needs.

Possible extension question: "How many more counters would X need so each partner has the same amount of counters?"

Teacher Reflection (to be completed after lesson)
What went well in this lesson?
Did all students accomplish the outcome(s))? What evidence do I have?
What would I do differently next time?

Name _____



I have ____ full ten-frames.

I have ____ left over.

My number is _____.

Lesson 6: Ten-Frame Compare

Brief Overview: In this lesson the children need to first make a judgment of equality or inequality using a concrete place-value model and then translate that understanding to the appropriate symbol system. They must also interpret the inequality symbols in two directions: 57 > 39 means both that 57 is greater than thirty-nine and that thirty-nine is less than fifty-seven. As you plan, consider the variability of learners in your class and make adaptations as necessary.

Prior Knowledge Required:

- Students understand the difference and can demonstrate knowledge of "more," "less" and "equal."
- Students can read a two digit number and identify the value of each digit in the number.

Estimated Time: 45 minutes

Resources for Lesson:

- Center Directions
- Ten Frame Compare directions and set of ten frame cards -(please reference http://www.k-5mathteachingresources.com/support-files/tenframecompare.pdf)
- Recording Sheet

Content Area/Course: Math

Grade: One

Time: approximately 45 minutes

Unit Title: The Power of Ten

Lesson 6: Ten Frame Compare

Essential Question(s) to be addressed in this lesson:
How can you figure out if quantities are greater than, less than or equal to one another?

Standard(s)/Unit Goal(s) to be addressed in this lesson:

1.NTB.3 Compare two-two digit numbers based on meanings of the Tens and Ones digit, recording the results of comparisons with the symbols <, = and >.

SMP2 Reason abstractly and quantitatively

Assumptions about what students know and are able to do coming into this lesson (including language needs):
Students understand the difference and can demonstrate knowledge of "more," "less" and "equal."

Students can read a two digit number and identify the value of each digit in the number.

By the end of this lesson students will know and be able to:
Efficiently and effectively compare two two-digit numbers using the correct symbols to represent equivalency or inequality (<, > or =)

Use place-value knowledge to defend their work.

Instructional Resources/Tools

- Center Directions
- Ten Frame Compare directions and set of ten frame cards -(please reference http://www.k-5mathteachingresources.com/support-files/tenframecompare.pdf)
- Recording Sheet

Anticipated Student Preconceptions/Misconceptions
Misconception #1: Students may get confused with which sign is used in which situation. Displaying this information may help to clarify this issue:

< is the **less than** symbol because it is used when the sentence would look like: _____ is less than _____

> is the **greater than** symbol because it is used when the sentence would look like: _____ is greater than _____

Misconception #2: Students may think that the equal sign (=) means the answer. Suggested language to help clarify this issue:

3+5 is *equivalent to* or "the same as" 8. **=** is the symbol used when both expressions are *equivalent, or the same amount*.

Misconception #3: Students may understand the relationship represented by the number sentence but read it from right to left or use nonstandard language (e.g., reading 15 < 21 as "15 is smaller than 21" or"21 is greater than 15." Acknowledge the correctness of the relationship, but use indirect correction to emphasize the standard mathematical vocabulary: "Yes, 21 is greater than 15. Or, if we read the sentence from left to right, we would say 15 is less than 21."

Assessment

Formative: Exit Slip: Write this expression on the board: 52 > 47. Ask children if it is true or false. Tell them to explain how they know in words or pictures.

The lesson

- ☐ Review of "math talk" and symbols for comparing numbers (10-15 minutes)
- ☐ Pair children up (5 minutes)
- ☐ Activity (at least 20-30 minutes)
- ☐ Assessment (10 minutes)

Lesson Sequence and Description

Write this expression on the board: 52 > 47. Ask children if it is true or false. Tell them to explain how they know in words or pictures.

Tell students they will be comparing numbers represented on ten frames.

Provide examples of comparing numbers using appropriate academic language:

- ☐ 56 is greater than 25
- ☐ 12 is less than 34
- ☐ 47 is equal to 47

Show the symbols used to represent "greater than" (>), "less than (<)" and "equal to" (=).

Review previous examples of comparing numbers, this time using the symbols (SMP2- *Reason abstractly and quantitatively):*

- ☐ 56 > 2
- ☐ 12 < 34
- ☐ 47 = 47

To help the children differentiate the inequality symbols, give them a visualization technique:

- point out that the symbol is wide on the side of the bigger number and narrow on the side of the smaller number.
- have the children visualize an alligator's open mouth, and say: "Would the hungry alligator want to eat the smaller number or the bigger number?...So the alligator's open mouth is always pointing to the bigger number."
- The standard way of reading number sentences is from left to right. In the symbol **>** the gap is greater on the left, so it is called the *greater than* symbol; therefore you would read 56 > 2 as *56 is greater than 2*. In the symbol **<** the gap is less on the left, so it is called the *less than* symbol; therefore you would read 12 < 34 as *12 is less than 34*.

