Skill: determine an approximate value of a radical expression using a variety of methods.

N.RN.A.2 Extend the properties of exponents to rational exponents. Rewrite expressions involving radicals and rational exponents using the properties of exponents.

A.SSE.A.2 Interpret the structure of expressions. Use the structure of an expression to identify ways to rewrite it.

Previous knowledge/skills (Review if needed):

Square Root of a Number: If $b^2 = a$, then *b* is a square root of *a*

Principal Square Root: the positive value of *b*, using the definition of *b* above

Radical Symbol: $\sqrt{\ }$

Radicand: the number or expression inside the radical symbol

Ex: Evaluate $\sqrt{16}$, $-\sqrt{16}$, and $\pm\sqrt{16}$

$\sqrt{16}$ denotes the principal (positive) square root of 16. The answer is $\boxed{4}$, because $4^2 = 16$.

$-\sqrt{16}$ denotes the negative square root of 16. The answer is $\boxed{-4}$, because $(-4)^2 = 16$.

$\pm\sqrt{16}$ denotes the positive and negative square roots of 16. It is read "plus or minus the square root of 16." The answer is $\boxed{\pm 4}$, because $(-4)^2 = 16$ and $4^2 = 16$.

Special Cases:

Square Root of Zero: 0 has only one square root. $\sqrt{0} = 0$, because 0 cannot be positive or negative.

Square Root of Negative Numbers: negative numbers have **no real square roots**, because the square of every real number is nonnegative.

Note: Have students try to think of a number when multiplied by itself equals -16. Emphasize that negative numbers have no REAL square roots. (Not that negative numbers have no square roots.)

Ex: Evaluate the expressions.

1. $\sqrt{169} = \boxed{13}$
2. $\sqrt{\dfrac{25}{49}} = \boxed{\dfrac{5}{7}}$
3. $-\sqrt{2.25} = \boxed{-1.5}$
4. $\pm\sqrt{196} = \boxed{\pm 14}$
5. $\sqrt{-25}$ no real square root

The radicands of examples 1-4 are all examples of **perfect squares**.

Perfect Squares: numbers whose square roots are rational numbers (integers or quotients of integers)

Ex: List all of the perfect squares created by square the whole numbers up to 20.

1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225, 256, 289, 324, 361, 400

Note: You should *memorize* all of these perfect squares.

Estimating the Square Root of a Number: To estimate the square root of a number that is not a perfect square, we will find the two perfect squares that the number lies between. The square root of the number will lie between the square roots of the perfect squares.

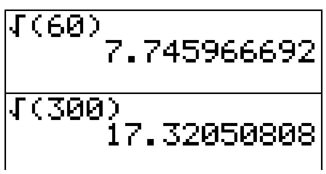
Ex: Estimate the value of the square roots.

1. $\sqrt{60}$ 60 lies between the perfect squares 49 and 64, closer to 64. Therefore the square root of 60 is between $\sqrt{49}=7$ and $\sqrt{64}=8$. So we estimate that $\sqrt{60}$ is between 7 and 8, closer to 8.

2. $\sqrt{300}$ 300 lies between the perfect squares 289 and 324, closer to 289. Therefore the square root of 300 is between $\sqrt{289}=17$ and $\sqrt{324}=18$. So we estimate that $\sqrt{300}$ is between 17 and 18, closer to 17.

Estimating Square Roots on the Calculator: We will use the $\sqrt{}$ key to evaluate the approximate value of the square roots in the examples above.

1. $\sqrt{60} \approx \boxed{7.746}$
2. $\sqrt{300} \approx \boxed{17.321}$



Note: These answers confirm our estimates from above.

Irrational Number: a real number that cannot be written as the quotient (fraction) of two integers. The digits after the decimal will never terminate and never repeat

Examples of irrational numbers: π, $\sqrt{2}$, $-\sqrt{48}$

Note: Any square root of a non-perfect square is irrational.

Ex: Label the following as rational or irrational.

