Representing Proportional Relationships and Slope (page 1)

A proportional relationship between two quantities exists if they have a constant ratio and a constant rate of change. This relationship is also called a direct variation. The equations of such relationships are always in the form $y = mx$. When graphed, they produce a line that passes through the origin. In this equation, $m$ is the slope of the line; it is also called the unit rate, the rate of change, or the constant of proportionality of the function.

**Example:** Graph the proportional relationship between the two quantities, write the equation representing the relationship, and describe how the unit rate or slope is represented on the graph.

**Gasoline cost $3.50 per gallon**

We can start by creating a table to show how these two quantities, gallons of gas and cost, vary. Two things show us that this is definitely a proportional relationship. First, it contains the origin, $(0, 0)$, and this makes sense: if we buy zero gallons of gas it will cost zero dollars. Second, if the number of gallons is doubled, the cost is doubled; if it is tripled, the cost is tripled.

The equation that will represent this data is $y = 3.50x$, where $x$ is the number of gallons of gasoline and $y$ is the total cost ($y = mx$). Slope is 3.5 as indicated in the table as the unit rate.

The graph is shown. (Note: The equation does extend into the third quadrant because this region does not make sense for the situation. We will not buy negative quantities of gasoline, nor pay for it with negative dollars!)

We can find the slope by creating a “slope triangle” which represents

$$\frac{\text{rise}}{\text{run}} = \frac{7}{2} = 3.5,$$

which confirms the slope we show in the equation.

Either way, the constant of proportionality is the slope, which is 3.5.

**Example:** Graph the proportional relationship between the two quantities, write the equation representing the relationship, and describe how the unit rate or slope is represented on the graph.

**Five Fuji Apples cost $2**

Again, we can begin by creating a table relating the number of apples to their cost. We can use this table to plot the points and determine the slope of the line.

Using the slope triangle, we can see that

$$\frac{\text{rise}}{\text{run}} = \frac{2}{5}.$$

Using $y=mx$, the equation for the line is $y = \frac{2}{5}x$.

For unit rate: if five apples cost $2.00, then one apple costs $\frac{2.00}{5} = .40$ or 40 cents per apple. (It is also represented on the graph: for one apple, the graph rises .40.)
Representing Proportional Relationships and Slope (page 2)

For each of the problems, graph the proportional relationship between the two quantities, write the equation representing the relationship, and describe how the unit rate or slope is represented on the graph.

1. **Mario walks at 3 miles per hour.**

   ![Graph](image1)

   **Equation:**
   
   **Description:**

2. **In a food eating contest, a contestant eats 60 hot dogs in 10 minutes.**

   ![Graph](image2)

   **Equation:**
   
   **Description:**

3. **Every six days, Draco receives four boxes of cauldron cakes.**

   ![Graph](image3)

   **Equation:**
   
   **Description:**