Pair children up and distribute a set of ten-frame cards to each group. Display directions for activity. Shuffle cards and deal ten cards to each player (See direction page, recording sheet, and cards at http://www.k-5mathteachingresources.com/support-files/tenframecompare.pdf).

Players stack their cards into one pile. Each player turns over the top card in their stack.

Players use the ten frames to determine the number represented. Each number is written on the recording sheet in standard form the appropriate box. Partners compare their cards using one of the math talk sentences introduced at the beginning of the lesson:

_____ is greater than _____
_____ is less than _____
_____ is equal to _____

Next, partners use a symbol (<, >, =) to compare their numbers. They place the symbol in the appropriate box on the sheet. Players determine who has the greater number and provide a reason for their thinking. The person with the greater number takes the cards. If the cards are

equal players turn over another card and compare these cards. Once their recording sheets are complete the children take turns "reading" the comparisons they generated during the activity two different ways (e.g., "24 is less than 63" and "63 is greater than 24", and decide which would be the standard way of reading the number sentence.

Worksheets can be used in a center for additional practice.

Guiding Questions:
These questions should be asked throughout the activity as students are working. (SMP2: *Reason abstractly and quantitatively*)

- How can you determine if one number is greater than another? What symbol do you use to show that?
- How can you determine if one number is less than another? What symbol do you use to show that?
- How can you tell if two numbers are equal? What symbol do you use to show that?

Content background needed:
☐ Numbers are in order according to the amount they represent
☐ 5 is more than 3
☐ 7 is less than 10
☐ 6 is equal to

Instructional practices that support academic language development:
☐ *Greater Than* – more than/bigger than
☐ *Less Than* – smaller than
☐ *Equal to* – the same as/equivalent

Specific accommodations for students with disabilities, ELLs, advanced students:
☐ Have a set of cards that is labeled with the numbers that is represented on the ten frame card (ELLs and students with disabilities)
☐ Model/demonstrate expectations for group. Teacher does "think alouds" as they go through the activity with partner – filling in the worksheet and using the expected language. (ELL and students with disabilities)
☐ Have children add two cards together to make a larger number (advanced students)
☐ Review of "math talk" and symbols for comparing numbers (10-15 minutes)
☐ Pair children up (5 minutes)
☐ Activity (at least 20 -30 minutes)
☐ Assessment (10 minutes)

Additional Resources

IXL Comparing Numbers up to 100: for practice with comparing numbers.
http://www.ixl.com/math/grade1/comparing-numbers-up-to-100

Please note: Students could use number lines, place-value materials or ten-frames while they are playing the game on the computer.
More or Less by Stuart Murphy

_____ is greater than _____
_____ is less than _____
_____ is equal to _____

Next, partners use a symbol (<, >, =) to compare their numbers. They place the symbol in the appropriate box on the sheet. Players determine who has the greater number and provide a reason for their thinking. The person with the greater number takes the cards. If the cards are equal players turn over another card and compare these cards. Once their recording sheets are complete the children take turns "reading" the comparisons they generated during the activity two different ways (e.g., "24 is less than 63" and "63 is greater than 24", and decide which would be the standard way of reading the number sentence.

Worksheets can be used in a center for additional practice.

Closure

Review outcomes of this lesson:

- Students will use the symbols <, > and = to show the relationship between two numbers represented with ten frames.
- ☐ Each pair of students will have a recording sheet that depicts the results of the activity.
- ☐ Each child in the pair will read the comparison of the numbers represented with the symbols <, > and =.

Teacher Reflection (to be completed after lesson)

What went well in this lesson?
Did all students accomplish the outcome(s))? What evidence do I have?
What would I do differently next time?

Lesson 6 Worksheet

Decide who will be player A and who will be player B. Both players turn over the card on top of their pile. First, use the "math talk" sentences introduced in the lesson to compare your two numbers. Next, write down the numbers represented by the ten-frames on your card under your name, then circle the appropriate symbol in the center box: <, = or >. When you are done, take turns reading the comparisons using both "greater than" and "less than" vocabulary, and decide which would be the standard way to read the number sentence.

Player A	Comparison < = >	Player B

Lesson 7: Comparing Numerals on a Decade Number Line

Brief Overview: In this lesson children will justify their judgments of greater than/less than using various mathematical place-value models– bean sticks, ten-frames, a blank hundreds board, and a decade number line. By making a connection between their conceptual understanding and various physical models, they will get different perspectives on the concept, thereby making their understanding broader and deeper. As you plan, consider the variability of learners in your class and make adaptations as necessary.

Prior Knowledge Required:

- Students understand the difference and can demonstrate knowledge of "more than", "less than", and "equal to".
- Students can read and identify the values of digits in a two-digit number.