1. 0 Rational: Can be written as a fraction: $\frac{0}{1}$

2. $\sqrt{12}$ Irrational: Cannot be written as a fraction (12 is not a perfect square)

3. 0.5 Rational: Can be written as a fraction: $\frac{1}{2}$

4. $0.\overline{12}$ Rational: Can be written as a fraction: $\frac{12}{99}$

5. $\sqrt{2.89}$ Rational: $\sqrt{2.89} = 1.7$, which can be written as a fraction: $\frac{17}{10}$

6. $-0.989898...$ Rational: Can be written as a fraction: $-\frac{98}{99}$

7. 3.14159… Irrational: This is the approximation of π, which is a decimal that does not terminate and never repeats.

You Try:

1. Evaluate the following square roots. $\sqrt{\frac{400}{81}}$, $\pm\sqrt{19600}$, $-\sqrt{2.89}$

2. Estimate the value of $\sqrt{50}$ without a calculator.

QOD: Tell whether the statement is true or false. If it is true, give an example, and if it is false, give a counterexample. "The square root of the sum of two numbers is equal to the sum of the square roots of the numbers."

Sample Practice Question(s):

1. Which statement about $\sqrt{193}$ is true?

A. It lies between 13 and 14.
B. It lies between 14 and 15.
C. It lies between 100 and 200.
D. It lies between 169 and 196.

2. Evaluate the radical expression $-\sqrt{\frac{25}{100}}$.

A. $-\sqrt{\frac{1}{2}}$

B. $-\sqrt{\frac{5}{10}}$

C. $-\frac{1}{2}$

D. $-\frac{1}{4}$

<u>Sample Nevada High School Proficiency Exam Questions (taken from 2009 released version H):</u>

1. Which integer is **closest** to the value of $\sqrt{5^3}$?

A 3
B 8
C 11
D 15

2. The value of $3\sqrt{180}$ is between which two integers?

A 12 and 15
B 22 and 25
C 39 and 42
D 58 and 62

Skill: simplify and evaluate algebraic expressions with radicals.

Explore: Evaluate $\sqrt{a \cdot b}$ and $\sqrt{a} \cdot \sqrt{b}$, then evaluate $\sqrt{\frac{a}{b}}$ and $\frac{\sqrt{a}}{\sqrt{b}}$ for the given values of *a* and *b*.

1. $a = 4,\ b = 9$
2. $a = 64,\ b = 100$
3. $a = 25,\ b = 4$
4. $a = 100,\ b = 625$
5. $a = 36,\ b = 16$

What conclusion can you make about multiplying and dividing with radicals?

<u>Properties of Radicals</u> Let *a* and *b* be positive numbers.

Product Property: $\sqrt{a \cdot b} = \sqrt{a} \cdot \sqrt{b}$

Quotient Property: $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

Question: Do these properties apply to addition and subtraction? Give a counterexample of each case.

<u>Simplest Form of a Radical</u>: a square root is in simplest form if no perfect square factors (other than 1) are in the radicand and there are no radicals in the denominator of a fraction

<u>Simplifying a Radical Using the Product Property</u>

Ex: Simplify the square root $\sqrt{72}$.

Step One: Find the *largest* perfect square factor of 72. Rewrite 72 as a product using the perfect square as one of the factors. $\sqrt{72} = \sqrt{36 \cdot 2}$

Note: 4 is also a perfect square factor of 72, but 36 is the *largest* perfect square factor.

Step Two: Use the product property of radicals to rewrite as a product of two radicals. $\sqrt{36 \cdot 2} = \sqrt{36} \cdot \sqrt{2}$

Step Three: Take the square root of the perfect square. $\boxed{6\sqrt{2}}$

Note: There are no perfect square factors (other than 1) for the new radicand, 2, so we know we have completely simplified the radical.

Ex: Simplify the radical expression $4\sqrt{27}$.

Step One: Find the *largest* perfect square factor of 27. Rewrite 27 as a product using the perfect square as one of the factors. $4\sqrt{27} = 4\sqrt{9 \cdot 3}$

Step Two: Use the product property of radicals to rewrite as a product of two radicals. $4\sqrt{9 \cdot 3} = 4\sqrt{9} \cdot \sqrt{3}$

Step Three: Take the square root of the perfect square. $4 \cdot 3\sqrt{3} = \boxed{12\sqrt{3}}$

Note: There are no perfect square factors (other than 1) for the new radicand, 3, so we know we have completely simplified the radical.