Estimated Time: 45-65 minutes

Resources for Lesson:

- Supply of beans and bean sticks; visual displays of 10 ten-frames; blank hundreds board; decades number line
- Recording worksheet: *Is it Greater Than, Less Than or Equal?*

Content Area/Course: Place Value

Grade: 1

Time: 40 – 65 minutes

Unit Title: The Power of Ten

Lesson 7: Comparing Numerals on a Decade Number Line

Essential Question(s) to be addressed in this lesson:
How can you figure out if quantities are greater than, less than or equal to one another?

Standard(s)/Unit Goal(s) to be addressed in this lesson:
1.NBT.3: Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, =, and <.
SMP3. Construct viable arguments and critique the reasoning of others.

SMP4. Model with mathematics

Assumptions about what students know and are able to do coming into this lesson (including language needs):
Students understand the difference and can demonstrate knowledge of "more than", "less than", and "equal to".

Students can read and identify the values of digits in a two-digit number.

By the end of this lesson students will know and be able to:
Compare two two-digit numbers, using the correct symbol to represent equivalency or inequality (**>** greater than; **=** equal to ; **<** less than) and prove that relationship using various physical models (bean sticks; ten-frames; blank hundreds board; decades number line).

Instructional Resources/Tools
Supply of beans and bean sticks; visual displays of 10 ten-frames; blank hundreds board; decades number line

Recording worksheet: *Is it Greater Than, Less Than or Equal?*

Anticipated Student Preconceptions/Misconceptions
Misconception 1: Students may get confused with which sign is used in which situation. Helpful poster or sentence strips to use:

> **<** is the **less than** symbol because it is used when the sentence would look like:
> _____ is less than _____

> is the **greater than** symbol because it is used when the sentence would look like:
_____ is greater than _____

Misconception 2: Students may not know how to navigate on a hundreds boards or a number line where the numbers are not listed. Specifically, may not know on hundreds board that greater number of tens means moving down and greater number of units means moving right; on number line may use the wrong digit to determine decades placement or position within the decade, or they may move left from the decades mark to make the units judgment, rather than right.

Assessment

Pre-assessment/ Formative

These questions should be asked throughout the lesson as students are working:

1. How can you tell if a number is greater than another number using this model? Prove it!
2. How can you tell if a number is less than another number using this model? Prove it!

Lesson Sequence and Description

Lesson:

Tell students they will be comparing two two-digit numbers based on the tens and ones digits, recording the results of comparisons with the symbols <, >, and =, then prove their answer using different models.

Review the meaning of each symbol using the posters or sentence strips.

Write the following two numbers on the board, side by side: 57 & 46.

Give the students some time to think about which symbol they would use to show the relationship of the numbers (<, >, or =). Have students use thumb signals when ready to discuss. Ask for a volunteer to answer. Write answer on board.

Suggested Questions:

- How do you know that this is the correct symbol? Do you agree? Disagree? Why or why not?
- Ask the students to justify their answer.
- Teacher records students' thinking to make it visible to other students.
- How many thought of it that way?
- (If any of the following models were already used by the students to justify their choices, skip those sections).

I have some bean sticks and loose beans here. Who can model the first number, 57, for us on the overhead over here (SMP3. *Construct viable arguments and critique the reasoning of others*)? (point to left side; write 57 below the model)….Who can model the second number over here? (point to right side; write 46 below the model)….Thank you. And we said 57 is greater than 46, so who can write the correct symbol between the two models? Now who can prove 57 is greater than 46 using these two models? Facilitate discussion of difference between tens

and ones information (SMP3**.** *Construct viable arguments and critique the reasoning of others*, SMP4: *Model with mathematics*).

I have two different numbers: 39 and 61 (write on board). What symbol should I write between these two numbers?And how would I read this number sentence?

If I were doing to fill these ten frames with 39 beans, starting with this one (point to upper left corner) and using the top ten-frames before moving down to the bottom ten-frames, which box would the last bean fall? (SMP4: *Model with mathematics*). Have student come up and place bean, indicate which boxes are filled and discuss reasoning. Facilitate class discussion of answer. Write 39 in the box.

And which box would be the last one filled if I had 61 beans? Repeat the process, writing 61 in the chosen box.

Now who can prove to me 39 is less than 61, using this picture? Facilitate class discussion (SMP3: *Construct viable arguments and critique the reasoning of others* , SMP4 *Model with mathematics*).

This time my numbers are going to be 82 and 57. Write each, and have students insert inequality symbol and read number sentence.

Here I have a hundreds board with all the numbers removed. If the numbers were on the board, where would the 1 be?...Where would the 100 be?...So if I was filling this hundreds board with 87 beans, in which box would the 87th bean fall (SMP4 *Model with mathematics*)? Write number 87 in box. Facilitate discussion of response, emphasizing how tens information affects vertical placement and units information affects horizontal placement). (SMP4 *Model with mathematics*).

Repeat with number 57, and ask a student to prove 82 is greater than 57, using the numbers on the hundreds board. Emphasize fact that bean representing 82 is lower on the board, meaning it is a larger number, even though it has fewer units than 57 (SMP3, SMP4 *Model with mathematics*).

I have one more way of modeling numbers. Here is a 1-100 number line that has no numbers on it. The only marks on this number line are where the skip-counts of ten would go (point to hash marks). I call this a decades number line, because *decade* means ten.

What if I wanted to put the numbers 18 and 41 on this number line? Where would the 18 go?...Where would the 41 g? So is 18 greater than 41 or less than 41? Facilitate discussion of placements, emphasizing choice of correct decade and what units information tells about how far to the left or to the right number should fall within that decade (SMP3: *Construct viable arguments and critique the reasoning of others*, SMP4: *Model with mathematics*).

<u>Independent Activity</u>

Set up four work stations with 0-9 spinners, 2 place markers, and either:

- Bean sticks.
- Ten-frame sheets.
- Blank hundreds boards.
- Decades number line.

Each student at work station has 2-page worksheet to record spun numbers. Each child has one turn as leader. Leader:

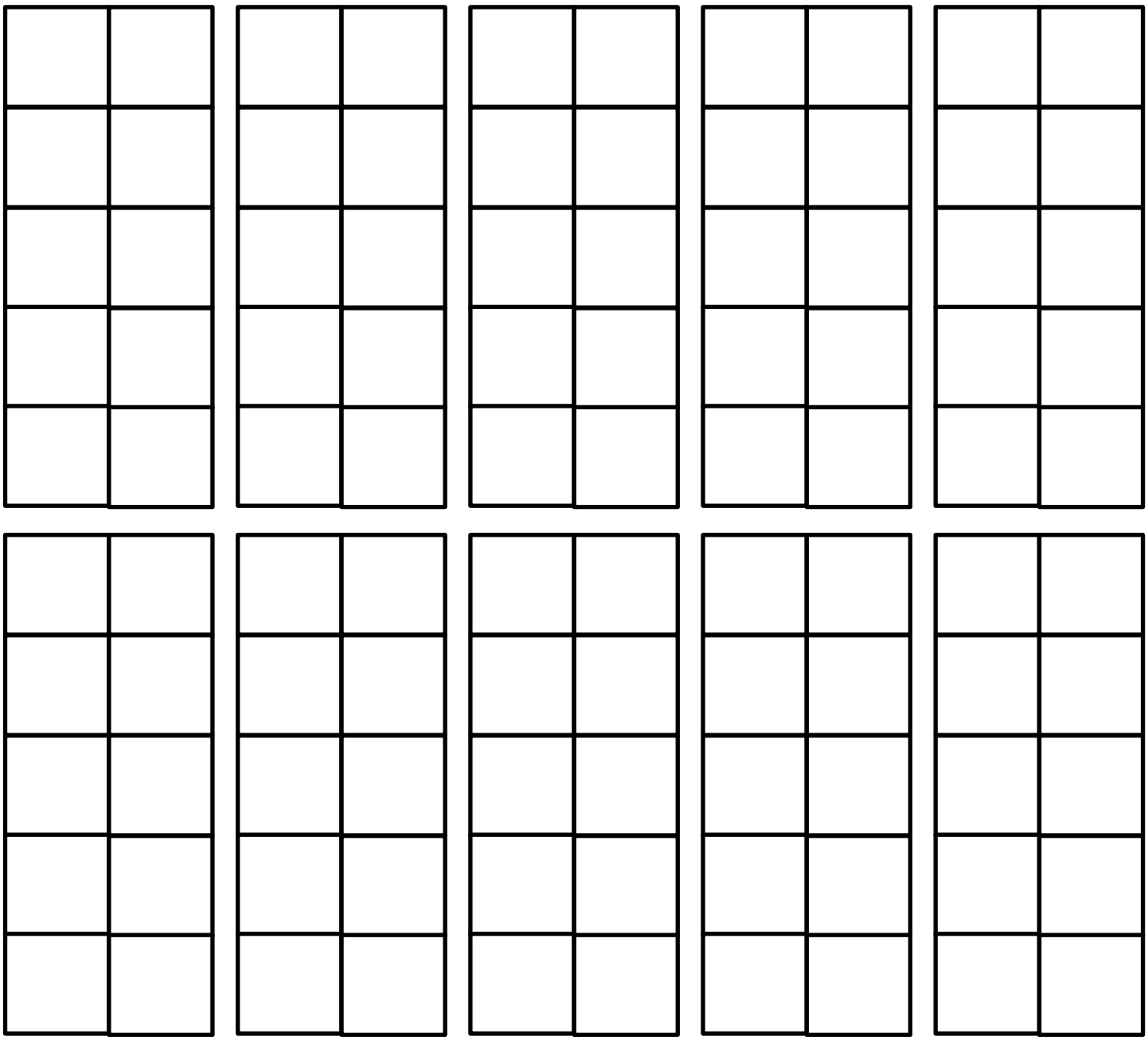
1. Spins the spinner four times to generate 2 two-digit numbers to record on worksheet (if 0 falls in tens digit, tell children to ignore it, since "That means there are no tens").
2. Places the two place markers on the model, representing where those numbers would fall on the model, and explains their choices.
3. Tells the group what symbol to circle between the two numbers: < >, or =.