Simplifying a Radical Expression by Rationalizing the Denominator

Ex: Simplify the expression $\frac{2}{\sqrt{3}}$.

Step One: Multiply the numerator and denominator of the fraction by the radical in the denominator.

$\frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$ Note: This does not change the value of the original expression because $\frac{\sqrt{3}}{\sqrt{3}} = 1$.

Step Two: Simplify using the product property of radicals. $\frac{2\sqrt{3}}{\sqrt{3 \cdot 3}} = \frac{2\sqrt{3}}{\sqrt{9}} = \boxed{\frac{2\sqrt{3}}{3}}$

Note: We have now *rationalized* the denominator. (3 is a rational number, where $\sqrt{3}$ is irrational.)

Ex: Simplify the expression $\sqrt{\frac{32}{75}}$.

Step One: Use the quotient property of radicals. $\sqrt{\frac{32}{75}} = \frac{\sqrt{32}}{\sqrt{75}}$

Step Two: Simplify each radical. $\frac{\sqrt{16 \cdot 2}}{\sqrt{25 \cdot 3}} = \frac{\sqrt{16} \cdot \sqrt{2}}{\sqrt{25} \cdot \sqrt{3}} = \frac{4\sqrt{2}}{5\sqrt{3}}$

Step Three: Rationalize the denominator. $\frac{4\sqrt{2}}{5\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{4\sqrt{6}}{5\sqrt{9}} = \frac{4\sqrt{6}}{5 \cdot 3} = \boxed{\frac{4\sqrt{6}}{15}}$

Ex: Simplify the expression $\frac{3\sqrt{36} \cdot \sqrt{63}}{8}$.

Step One: Simplify each radical. $\frac{3\sqrt{36}\cdot\sqrt{63}}{8}=\frac{3\cdot 6\sqrt{9\cdot 7}}{8}=\frac{3\cdot 6\cdot 3\sqrt{7}}{8}=\frac{54\sqrt{7}}{8}$

Step Two: Simplify the fraction. (Divide numerator and denominator by 2.) $\frac{54\sqrt{7}}{8}=\boxed{\frac{27\sqrt{7}}{4}}$

You Try: Simplify the expression: $\sqrt{7}\cdot\frac{\sqrt{27}}{\sqrt{2}}$

QOD: An alternate method for simplifying radicals is to make a factor tree. Try this method with $\sqrt{800}$.

Sample Practice Question(s):

Simplify the radical $\sqrt{40}$.

A. $2\sqrt{10}$

B. $2\sqrt{20}$

C. $4\sqrt{10}$

D. $8\sqrt{5}$

Sample Nevada High School Proficiency Exam Questions (taken from 2009 released version H):

Simplify:

$$\sqrt{3^2+3^2}$$

A $2\sqrt{3}$
B $3\sqrt{2}$
C 6
D 9

Review Radicals

Skills:

- **simplify and evaluate algebraic expressions with radicals.**
- **solve practical problems using radicals.**

A.REI.B. 4b Solve equations and inequalities in one variable. Solve quadratic equations in one variable by taking square roots.

Operations with Radicals

Sum and Difference: To add or subtract radicals, they must be *like* radicals (same root, same radicand). Add or subtract the coefficient, keep the *like* radical.

Ex: Find the sum: $3\sqrt{5}+8\sqrt{5}$

Using the distributive property, we can rewrite this as $(3+8)\sqrt{5}=\boxed{11\sqrt{5}}$

Product and Quotient: To multiply or divide radicals with the same root, multiply or divide the radicands.

Ex: Simplify the expression $\sqrt{3}\cdot\sqrt{27}$

Multiply and simplify: $\sqrt{3}\cdot\sqrt{27}=\sqrt{3\cdot 27}=\sqrt{81}=\boxed{9}$

Ex: Expand the expression. $\left(1+\sqrt{3}\right)^2$

Rewrite the exponent as a product: $\left(1+\sqrt{3}\right)\left(1+\sqrt{3}\right)$

Multiply using the distributive property: $1+\sqrt{3}+\sqrt{3}+\sqrt{3}\cdot\sqrt{3}$

Simplify: $1+2\sqrt{3}+\sqrt{9}=1+2\sqrt{3}+3=\boxed{4+2\sqrt{3}}$

Multi-Step Problems

Ex: Simplify the expression $\sqrt{24}-\sqrt{96}+\sqrt{6}$.

First simplify each radical term: $\sqrt{4\cdot 6}-\sqrt{16\cdot 6}+\sqrt{6}=2\sqrt{6}-4\sqrt{6}+\sqrt{6}$

Add/Subtract like radicals: $=(2-4+1)\sqrt{6}=-1\cdot\sqrt{6}=\boxed{-\sqrt{6}}$

Ex: Simplify the expression $\dfrac{3}{\sqrt{48}}$

Simplify the denominator: $\dfrac{3}{\sqrt{16\cdot 3}}=\dfrac{3}{4\sqrt{3}}$

Rationalize the denominator: $\dfrac{3}{4\sqrt{3}}\cdot\dfrac{\sqrt{3}}{\sqrt{3}}=\dfrac{3\sqrt{3}}{4\sqrt{9}}=\dfrac{3\sqrt{3}}{4\cdot 3}$

Simplify: $\dfrac{3\sqrt{3}}{4\cdot 3}=\boxed{\dfrac{\sqrt{3}}{4}}$

<u>You Try:</u> Find the area of a rectangle with a length of $\left(\sqrt{2}+9\right)$ and width of $\sqrt{68}$

<u>QOD</u>: Show numerically why $\sqrt{a}\cdot\sqrt{b}=\sqrt{a\cdot b}$, but $\sqrt{a}+\sqrt{b}\neq\sqrt{a+b}$.

<u>Sample Practice Question(s):</u>

1. Simplify the radical $\sqrt{\dfrac{90}{2}}$.

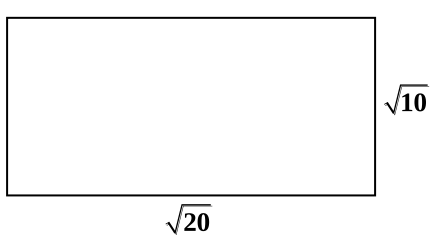
A. $3\sqrt{5}$

B. $5\sqrt{3}$

C. $3\sqrt{15}$

D. $15\sqrt{3}$

2. Find the area of the rectangle. Give the exact answer in *simplest* form.



A. $\sqrt{30}$

B. $\sqrt{200}$

C. $2\sqrt{50}$

D. $10\sqrt{2}$

REVIEW Pythagorean Theorem and radicals

8.G.B. 6 Explain a proof of the Pythagorean Theorem and its converse.

8.G.B. 7 Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.

Skill: apply the Pythagorean theorem and its converse in mathematical and practical situations (with and without technology).

Pythagorean Theorem: the sum of the squares of the two *legs* of a right triangle is equal to the square of the *hypotenuse*

Legs of a Right Triangle: the two sides of a right triangle that form the right angle

Hypotenuse: the longest side of a right triangle; the side opposite the right angle

Activity: Draw a right triangle on a sheet of paper. Label the two legs *a* and *b*, and the hypotenuse *c*. Create squares using the lengths of each side of the triangle as the length of the sides of the squares. Write expressions for the areas of these squares. Cut out the squares, and rearrange the areas that represent the squares of *a* and *b* to see if they fit exactly into the area that represents the square of *c*.

Extension: Try the same activity with an obtuse and acute triangle.

Note: The Pythagorean Theorem is usually written as $a^2 + b^2 = c^2$, just remember that *a* and *b* are the **legs** and *c* is the **hypotenuse** of the right triangle.