Teacher circulates, listening to the explanations, and signals when groups should move to the next work station. If group sizes are too large, multiple work stations can be set up for each model. If children are confused by one of the work stations, it can be eliminated from the rotation.

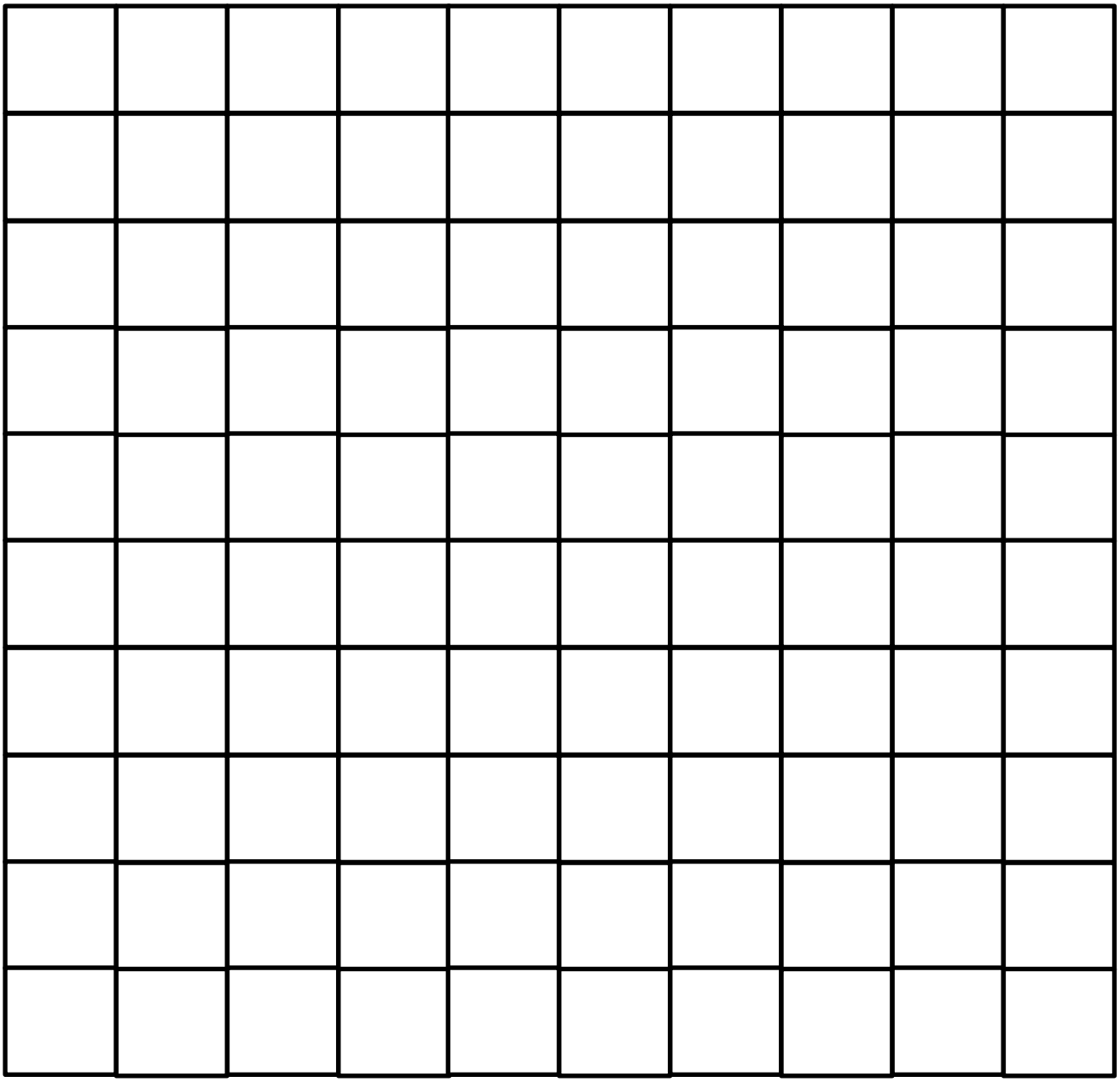
Can be set up in a center, for those children who need more practice.

Teacher Reflection (to be completed after lesson)
What went well in this lesson?
Did all students accomplish the outcome(s))? What evidence do I have?
What would I do differently next time?

Ten-Frames



Blank Hundreds Board



Decades Number Line

Greater Than, Less Than, or Equal? Prove it!

______ ______ digits spun	Comparison < = >	______ ______ digits spun
______ ______ digits spun	Comparison < = >	______ ______ digits spun
______ ______ digits spun	Comparison < = >	______ ______ digits spun
______ ______ digits spun	Comparison < = >	______ ______ digits spun
______ ______ digits spun	Comparison < = >	______ ______ digits spun
______ ______ digits spun	Comparison < = >	______ ______ digits spun
______ ______ digits spun	Comparison < = >	______ ______ digits spun
______ ______ digits spun	Comparison < = >	______ ______ digits spun

_____ _____ digits spun	Comparison < = >	_____ _____ digits spun
_____ _____ digits spun	Comparison < = >	_____ _____ digits spun
_____ _____ digits spun	Comparison < = >	_____ _____ digits spun
_____ _____ digits spun	Comparison < = >	_____ _____ digits spun

Lesson 8: Comparing Two-Digit Numbers

Brief Overview: In this lesson, students are expected to be able to make judgments of relative quantity without the aid of visuals or manipulatives. They must rely solely on their understanding of place value and the relative impact on quantity of changes in the units place versus changes in the tens place to make their judgments. As you plan, consider the variability of learners in your class and make adaptations as necessary.

Prior Knowledge Required:

- Students understand the difference and can demonstrate knowledge of "more", "less", and "equal".
- Students can read and identify the values of digits in a two-digit number.

Estimated Time: 45-65 minutes

Resources for Lesson:

- numeral cards (0-9)
- cards with symbols <, >, and =
- Recording Sheet

Content Area/Course: Place Value

Grade: 1

Time: 40 – 65 minutes

Unit Title: The Power of Ten

Lesson 8: Comparing Two-Digit Numbers

Essential Question(s) to be addressed in this lesson:
How can you figure out if quantities are greater than, less than or equal to one another?

Standard(s)/Unit Goal(s) to be addressed in this lesson:
1.NBT.3: Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, =, and <.

SMP2. Reason abstractly and quantitatively.

Assumptions about what students know and are able to do coming into this lesson (including language needs):
Students understand the difference and can demonstrate knowledge of "more", "less", and "equal".

Students can read and identify the values of digits in a two-digit number.

By the end of this lesson students will know and be able to:
Compare two two-digit numbers and use the correct symbols to represent equivalency or inequality
(**>** greater than; **=** equal to ; **<** less than).

Instructional Resources/Tools
numeral cards (0-9), cards with symbols <, >, and = Recording Sheet

Anticipated Student Preconceptions/Misconceptions
Misconception 1: Students may get confused with which sign is used in which situation. Helpful poster or sentence strips to display:

> **<** is the **less than** symbol because it is used when the sentence would look like:
> _____ is less than _____

> **>** is the **greater than** symbol because it is used when the sentence would look like:
> _____ is greater than _____

Misconception 2: Students may think that the equal sign (=) means the answer. Suggested language to use:

3 + 5 = 8, "three plus five is *the same as* 8" or "three plus five is *equivalent to* (same amount as) 8"..

Formative Assessment

These questions should be asked throughout the lesson as students are working (SMP2: *Reason abstractly and quantitatively)*:

1. How can you tell if a number is greater than another number? Prove it!
2. How can you tell if a number is less than another number? Prove it!
3. How do you know if two numbers are equal? Prove it!

CEPA B should be administered after this lesson.

Preparation:

Make number cards displaying 39 and 52. Show student the numeral cards with 39 and 52. Teacher:

a. Tell me which number is more. *Record student response (□or -).*
b. How do you know? *Record student response (□or -).*
(Ex: 5 tens is more than 3 tens)
c. Write the symbol in the box to show which is more.

Show student the card with the >, =, < symbols.

Record students' response (□or -).

□□or - Student correctly states the number that is more
□□or - Explanation supports understanding of place value
□□or - Student writes the correct symbol to show which is more

Lesson Sequence and Description

Students will compare two two-digit numbers based on the tens and ones digits, recording the results of comparisons with the symbols <, >, and =. Review the meaning of each symbol using the posters or sentence strips.

Teacher will choose 4 cards to make two two-digit numbers and then record them on the board with a gap between them (e.g., 23 64). Ask students to think about what symbol belongs in the blank.

Give the students some time to think about which symbol they would use to show the relationship of the numbers (<, >, or =). Have students use thumb signals when ready to discuss. Ask for a volunteer to answer. Write answer on board.

Does anyone have a different answer? (Record it also if there is an additional answer.)

Ask students to justify their answer. Ask questions such as:

- How do you know that this is the correct symbol? Do you agree? Disagree? Why or why not? Teacher records students' thinking to make it visible to other students.

- How many thought of it that way?
- Do you have a different way to think about it? (if they choose to do so, students can refer to models to prove their thinking, such as ten frames or a number line). *(SMP: Reason abstractly and quantitatively)*

Independent activity:
Use your 0-9 cards. Turn over 4 cards and make two different two digit numbers.