Using the Pythagorean Theorem to Find Missing Sides of a Right Triangle

Ex: Given that the two legs of a right triangle are 3 and 4, find the hypotenuse.

Use the Pythagorean Theorem: $3^2 + 4^2 = c^2$

Solve for *c*:

$$9 + 16 = c^2$$
$$25 = c^2$$
$$5 = c$$

The hypotenuse is $\boxed{5}$.

Ex: Given that the hypotenuse of a right triangle is 6, and one of the legs is 5, find the length of the other leg.

Use the Pythagorean Theorem: $5^2 + b^2 = 6^2$

Solve for *c*:

$$25 + b^2 = 36$$
$$b^2 = 11$$
$$b = \sqrt{11}$$

The leg is $\boxed{\sqrt{11}}$.

Converse of the Pythagorean Theorem: If a triangle has side lengths *a*, *b*, and *c* such that $a^2 + b^2 = c^2$, then the triangle is a right triangle.

Ex: Determine whether the given lengths are sides of a right triangle.

1. 5, 12, 13

Use the Pythagorean Theorem: $a^2 + b^2 = c^2 \Rightarrow 5^2 + 12^2 = 13^2$
$25 + 144 = 169$ true

The lengths satisfy the Pythagorean Theorem, so this is a right triangle.

2. 4, 6, 8

Use the Pythagorean Theorem: $a^2 + b^2 = c^2 \Rightarrow 4^2 + 6^2 = 8^2$
$16 + 36 = 64$ false

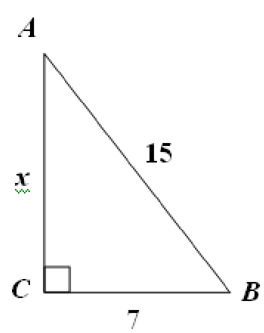
The lengths do not satisfy the Pythagorean Theorem, so this is not a right triangle.

You Try: The distance from home base to first base on a baseball diamond is 90 feet. What is the distance from home plate to second base? (Note: The distances between the bases are the same for all.)

QOD: Sets of 3 numbers that can represent the sides of right triangles are called Pythagorean Triples. List as many Pythagorean Triples as you can.

Sample Practice Question(s):

1. **Triangle *ABC* is a right triangle. Which equation could be used to determine the value of *x*?**



A. $x = 7^2 + 15^2$

B. $x = 15^2 - 7^2$

C. $x = \sqrt{7^2 + 15^2}$

D. $x = \sqrt{15^2 - 7^2}$

2. **Use the converse of the *Pythagorean Theorem* to determine if the 3 numbers could represent the sides of a right triangle.**

   I. 5, 7, 9

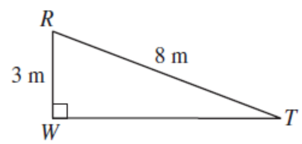
   II. 5, 12, 13

   Which of these sets of 3 numbers could represent the sides of a right triangle?

A. Neither I nor II
B. I only
C. II only
D. Both I and II

<u>Sample Nevada High School Proficiency Exam Questions (taken from 2009 released version H):</u>

1. In right triangle *RTW*, shown below, then length of side $\overline{WR}$ is 3 meters (m), and the length of side $\overline{RT}$ is 8 m.



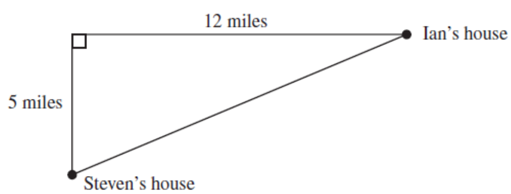
What is the length of side $\overline{TW}$?

A $\sqrt{5}$ m
B $\sqrt{11}$ m
C $\sqrt{55}$ m
D $\sqrt{73}$ m

2. There are two routes that may be used to drive from Steven's house to Ian's house. The routes are described below.

- Route 1: Drive 5 miles north and then 12 miles east.
- Route 2: Drive the straight road that goes directly to Ian's house.

The two routes are shown in the diagram below.



How much longer is route 1 than route 2?
A 4 miles
B 7 miles
C 13 miles
D 17 miles