Use the symbols <, > or = to compare the numbers you make. Record and explain your thinking on the worksheet provided.

Reminder: CEPA B should be administered after this lesson.

Teacher note:
As students work, observe what strategies do students use for determining which number is greater than or less than.

Teacher Reflection (to be completed after lesson)
What went well in this lesson?
Did all students accomplish the outcome(s))? What evidence do I have?
What would I do differently next time?

Is it Greater Than, Less Than or Equal To?

Compare the numbers using the symbols: <, = or >.

First Number		Second Number
	Comparison < = >	
	Comparison < = >	
	Comparison < = >	
	Comparison < = >	
	Comparison < = >	
	Comparison < = >	

Choose one example, and explain how you know you used the correct symbol.

Lesson 9 – Ordering Numbers from Least to Greatest

Brief Overview: In this lesson, children must make greater than/less than judgments relative to random benchmark numbers and to the other numbers they are placing. The task requires them not just to differentiate the tens from the units digit but to strategize when the tens digit alone gives them all the information they need to make a greater than/less than judgment and when they must also take the units digit into account. As you plan, consider the variability of learners in your class and make adaptations as necessary.

Prior Knowledge Required:

- Can read 2-digit numerals.
- Can model quantities up to 100 with place-value materials.

Estimated Time: 45-65 minutes

Resources for Lesson:

- Hundreds board
- Ten-frame cards
- 3 separate number lines: 1 – 10; 1 – 20: 1 -100
- Place-value materials

Content Area/Course: Math

Grade: 1

Time: 40 - 65 minutes

Unit Title: The Power of Ten

Lesson 9 – *Ordering Numbers from Least to Greatest*

Essential Question(s) to be addressed in this lesson:
How can you figure out if quantities are greater than, less than or equal to one another?

Standard(s)/Unit Goal(s) to be addressed in this lesson:
1.NBT.3 Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, =, and <.

SMP2. Reason abstractly and quantitatively.

SMP3. Construct viable arguments and critique the reasoning of others.

Assumptions about what students know and are able to do coming into this lesson (including language needs):
Can read 2-digit numerals.

Can model quantities up to 100 with place-value materials.

By the end of this lesson students will know and be able to:
Put a series of two-digit numerals in order from least to greatest.

Instructional Resources/Tools
Hundreds board

Ten-frame cards

3 separate number lines: 1 – 10; 1 – 20: 1 -100

Place-value materials

Anticipated Student Preconceptions/Misconceptions
Reversals (e.g., reads 24 as forty-two)

Larger digits automatically make the numeral larger (e.g., 19 is greater than 23 because it contains a 9).

Places numbers with the same number of tens next to each other but doesn't use the units information to judge which ones are greater (e.g., arranges thirties numbers as 36 39 35).

Formative Assessment
Worksheet with the following numerals printed in order:

10 ____15 ___25 ___55 ___70 ___85 ___100

Child's task: To insert the following numerals where they belong so all the numbers are arranged from least to greatest:

42 17 95 24 59 71

Lesson Sequence and Description
Launch:
Teacher writes 12 and 21 on the board. Someone told me these numbers are the same. Do you agree? Turn to your partner and discuss how you know.
Follow up with a whole-group discussion, emphasizing the difference between the tens place and the units place.

Explain the rationale for this lesson: "We've been working on greater than and less than. Which of these two numbers is greater than the other? How would you write that? Which one is less than the other? How would you write that?

Sometimes we want to compare more than two numbers. Write the following numerals on the board: 7 6 8 5.

Can we put these numerals in correct order, from least to greatest?"

Have child suggest the correct order. Discuss reasons. (SMP2 -Reason abstractly and quantitatively).

Display a 1 – 10 number line.

Have children show where they would put the four numbers on the number line and justify the choices they made. (SMP3- Construct viable arguments and critique the reasoning of others).

Next display the following numerals: 8 15 10 4. Repeat discussion questions and modeling from above this time with the 1 – 20 number line.

Repeat using: 31 23 35 42, emphasizing how tens digit affects magnitude compared to units digit this time using the 1 – 100 number line.

Repeat using: 31 13 27 72. Emphasize how the tens digit affects magnitude.
When ready, distribute assessment worksheet and explain directions.

Note: Can use these materials as alternate models:
- Hundreds board
- Place-value materials

Suggested resources for reinforcement

Library of Virtual Manipulatives: **PreK-2, Base Blocks:** This activity allows students to work with a virtual place value mat. (http://nlvm.usu.edu/en/nav/vlibrary.h tml. Note to teacher: Set "Dec. Places" to zero; set columns to two).

Teacher Reflection (to be completed after lesson)

What went well in this lesson?
Did all students accomplish the outcome(s))? What evidence do I have?
What would I do differently next time?

Name _____

The following numbers are in order from least to greatest.

10 15 25 55 70 85 100

Add the numbers below to the line above, keeping all the numbers in order from least to greatest.

42 17 95 24 59 71

Embedded Performance Assessment (CEPA) B
"Ages of People I Know" Display

Brief Overview: This CEPA requires students to make judgments of relative quantity for two-digit numbers, with a minimal set of cues provided. They must draw heavily on their understanding of numbers in 1-100 space, differentiating between tens digit information and ones digit information.

Estimated Time: 45-65 minutes

Resources for Lesson:

- Open number line work sheet.
- Decades number line work sheet.

Embedded Performance Assessment (CEPA) B
"Ages of People I Know" Display

Content Area: Math

Grade: 1

Massachusetts Learning Standards

1.NBT.2. Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following special cases:

a)10 can be thought of a bundle of ten ones—called a "ten"
b) the numbers from 11-19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones
c) The numbers 10, 20, 30, 40, 50, 60,70, 80, 90 refer to one, two, three, four, five six, seven, eight, or nine tens (and 0 ones).

1.NBT.3. Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >,= and <.

SMP2. Reason abstractly and quantitatively

SMP3. Construct viable arguments and critique the reasoning of others

SMP7. Look for and make use of structure

SL1.4. Describe people, places, things, and events with relevant details, expressing ideas and feelings clearly

Activity	Purpose	Time Required	Materials Provided	When to Administer
See Above	• Accurately read two- digit numerals. • Understand the meanings of the digits in terms of tens and units. • Compare, order, and sequence two-digit numerals. • Use base-ten reasoning to justify their choices. • Know the relative magnitude of numbers on a scale of 1 to 100.	• 45 minutes (whole-class activity) • Individual interviews (<5 minutes each), while another activity is in process.	• Open number line work sheet. • Decades number line work sheet.	End of unit

Overview of Activities

Help Mrs. Smith place her family members' ages on a number line.

Part One: Give each child a handout showing an open number line (a horizontal line with 0 on one end and 100 on the other). Child chooses one numbered card from each of 4 bags: child (single digits), cousin (teen numbers), parent (a number within the 30's and 40's), and grandparent (a number within the 60's and 70's) and places each card on the number line where he or she thinks each person's age belongs (the most accurate place). When child is satisfied with the placement, the child writes the numbers on the number line.

Order the family members on a number line that has been divided into 10 sections.

Part Two: Give each student a decade ruler page (same number line with decade increments marked). Have them place these number lines directly below their current layout. Point out that they are both the same length and go from 0 to 100. "This line can help you decide where

the numbers belong. Look at the numbers are on your cards and where you placed them on the number line.

Individual interview: Ask child: "Are you still satisfied with where you have each dot?" If they decide a dot should be moved, ask them to explain their reasoning. Child marks the placement points on the decade number line and writes the numerals (with assistance, if necessary).

- *Follow-up probe 1*: "What if you had one more person to add to your line that was this age (dot with 30 written on it). Where would you put it? Why there?"
- *Follow-up probe 2*: "And another dot was put here (place on the seventy divider), but the person forgot to write the age on it. What number should they have written on their dot? How do you know?"

Part Three: Mrs. Smith's kindergarten son, Ethan said that his uncle Joe who is 37 is the same age as his grandfather who is 73. Is Ethan correct? How can you prove your answer? (models, number lines, picture) (SMP3: *Construct viable arguments and critique the reasoning of others*)

Criteria for Evaluating Student Products and Performances

Criteria for exemplary rating:

1. In Step 2 (open number line):
   - Are the numbers in proper sequence (least to greatest)?
   - Are numbers in approximately the right positions? Is this true for the larger numbers, or just for the smaller numbers?
2. In Step 3 (decade number line):
   - Are they counting by ones to make their judgments?
   - Are they using the decade markers to precisely place their numbers (i.e., numbers in the 30's are placed to the right of the 30 decade mark)?
3. In Follow-up probe 1 (card with 30 written on it):
   - Do they place the card directly on the decade mark, and use base-ten language to justify their choice (e.g., "ten, twenty, thirty – right here"). [Note: Child might answer "One ten, two tens, three tens").
4. In Follow-up probe 2 (blank card put on the 70 decade mark)
   - Do they tell you to write the number 70? Do they use base-ten language to justify their answer (e.g., "ten, twenty, …, seventy"). [Note: Child might answer "Seven tens" rather than "Seventy").

Misconceptions to look for:

1. Students might be tempted to make decisions counting by ones rather than by tens.

2. In reading numerals, students may mistake tens digits for units digits (e.g., may read 31 as thirteen).
3. In comparing two or more numbers, children may not know how to use the tens information to make judgments of greater than or less than.
4. Children may know how to read two-digit numbers without having a sense of the quantities involved relative to 100 (i.e., whether the quantity is closer to 0, 50, or 100).